Cosmology and Non Linear Relativistic Field Theory

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2 Participants

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- S. E. PEREZ BERGLIAFFA

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- JOAO BRAGA (INPE, Sao Paulo, Brazil) (25/02/2007 to 27/02/2007).
- THYRSO VILLELA NETO (INPE, Sao Paulo, Brazil) (17/04/2007 to 18/04/2007).
- WINFRIED ZIMDAHL (07/05 to 09/05/2007and 14/04 to 18/04/2008).
- A. GUIMARAES JUNIOR (28/05 to 31/05/2007).
- RICHARD KERNER (U. Paris VI-VII, France) (31/05 to 03/06/2007).
- JEAN FRANCOIS EISENSTAEDT (U. Paris VI-VII) (28/06 to 30/06/2007).
- LAWRENCE H. FORD (Tufts U., USA) (12/08 to 24/08/2007 and 15/06 to 22/06/2008).
- CHRISTIAN CORDA (U. Pisa, Italia) (25/11 to 31/12/2007).
- ROBERTO COLISTETE JUNIOR (UFES, Brasil) (27/02 to 28/02/2008).

- C.A. WUENSCHE (INPE, Brasil) (28/02 to 29/02/2008).
- L. R. W. ABRAMO (IF-USP, Brazil) (12/03 to 13/03/2008).
- LUC CHRISTIAN BLANCHET (IAP, France) (12/05 to 31/05/2008).
- REMO RUFFINI (U. La Sapienza and ICRANET, Italia) (20/07 to 02/08/2008).
- A. STAROBINSKI (Landau Institute, Russia) (07/07 to 29/07/2008).
- V. MELNIKOV (Institute of Metrology, Russia) (08/07 to 02/08/2008).
- R. C. TRIAY (U. Marseille, France) (18/07 to 03/08/2008).
- A. PEREZ (U. Marseille, France) (19/07 to 30/07/2008).
- A. CHALLINOR (U. Cambridge) (20/07 to 24/07/2008).
- F. BERNARDEAU (IAP, France) (26/07 to 02/08/2008).
- D. P. CHARDONNET (U. De Haute Savoie) (26/07 to 02/08/2008).
- J. NARLIKAR (IUCAA, India) (01/07 to 30/07/08).
- AURORA M. P. MARTINEZ (ICIMAF, Cuba) (01/09 to 30/10/07 and 01/09 to 30/10/08)
- WOLFGANG KUNDT (U. Bonn, Germany) (17/03 to 18/03/2008 and 23/03 to 24/03/2008).
- UGO MOSCHELLA (U. Como, Italia) (06/07 to 02/08/2008).

3 ICRA-BR activities

In the years 2007-2008 ICRA-Br developped an intense activity of Conferences, Workshops, Visiting scientists and Schools of advanced topics. We present here a list of these main meetings.

- During the period from 26 February to 03 March (2007) was held in Rio the First Cesare Lattes Meeting on Gamma-Ray Bursts which was sponsored by ICRANet, and took place in Mangaratiba (Rio de Janeiro). For further details see the homepage of ICRANet (www.icra.it).

- The IV School on Cosmology and Gravitation was held from 16 to 21 July (2007). This event has its focus on students from Brazilian universities and young beginners in pos-graduate programs in Brazil and South America. This school takes place in years in which no Brazilian School of Cosmology and Gravitation is to be held, and has as main purpose to present to undergraduate, M.Sc. and Ph.D. students in Physics and related areas, an introductory overview to both Cosmology and Gravitation.

- The Conference Goedel: Logic and Time took place during August 27, 28 (2007) at ICRA/CBPF to homage the great mathematician Kurt Goedel, who performed a deep reform in the structure of Logics and contributed in a singular fashion to the examination of the notion of global cosmic time. This work triggered a series of questions about the concept of time in General Relativity.

