Hot White Dwarf Stars

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Supernovae, Hypernovae and Binary Driven Hypernovae
An Adriatic Workshop
June 20-30, 2016, ICRANet, Pescara

Outline

1. Motivations
2. Equations of Stellar Structure: General Relativistic and Newtonian
3. Equation of State at $T \neq 0$ and $T = 0$
4. Some Numerical Calculations
5. Analytic Expression for Mass-Radius Relation
6. Summary and Future Prospects
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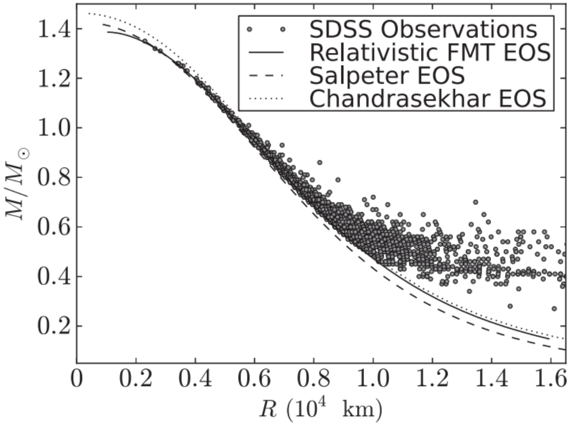

Figure: Mass-radius relation for $T = 0$ white dwarf stars vs observational data (S.M. Carvalho et al, 2014),(P.-E. Tremblay et al., 2011)

From spherically symmetric metric

$$ds^2 = e^{\nu(r)}c^2dt^2 - e^{\lambda(r)}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2\,, \qquad (1)$$

the equations of equilibrium can be written in the TOV form,

$$\frac{d\nu(r)}{dr} = \frac{2G}{c^2}\frac{4\pi r^3P(r)/c^2 + M(r)}{r^2\left[1 - \frac{2GM(r)}{c^2r}\right]}\,, \qquad (2)$$

$$\frac{dM(r)}{dr} = 4\pi r^2\frac{\mathcal{E}(r)}{c^2}\,, \qquad (3)$$

$$\frac{dP(r)}{dr} = -\frac{1}{2}\frac{d\nu(r)}{dr}[\mathcal{E}(r) + P(r)]\,. \qquad (4)$$

From the Eqs. (2) and (4) the total pressure can be rewritten in the following form

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}\left[1+\frac{P(r)}{\rho(r)c^2}\right]\left[1+\frac{4\pi r^2 P(r)}{M(r)c^2}\right]\left[1-\frac{2GM(r)}{rc^2}\right]^{-1} \tag{5}$$

The Tolman-Oppenheimer-Volkoff equation completely determines the structure of a spherically symmetric body of isotropic material in equilibrium.

If terms of order $1/c^2$ are neglected, the TOV equation becomes the Newtonian hydrostatic equation,

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}, \tag{6}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r) \tag{7}$$

and

$$\frac{d\Phi(r)}{dr} = \frac{GM(r)}{r^2} \tag{8}$$

used to find the equilibrium structure of a spherically symmetric body of isotropic material when general-relativistic corrections are not important.

The Chandrasekhar EoS is given by

$$\mathcal{E}_{Ch} = \mathcal{E}_N + \mathcal{E}_e \approx \mathcal{E}_N = \frac{A}{Z} M_u c^2 n_e, \tag{9}$$

$$P_{Ch} = P_N + P_e \approx P_e, \tag{10}$$

where A is the average atomic weight, Z is the number of protons, $M_u = 1.6604 \times 10^{-24}$ g is the unified atomic mass, c is the speed of light and n_e is the electron number density. In general, the electron number density follows from the Fermi-Dirac statistics and is determined by

$$n_e = \frac{2}{(2\pi\hbar)^3} \int_0^\infty \frac{4\pi p^2 dp}{\exp\left[\frac{\tilde{E}(p) - \tilde{\mu}_e(p)}{k_B T}\right] + 1}, \tag{11}$$

where k_B is the Boltzmann constant, $\tilde{\mu}_e$ is the electron chemical potential without the rest-mass, and $\tilde{E}(p) = \sqrt{c^2 p^2 + m_e^2 c^4} - m_e c^2$, with p and m_e the electron momentum and rest-mass, respectively.

