Exact solutions of Einstein and Einstein-Maxwell equations

0.1 Topics

- Exact solutions in General Relativity
- Cosmology and Astrophysics
- Quantum Fields
- Alternative theories

0.2 Participants

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- V.A.Belinski
- T.Damour
- M.Novello
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0.3 Research activity

0.3.1 Exact solutions in General Relativity

• In the non-relativistic physics two particles can be in equilibrium if the product of their masses is equal to the product of their charges. However, the question on the existence of an analogue of such equilibrium state in General Relativity is far to be trivial. Besides the natural mathematical complications, in General Relativity arise two different types of the "point" centers, namely Black Hole (BH) and Naked Singularity (NS) and one need to consider all three configurations BH - BH, NS - NS and BH - NS separately. Yet in each case the notion of a physically sensible distance between these objects should be defined.

When the Inverse Scattering Method (ISM) have been adopted for integration of the Einstein and Einstein-Maxwell equations it was shown on the exact mathematical level that Black Holes and Naked Singularities represent nothing else but stationary axially symmetric solitons. Then by the ISM machinery one can obtain the infinite families of exact stationary axially symmetric solutions of these equations containing such solitons centralized at different points of the symmetry axis. The formal construction of such solutions do not represents any difficulties apart of the routine calculations in the framework of the well developed procedure how to insert a number of solitons into a given background spacetime. However, it is quite intricate task to single out from these families the physically reasonable constructions which correspond to a real equilibrium states of charged Black Holes and Naked Singularities interacting with each other. The point is that in general the stationary axially symmetric solitonic solutions possess some features which are unacceptable from the physical point of view. These unwanted traits are due the presence in the solutions exotic peculiarities of the following four types: (i) NUT parameters, (ii) angle deficit at the points of the symmetry axis, (iii) closed time-like curves around that parts of symmetry axis which are out of the sources and (iv) magnetic charges. The NUT parameters are incompatible with asymptotic flatness of the spacetime at spatial infinity. The angle deficit is the well known conical singularity violating the local Euclidness of space at the points of symmetry axis (it can be treated as some singular external strut or string preventing the sources to fall onto or to run away each other). Keeping in mind the physical applications we also should avoid of any excess of closed timelike curves with respect to those already existing inside the sources as an inseparable part of their inner structure. Also magnetic charges should be excluded since their presence contradicts the Maxwell theory. All four aforementioned phenomena have nothing to do with a real equilibrium of the physical bodies and the corresponding equilibrium solution should be free of such pathologies. To deliberate solution from them one need to place the set of the free parameters of the solution under some additional restrictions which can be written in the form of some system of algebraic equations. The problem is that these equations, even for the simplest case of two objects, are extremely complicated and it is difficult to resolve them in an exact analytical form in order to see directly whether they have physically appropriate solutions compatible with the existence of a positive definite distance between the sources.

The explicit analytical resolution of the problem have been achieved in our sector and the final results have been published in two papers (see citation below) where it was constructed the exact analytic solution of the Einstein-Maxwell equations for two charged rotating sources separated by the well defined positive distance and free of struts or of any other unphysical properties. We showed also that such solution exists only for the BH-NS system and it is impossible to have the similar equilibrium state for the pair BH-BH or NS-NS. These results have been published in [1, 2].

• It was found the new and more solid way (with respect to our past investigations) of derivation of the Kerr solution by adding to the Schwarzchild black hole a solitonic whirl. The main problem here were to integrate the Lax pair equations for the Schwarzchild background in order to find the background spectral matrix. The corresponding calculations were performed successfully (this was the main technical achievement) and with this approach we can have a new interpretation of the distribution of energy of the Kerr black hole between its rotational and rest parts. These results have been published in [3].