- During August 8, 9 and 10 (2007) was held in Sobral (Ceara) the Sobral First Conference on Cosmology, Relativity and Astrophysics, which aimed at celebrating the most important scientific mission ever realized in the country involving gravitational processes, that is the Eddington Mission.

- From October 9 to 11 (2007), the First ICRA-BR Internal Workshop took place at CBPF, with 33 oral presentations by researchers and students members of ICRA-BR. They had the opportunity to learn about the research activities that are being developed by the other members of the group. Such an experience was so successful that it was decided to transform it in one of the permanent activities, to be held each year.

- In November 27 and 28 Prof. M. Novello participated of the presentation of The 2005-2007 Scientific Report ICRANet held in Pescara, Italia, for the ICRANet Scientific Committee. In this meeting Prof. Novello described the activities organized by ICRA-BR/CBPF.
- In 2007, an ICRA-BR team of researchers performed a historical-scientific study on the evolution of Cosmology along the XX century. The main topics of this path were included in a poster prepared as a piece of popular science. It is being distributed in technical schools of the Brazilian Ministry of Science and Technology and also in the public schools of the city of Rio de Janeiro. This experience will be extended to cover other Brazilian states. To continue this endeavour a review book is being prepared containing reading material and pictures to serve as a guide for the teaching of Cosmology in schools and universities.
- During the 2nd semester of 2007 and the first months of 2008, the 3rd edition of the itinerant program of Cosmology (Programa Minimo de Cosmologia, PMC, in Portuguese) was held in the University of the State of Ceara (while the first was in Rio Grande do Sul and the second in the state of Rio de Janeiro). Its goal is to communicate basic and advanced knowledge in Cosmology and Relativity throughout Brazil.
- The XIII Brazilian School of Cosmology and Gravitation was held from July 20 to August 3, 2008. The 30th anniversary of the School was commemorated in this edition. The School started in 1978 under the initiative of the Cosmology and Gravitation Group (CBPF). Its efforts are dedicated to the diffusion of different aspects of Cosmology, Gravitation and Astrophysics. Details can be found bellow in `http://mesonpi.cat.cbpf.br:8080/esccosmologia/site_in/index.html`.
- Visits of professor M Novello, Herman Mosquera Cuesta and S E P Bergliaffa to ICRANet in Pescara.

4 BSCG (Brazilian School of Cosmology and Gravitation): 30 years

MARIO NOVELLO

A Revolution in Science: The Expansion of Cosmology

In the second half of the 1970's, the attention of physicists was drawn to processes of a global nature, namely cosmic processes. This was ensued by intense activity throughout the community of physicists in various areas, many of whom were led to migrate to Cosmology. Such a broad and intense displacement, involving so many scientists, requires proper sociological analysis of the scientific practice in order to provide insights into the transformation Cosmology was going through and to the changes in the traditional mode cosmological studies had been conducted until then.

This activity produced numerous proposals of solutions to some cosmological problems and prompted a reformulation of traditional questions of Physics, thanks to the reliability that could be attributed to the cosmic way of investigating nature, a fact acknowledged by the international scientific community

Up until the late 1960s, Cosmology attracted very little interest, apart from a small group of scientists working in the area. There are several reasons one could attribute to the causes of this lack of interest. Though dissemination of activities in Cosmology had started in that decade, the 1970s could be considered the split between one attitude and the other, and the popularization of Cosmology in the overall community of physicists was achieved in the 1980s. In fact, it was in this decade that major conferences brought cosmologists, astronomers, relativist astrophysicists (traditionally, those who dealt with the Universe in its totality), and theoretical high energy physicists (who examined the microcosmos of elementary particles) together in a single event. One remarkable example was the US Fermilab 1983 Conference, which was given the suggestive title of Inner Space / Outer Space.