It is possible to show that can be written in an alternative form as

$$n_e = \frac{8\pi\sqrt{2}}{(2\pi\hbar)^3} m^3 c^3 \beta^{3/2} \left[F_{1/2}(\eta,\beta) + \beta F_{3/2}(\eta,\beta)\right], \tag{12}$$

where

$$F_k(\eta,\beta) = \int_0^\infty \frac{t^k \sqrt{1+(\beta/2)t}}{1+e^{t-\eta}}\, dt \tag{13}$$

is the relativistic Fermi-Dirac integral, $\eta = \tilde{\mu}_e/(k_B T)$, $t = \tilde{E}(p)/(k_B T)$ and $\beta = k_B T/(m_e c^2)$ are degeneracy parameters. Consequently, the total electron pressure for $T \neq 0$ K is given by

$$P_e = \frac{2^{3/2}}{3\pi^2\hbar^3} m_e^4 c^5 \beta^{5/2} \left[F_{3/2}(\eta,\beta) + \frac{\beta}{2} F_{5/2}(\eta,\beta)\right]. \tag{14}$$

When $T = 0$ one can write for the number density of the degenerate electron gas the following expression from the Eq. (11)

$$n_e = \int_0^{P_e^F} \frac{2}{(2\pi\hbar)^3} d^3p = \frac{8\pi}{(2\pi\hbar)^3} \int_0^{P_e^F} p^2 dp = \frac{(P_e^F)^3}{3\pi^2\hbar^3} = \frac{(m_e c)^3}{3\pi^2\hbar^3} x_e^3 \tag{15}$$

The total electron energy-density and electron pressure

$$\begin{aligned} P_e &= \frac{1}{3}\frac{2}{(2\pi\hbar)^3} \int_0^{P_e^F} \frac{c^2p^2}{\sqrt{c^2p^2 + m_e^2c^4}} 4\pi p^2 dp \\ &= \frac{m_e^4 c^5}{8\pi^2\hbar^3} \left[x_e\sqrt{1 + x_e^2}(2x_e^2/3 - 1) + \mathrm{arcsinh}(x_e)\right], \end{aligned} \tag{16}$$

where $x_e = P_e^F/(m_e c)$ is the dimensionless Fermi momentum.

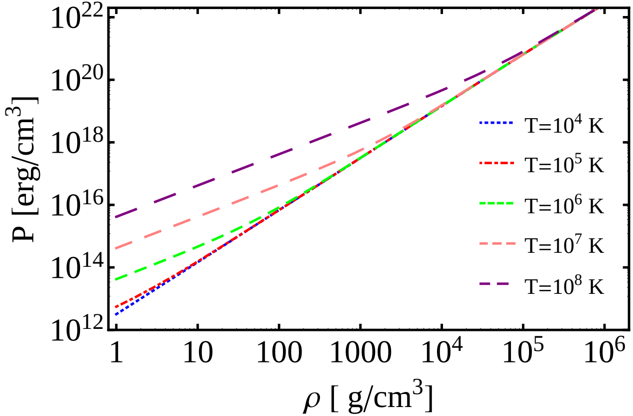

Figure: Total pressure as a function of the mass density in the case of $\mu = \frac{A}{Z} = 2$ white dwarf for selected temperatures in the range $T = 10^4 - 10^8$ K.