• The monodromy transform and corresponding integral equation method have been described which give rise to a general systematic approach for solving integrable reductions of field equations for supergravity as well as for pure vacuum gravity in four and higher dimensions. For physically different types of fields in space-times of $D \geqslant 4$ dimensions with $d = D - 2$ commuting isometries (stationary fields with spatial symmetries, interacting waves, evolution of inhomogeneous cosmological models) the string gravity equations govern the dynamics of interacting gravitational, dilaton, antisymmetric tensor and any number of Abelian vector gauge fields (all depending only on two coordinates). The spectral problem, equivalent to one constructed earlier, allows to parameterize the entire infinite-dimensional space of local solutions of these equations by two pairs of coordinate-independent holomorphic matrix functions of a spectral parameter w which constitute a complete set of monodromy data for normalized fundamental solution of this spectral problem. The "direct" and "inverse" problems of such monodromy transform always admit the unique solutions. We construct the linear singular integral equations which solve this inverse problem. For any kind of monodromy data the solution of these integral equations and corresponding solution for string gravity equations can be found explicitly. Simple reductions of the space of monodromy data leads to the similar constructions for solving of other integrable symmetry reduced gravity models, e.g. $5D$ minimal supergravity or vacuum gravity in $D \geqslant 4$ dimensions [4].

0.3.2 Cosmology and Astrophysics

Cosmology

• The work on the book "Cosmological Singularity" has been continued. The project is in progress and an official agreement with Cambridge University Press will be accomplished soon [5].

• The formalism of geometrothermodynamics to derive fundamental thermodynamic equations has been used to construct general relativistic cosmological models. In particular, it were shown that the simplest possible fundamental equation, which corresponds in geometrothermodynamics to a system with no internal thermodynamic interaction, describes the different fluids of the standard model of cosmology. In addition, a particular fundamental equation with internal thermodynamic interaction were shown to generate a new cosmological model that correctly describes the dark sector of the Universe and contains as a special case the generalized Chaplygin gas model [7].

• Thanks to its fitting triumph, the Lambda CDM paradigm is assumed to be the most powerful model, for describing the Universe dynamics, over much the myriad of cosmological models. Unfortunately, the quest of a self-consistent model remains not well explained, because it is not clear how to solve the problems of fine-tuning and coincidence, afflicting the Lambda CDM framework; as a matter of fact, these theoretical drawbacks do not allow to consider the Lambda CDM model, as the final picture of the modern cosmological scenario. It is shown that the simplest model, which provides a constant equation of state for the pressure, leads to a generalization of Lambda CDM, reducing to it in a particular case. Moreover, it were found the physical mechanisms of this model, describing the thermodynamical reasons why a constant pressure should be negative in an expanding Universe [8].

• It is possible to use cosmography to present constraints on the kinematics of the Universe, without postulating any underlying theoretical model. To this end, it can be used a Monte Carlo Markov Chain analysis to perform comparisons to the supernova Ia Union 2 compilation, combined with the Hubble Space Telescope measurements of the Hubble constant, and the Hubble parameter datasets. We introduce a sixth order cosmographic parameter and show that it does not enlarge considerably the posterior distribution when comparing to the fifth order results. We also propose a way to construct viable parameter variables to be used as alternatives of the redshift z. These can overcome both the problems of divergence and lack of accuracy associated with the use of z. Moreover, we show that it is possible to improve the numerical fits by re-parameterizing the cosmological distances. In addition, we constrain the equation of state of the Universe as a whole by the use of cosmography. To this end, we derive expressions which can be directly used to fit the equation of state and the pressure derivatives up to fourth order. All these results are consistent with the Lambda CDM model, although alternative fluid models, with nearly constant pressure and no cosmological constant, match the results accurately as well [9].

Astrophysics

• One of the main problems of classical general relativity consists in finding an interior solution of Einstein's equations which could be matched with an exterior axisymmetric stationary solution. During the last few years we developed an approach how to use the interior and the exterior multipole moments of the corresponding metrics in order to reach a physical match at the surface of the interior mass distribution. An updated description of such procedure can be find in [10].

• Some exact and approximate solutions of Einstein's equations that can be used to describe the gravitational field of astrophysical compact objects in the limiting case of slow rotation and slight deformation have been studied. First, it was shown that none of the standard models obtained by using Fock's method can be used as an interior source for the approximate exterior Kerr solution. We then use Fock's method to derive a generalized interior solution, and also an exterior solution that turns out to be equivalent to the exterior Hartle-Thorne approximate solution that, in turn, is equivalent to an approximate limiting case of the exact Quevedo-Mashhoon solution. As a result we obtain an analytic approximate solution that describes the interior and exterior gravitational field of a slowly rotating and slightly deformed astrophysical object [6].