There have been concerted reasons contributing for this growth in Cosmology, some of which are intrinsic to this science while others are totally independent of it. This is not the place for such an inventory, but, just for clarifying purposes, one could give two examples. One, internal to Cosmology, is related to the success of the new telescopes and space probes, which yielded a huge amount of highly-reliable new data. Another, of an extrinsic nature, was the crisis of elementary particles physics in the 1970s, which, for the purposes of its own development, required the construction of huge and extraordinarily expensive high-energy accelerators, which faced political hindrances in Europe and in the United States.

The evolutionary character associated to the geometry discovered by Russian mathematician A. Friedmann, who described a dynamic expanding Universe, was thus the territory of choice to substitute in the minds of high-energy physicists, for the lack of particle accelerators, machines that could not be accomplished due to financial reasons. Such displacement was associated with the successes of Cosmology. Indeed, the standard model of the Universe was based on the existence of a configuration that described its material content as a perfect fluid in thermodynamic balance, whose temperature scaled as the inverse of the expansion; that is, the smaller the Universes total spatial volume, the greater the temperature. So, in the early times of the current expansion phase, the Universe would have experienced fantastically high temperatures, thereby exciting particle states and requiring the knowledge of the behavior of matter in situations of very high energies for its description. And, most conveniently, for free, without costs: all it took was to look at the skies.

It was within this context that the Brazilian School of Cosmology and Gravitation, BSCG, became a national and international endeavor , promoting the interaction between different physicists communities, involving astronomers, relativists, cosmologists, and theoretical high-energy physicists. It may not be an overstatement to say that the history of Cosmology in our country may be revealed through the analysis of the history of the BSCG.

Moving Toward a Second Copernican Revolution?

The booming interest for Cosmology, as recorded in the past few decades, has yielded several consequences, but perhaps the most remarkablethough not yet recognized as such shall be that it is inducing an effort to re-found Physics. To mention but one example that can help us understand the meaning of this re-founding, we could refer to Electrodynamics.

The success of Maxwells linear theory in describing electromagnetic processes was remarkable along the 20th Century. The application of this theory to the Universe, within the standard scenario of spatial homogeneity and isotropy, produced a number of particular features , including some unexpected ones. Among the latter, the one with most formidable consequences was the demonstration that the linear theory of Electromagnetism inevitably leads to the existence of a singularity in our past. That is, the Universe would have had a finite time to evolve and reach its current state.

This was the single most important characteristic of the linear theory since it led to the acceptance, in the scientists imagination, that the so-called theorems of singularity discovered in the late 1960s would, in effect, be applicable to our Universe.

However, in the following decade, a slightly more profound criticism changed this interpretation, thus rendering the consequences of theorems less imposing. This involved a lengthier r analysis of the mode through which the electromagnetic field is affected by the gravitational interaction. That it is affected, there had been no doubt, because this property was at the basis of the very theory of General Relativity, given that the field carries energy. What was yet to be learned, in detail, , was how to describe this action and which qualitative differences t the participation of the gravitational field could provoke. It soon came out that there was no single mode to describe this interaction. This is due to the vectorial and tensorial nature of electromagnetic and gravitational fields, respectively. Several proposals for this interaction were then examined.

One of these changes to Electromagnetism, motivated by the gravitational field, seemed to be somehow unrealistic because it could be naively interpreted as if the field transporter, the photon, acquired a mass in this process of interaction with the geometry of space-time, through its curvature. Moreover: this mass would depend on the intensity of this curvature. In fact, to adhere to the technical terminology, it was a non-minimal coupling between both fields: a mode of interaction that does not allow the behavior of the electromagnetic field to be reducedby using the Principle of Equivalenceto the structure that this field possesses in the idealized absence of a gravitational field. , This coupling radically changes the properties of the geometry of the Universe in the spatially homogeneous and isotropic scenario. Just to mention a new and remarkable characteristic, the electromagnetic field, under this mode of interaction with the gravitational field, produces an Eternal Universe, without singularity, without beginning, extending indefinitely to the past. It is not difficult to show that this interaction also generates a non-linearity of the electromagnetic field.