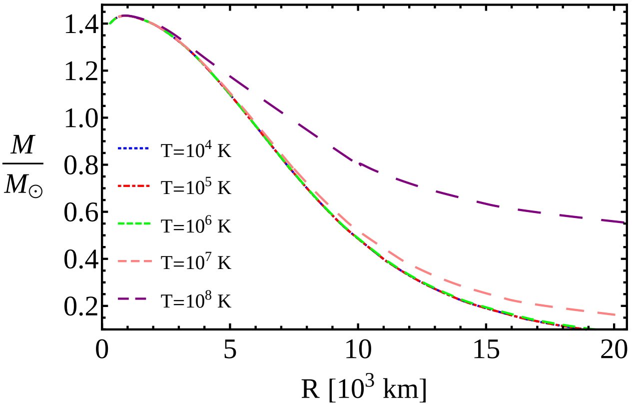

Figure: Mass versus radius for $\mu = 2$ white dwarfs at temperatures $T = [\ 10^4, 10^5, 10^6, 10^7, 10^8]$ K.

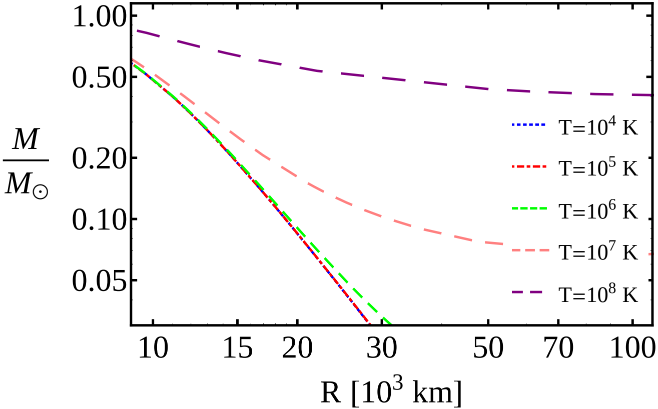

Figure: Mass versus radius for $\mu = 2$ white dwarfs at temperatures $T = [\,10^4, 10^5, 10^6, 10^7, 10^8]$ K in the range $R = 10 - 100 \times 10^3$ km.

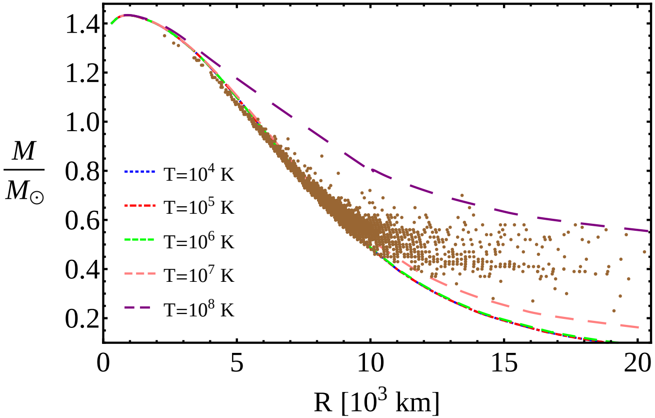

Figure: Mass-radius relations of white dwarfs obtained with the Chandrasekhar EoS (dashed lines) for selected finite temperatures from $T = 10^4$ K to $T = 10^8$ K and their comparison with the masses and radii of white dwarfs taken from the Sloan Digital Sky Survey Data Release 4 (brown dots).

Next, we will consider static and $T = 0$ temperature white dwarfs.

Pressure and Density Relation

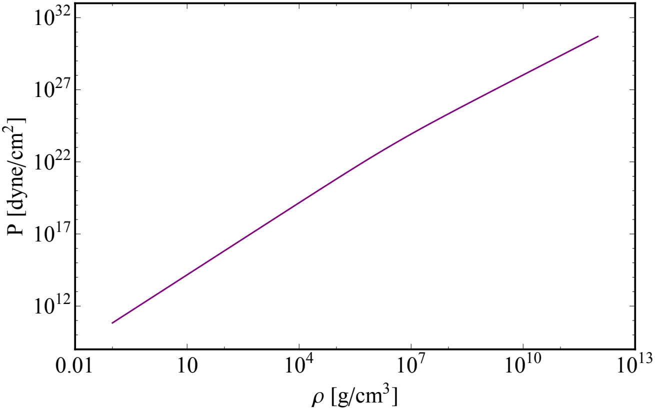

Figure: Pressure and Density Relation for Degenerate Electron Gas.