• During the last decade a ground-based network of interferometric gravitational wave (GW) detectors has been developed and is expected, thanks to an improved sensitivity, to detect, within a few years, the GW signals emitted during the inspiral and merger of compact binaries. The realization of this exciting observational prospect depends, however, on our theoretical ability to accurately compute, within Einstein's theory of general relativity, the motion of compact binaries and its associated GW emission. Recent developments have made it clear that the most efficient way to theoretically understand the late stages of the dynamics of compact binaries is to combine the knowledge coming from analytical relativity techniques, such as traditional post-Newtonian expansions, or the newer effective-one-body (EOB) formalism (developed by T.Damour et al. around ten years ago), with the knowledge coming from numerical relativity simulations. In EOB formalism one maps the general relativistic two-body problem onto that of a test particle moving in an effective external metric. This effective-one-body approach defines, in a non-perturbative manner, the late dynamical evolution of a coalescing binary system of compact objects. During the 2011-2012 analyzing the number of concrete physical models of black hole and neutron star binaries it was managed to show that the prediction of the effective one-body formalism, based purely on known analytical results, agrees strikingly well with the numerical results. Both by numerical calculations and analytical approach the number of relations for the physical parameters was established for the binaries (e.g. the gauge invariant relation between the binding energy and the angular momentum and the expected waveforms essentially up to the merger). All these results show that effective-one-body approach can be safely applied for the calculation of the gravitational radiation from the binary systems. See below the most recent results in this promising direction.

1. The gravitational wave signal from a binary neutron star inspiral contains information on the nuclear equation of state. This information is contained in a combination of the tidal polarizability parameters of the two neutron stars and is clearest in the late inspiral, just before merger. We use the recently defined tidal extension of the effective one-body formalism to construct a controlled analytical description of the frequency-domain phasing of neutron star inspirals up to merger. Exploiting this analytical description we find that the tidal polarizability parameters of neutron stars can be measured by the advanced LIGO-Virgo detector network from gravitational wave signals having a reasonable signal-to-noise ratio of $\rho = 16$. This measurability result seems to hold for all the nuclear equations of state leading to a maximum mass larger than $1.97M\odot$. We also propose a promising new way of extracting information on the nuclear equation of state from a coherent analysis of an ensemble of gravitational wave observations of separate binary merger events [11].

2. The gravitational-wave signal from inspiralling neutron-star–neutron-star (or black-hole–neutron-star) binaries will be influenced by tidal coupling in the system. An important science goal in the gravitational-wave detection of these systems is to obtain information about the equation of state of neutron star matter via the measurement of the tidal polarizability parameters of neutron stars. To extract this piece of information will require to have accurate analytical descriptions of both the motion and the radiation of tidally interacting binaries. We improve the analytical description of the late inspiral dynamics by computing the next-to-next-to-leading order relativistic correction to the tidal interaction energy. Our calculation is based on an effective-action approach to tidal interactions, and on its transcription within the effective-one-body formalism. We find that second-order relativistic effects (quadratic in the relativistic gravitational potential significantly increase the effective tidal polarizability of neutron stars by a distance-dependent amplification factor. We argue that higher-order relativistic effects will lead to further amplification, and we suggest a Pad-type way of resumming them. We recommend to test our results by comparing resolution-extrapolated numerical simulations of inspiralling-binary neutron stars to their effective one

body description [12].

3. We derive the gravitational radiation-reaction force modifying the Effective One Body (EOB) description of the conservative dynamics of binary systems. Our result is applicable to general orbits (elliptic or hyperbolic) and keeps terms of fractional second post-Newtonian order (but does not include tail effects). Our derivation of radiation-reaction is based on a new way of requiring energy and angular momentum balance. We give several applications of our results, notably the value of the (minimal) 'Schott' contribution to the energy, the radial component of the radiation-reaction force, and the radiative contribution to the angle of scattering during hyperbolic encounters. We present also new results about the conservative relativistic dynamics of hyperbolic motions [13].