This property led the way to think about other non-linear feature of the electromagnetic field where this form of interaction with gravitation was not dominant. These features did not correspond to non-linear corrections to Maxwells Electromagnetism such as those obtained by Euler and Heisenberg, of quantum origin, though they could contain them. Regardless of these possibilities allowed by the quantum world, physicists started to think about other origins for the non-linearity: they should be thought as if Maxwells equations with which Electromagnetism had been treated this far would be nothing more than approximations of a more complex form associated to a non-linear description. This non-linearity should appear as a cosmic mode of the field, where linearity is locally an approximation, thereby inverting the traditional way of thinking non-linearity as corrections to the basic linear theory.

This simple example allows for the introduction of a fantastic situation that Cosmology would be producing and that we can synthesize in a small sentence of great formal consequences: the extrapolation of terrestrial Physics to the entire Universe should be reviewed.

The old generalization mode is a rather natural and common procedure among scientists. Thus, by extrapolation, even in conditions that have never been tested before, we go on legislating until new physics can stop, block, limit this extension of the local scientific knowledge .

In other words, the considerations above seem to point to the need of a new Copernican criticism. Not quite the one that removed us from the center of the Universe, but another, arguing against the extrapolation scientists have been resorting to. That is, to think that a global characteristic should not be attributed to the Laws of Physics and that, from this perspective, the action of discarding global cosmological processes in building a complete theory of natural phenomena would be legitimate.

That is, these Laws may take forms and modes that are different from those with which, in similar but not the same situations, ”terrestrial Physics was successfully developed. This analysis, that may lead to a description different from that physicists are used to, which becomes more and more necessary, even indispensable, is what we refer to as re-founding Physics through Cosmology. We may quote English physicist P.A.M. Dirac and Brazilian physicist C. Lattes as recent precursors of this way of thinking. Unfortunately, the practical mode they proposed for a particular re-foundation was too simple, thus allowing for a powerful reaction that shunned these ideas to the bordering and swampy terrain of speculation. Recent and formidable advances in Observational Cosmology allow us to accept that the time is coming when an analysis of this re-foundation, slightly more sophisticated than that simple modification of the fundamental constants as Dirac and others intended, may be seriously undertaken.

Antecedents of the Brazilian School of Cosmology and Gravitation

At the end of January, 1971, my post-doctorate supervisor in Oxford, the renowned scientist Denis Sciama, invited me for a meeting at the All Souls College to which some scientists who worked in his research group were also invited (R. Penrose, S. Hawking, G. Ellis, W. Rindler, among others–Apart from myself, of all these, only Penrose and Rindler showed up). The goal was to informally discuss some major issues of Physics, particularly those related to a science that was experiencing intense activity back then: Cosmology. In a given moment of that meeting, Sciama said he considered it important that we participated in the first major School of Cosmology that the French were organizing for the coming summer, possibly July, in a beautiful place in the Mediterranean, in the small island of Cargse, Corsica.

It was a very special situation and it came at a crucial moment of my decision to dedicate myself to Cosmology. One week before, when I had participated in a conference at the International Center for Theoretical Physics (ICTP) in Trieste, I had talked to a CBPF physicist who had just arrived from Brazil and made some comments on my decision to dedicate to Cosmology that caused me to become apprehensive. His comments were that a decision had been made that it would be very important for Brazil and the CBPF that I shifted my interests and started a program to guide my research efforts to a more useful area for the country, such as some sector of solid state physics. And, he added, renewal of my doctorates scholarship could depend on my decision. This type of action was not uncommon in those days. I dont know whether such interference would happen today. At least, not with that lack of subtlety! My decision had already been made and my scholarship was renewed, particularly thanks to a Brazilian physicist who worked in Geneva like myself, though he was not at the Geneva Universitys Institut de Physique but rather at CERN: Roberto Salmeron. After learning of the evolution of my dissertation work, he told me he would be supporting my decision to choose a path that looked totally estranged from the major motivation of most scientists, that is, Cosmology. If I allow myself to wander a bit into this incident, it is just to show the general state of affairs a scientist had to overcome back then in order to address Cosmology. Curiously enough, less than ten years later, Cosmology started a formidable phase of expansion, and has attracted an ever bigger number of scientists since then. Having said that, let us go back to Corsica.