Mass and Parameter of Compactness

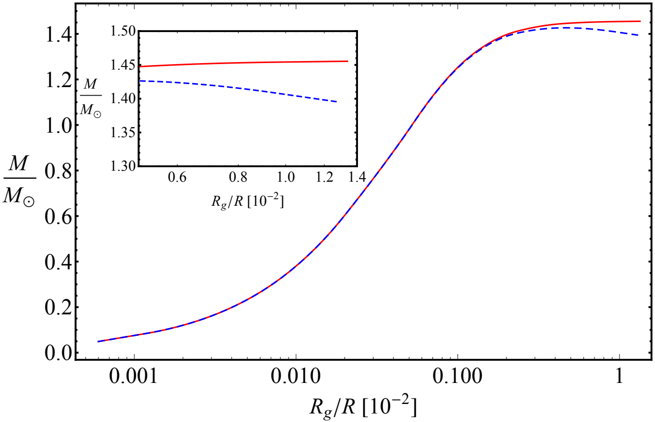

Figure: The solid red is NP, the dashed blue from GR.

Surface Gravitational Potential and Radius

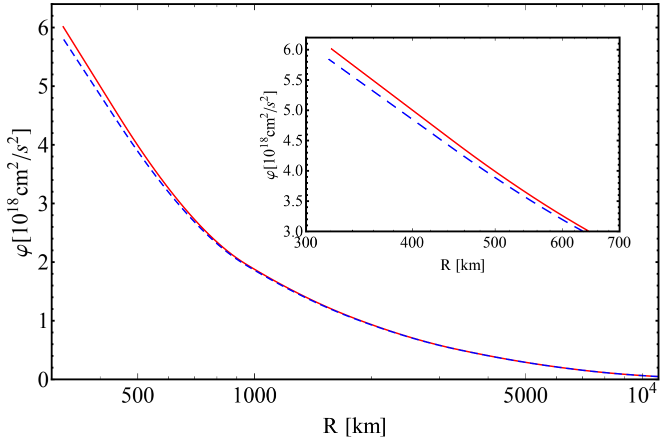

Figure: The solid red is NP, the dashed blue from GR.

Mass and Density Relation

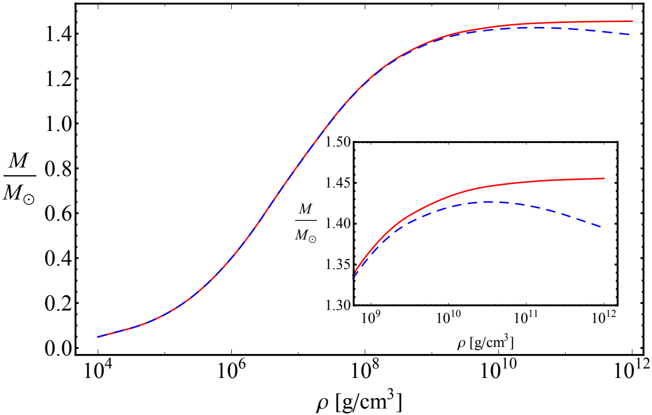

Figure: The solid red is NP, the dashed blue from GR.