4. The conservative piece of the gravitational self-force (GSF) acting on a particle of mass m_1 as it moves along an (unstable) circular geodesic orbit between the innermost stable circular orbit (ISCO) and the light ring of a Schwarzschild black hole of mass $m_2 \gg m_1$ have been computed numerically.[14].

0.3.3 Quantum Fields

• The previous work (V.Belinski, 2009) on the problem of particle creation by the physical Schwarzschild black hole created by the collapse have been elaborated and supplemented by the exact analysis of what is going on in case of the eternal black hole. It was shown that also for such case there are no way for appearing of any particle creation phenomenon [15].

• It was presented a unified description of mass generation mechanisms that are called the Mach and Higgs proposals. In our mechanism, gravity acts merely as a catalyst and the final expression of the mass depends neither on the intensity nor on the particular properties of the gravitational field. These two strategies provide mass for all bodies that operate independently and competitively can be combined into a single unified theoretical framework. As a consequence of this new formulation we are able to present an answer to the question about the origin of the mass of the Higgs boson [16].

0.3.4 Alternative theories

The main research in ICRANet follow the well-established Einstein General Relativity and conventional Quantum Theory, however, the alternative theories also are welcomed. This kind of activity is represented by our brazilian and mexican collaborators under the leadership of the members of ICRANet prof. M.Novello and prof. H.Quevedo. The 2012 activity in this fields can be outlined as follows.

• It was investigated the thermodynamic geometries of the most general static, spherically symmetric, topological black holes of the Horava-Lifshitz gravity. In particular, we show that a Legendre invariant metric derived in the context of geometrothermodynamics for the equilibrium manifold correctly reproduces the phase transition structure of these black holes. Moreover, the limiting cases in which the mass, entropy or Hawking temperature vanish are also accompanied by curvature singularities which indicate the limit of applicability of the thermodynamics and geometrothermodynamics of black holes. The Einstein limit and the case of a black hole with a flat horizon are also investigated[17].

• The thermodynamic properties of 5D static and spherically symmetric black holes in (i) Einstein-Maxwell-Gauss-Bonnet theory, (ii) Einstein-Maxwell-Gauss-Bonnet theory with negative cosmological constant, and in (iii) Einstein-Yang-Mills-Gauss-Bonnet theory have been investigated. To formulate the thermodynamics of these black holes we use the Bekenstein-Hawking entropy relation and, alternatively, a modified entropy formula which follows from the first law of thermodynamics of black holes. The results of both approaches are not equivalent. Using the formalism of geometrothermodynamics, we introduce in the manifold of equilibrium states a Legendre invariant metric for each black hole and for each thermodynamic approach, and show that the thermodynamic curvature diverges at those points where the temperature vanishes and the heat capacity diverges[18].

• I was shown that a generalized Born-Infeld electrodynamics responsible for regular configurations of the static field of a charged particle produces a nonsingular universe that contains a bouncing. This means that the Universe has a previous collapsing phase, attains a minimum value for its scale factor and then enters into an expanding phase. We exhibit such a scenario in the case of an average pure magnetic universe. At its infinity past as well as at its infinite future the distribution of the energy content of the magnetic fluid displays the form of a cosmological constant. Thus such a configuration is an intermediary between asymptotic vacuum states. Then this magnetic universe evolves from vacuum to vacuum[19].

0.4 Participation in the conferences

The Thirteenth Marcel Grossmann Meeting (MG13), Stockholm University, Sweden, July 1 - 7, 2012.

0.5 Teaching activity

• V.Belinski "Application of the Inverse Scattering Method to the General Relativity", the course of 4 lectures for Erazmus Mundus Joint Doctorate Program, Nice University "Sophia Antipolis", Nice, September 2012.

• T.Damour ”Gravitational waves”, the course of 4 lectures for Erazmus Mundus Joint Doctorate Program, Nice University “Sophia Antipolis”, Nice, September 2012.

• M. Novello, organization of the 15th Brazilian School of Cosmology and Gravitation (August 19 - September 1, 2012), Rio de Janeiro (Mangaratiba).

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