Mass and Radius Relation

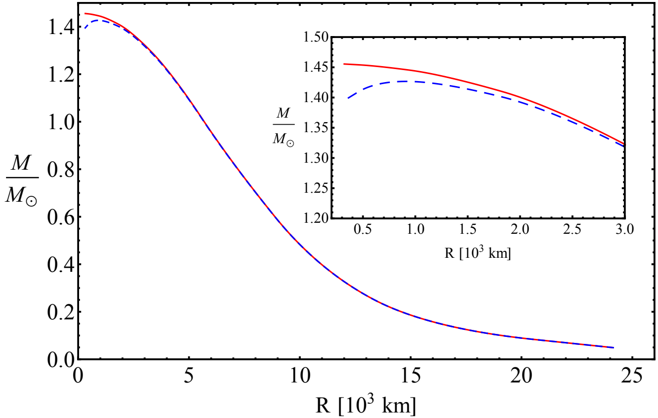

Figure: The solid red is NP, the dashed blue from GR.

Analytic Expression (AE) for Mass-Radius Relation

$$\frac{M}{M_{\odot}} = \frac{R}{a + bR + cR^2 + dR^3 + kR^4} \tag{17}$$

Newtonian Physics:
$a = 6.11$ km, $b = 0.664$,
$c = 2.2610^{-5} km^{-1}$, $d = -1.5010^{-9} km^{-1}$, $k = 1.3510^{-12}$

General Relativity:
$a = 14.65$ km, $b = 0.665$,
$c = 2.1710^{-5} km^{-1}$, $d = -1.3810^{-9} km^{-2}$, $k = 1.3410^{-12} km^{-3}$

Analytic Expression vs Structure Equations

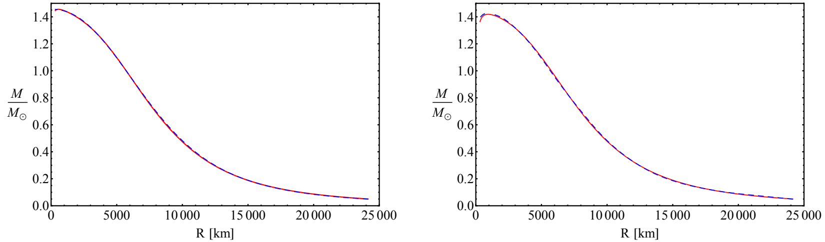

Figure: The left panel AE vs NSSE and the right panel AE vs GRSSE (the solid red is Analytic Expression, the dashed blue lines from Stellar Structure Equations (see Ref. Carvalho, Marinho, Malheiro, 2015))

Analytic Expression vs Structure Equations

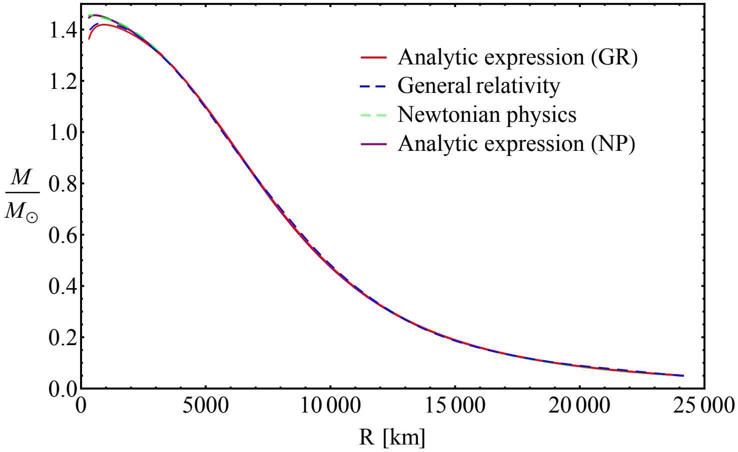

Figure: Mass and Radius Relation

Summary and Future Prospects

- The main parameters of static WDs have been found both in Newtonian Physics and General Relativity and compared. The importance of GR was shown for massive white dwarfs.
- The importance of finite temperatures has been shown.
- The analytic expression for mass-radius relation can be used. One who needs to plot mass-radius relation can use it.
- Work in progress...
- In the future we will consider effects of rotation, finite temperatures, general relativity together.

References

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