The Cargèse School was an enormous success. In attendance were great names of Cosmology coming not only from England and the United States, such as Schucking, Silk, Steigman, Harrison, Rees, Ellis, and others, but also some European ones, particularly professor Hagedorn, who was very successful at the time with his theory that postulated the existence of a maximum temperature inducing a new perspective on the singularity of the standard model.

For various reasons, the big names missing in that meeting were the representatives of the Soviet School of Cosmology who, nevertheless, attracted my attention because their approach seemed to be more imaginative than the conventional proposals by European and American physicists. However, they were the ones who eventually commanded the thoughts of the western community of scientists for the coming decades, with some beautiful exceptions.

A simple and superficial exam of the list of lecturers that participated in the BSCG shows that this Russian School has been really active, from the very first meeting to date. Thus, the BSCG have managed to popularize, especially among Brazilian scientists, many ideas from those Soviet physicists and, later, from the Russian community. The peculiarity and originality of this Russian School have marked this unique participation and often allowed it to become the main outlet for ideas that are alternative to the ones dominating the panorama of Cosmology. To share a particular and extremely relevant example, suffice it to mention the course program offered in 1979 at the II BSCG, in Joo Pessoa, by Professor Evgeni Lifshitz who, based on his previous efforts with V. Belinski and I. Khalatnikov, addressed the way in which the Universe behaved in the vicinities of a singularity, raising a daring hypothesis of the existence of a primordial anisotropic phase. Nearly thirty years later, in the most recent Marcel Grossmann Congress held in 2006 in Berlin, one of the plenary sessions conducted by the French physicist Thibault Damour attempted to revive the original ideas by Belinski-Lifshitz-Khalatnikov , adapting them to modern proposals of cosmological investigation.

The Cargèse School lasted two wonderful weeks, under the happy and casual coordination of Professor E. Schatzman. Himself an enthusiast of scientific communication, he brought together the young participants , sometimes at the beach and others at tiny Cargèse's downtown area, for some beautiful starry evenings of explanations to awed locals about recent discoveries in Astrophysics and Cosmology . After a brief introduction to the behavior and structure of stars and galaxies, our coordinator urged listeners to ask questions of all sorts to the scientists. Those questions were never limited to Astrophysics, Cosmology, and Physics in general; they rather and inevitably overflowed into a scientists social role, a theme Schatzman was passionate about.

In one such evening, feeling the cold breeze from the sea, concentrated around a small bonfire, I told him that the meeting had been so exciting to me, so informative, and such a unique experience, that I would try to organize similar meetings as soon as I got back to my country. Being so kind and heedful of others, as usual, he committed himself by saying that I could certainly count on his support, adding one question about the number of scientists working in that area in Brazil. I answered that though there were d a few physicists working in isolation who could follow up on the development of modern properties of the gravitation theory, there was nothing systematic going on in my country. He then added that if the idea was to be successful, I should try to create first a small core composed by young scientists who were to receive solid training in the theory of gravitation and r one or two years of Cosmology studies. When I returned to CBPF, in the second semester of 1972, that was exactly what I did, creating the Gravitation and Cosmology Group of CBPF, which turned up to be the seed of todays Institute of Cosmology Relativity and Astrophysics (ICRA).

First School: Success of the Teacher-Student Interaction

During the year of 1976, the Brazilian Center for Research in Physics went through a radical change. Aware of the constant difficulties posed to a special institution such as the CBPF, focused on fundamental research, the federal government finally accepted to integrate this center to a federal agency. The CBPF thus became the first physics research institute to be directly incorporated to the National Research Council (CNPq), currently the National Council for Scientific and Technological Development.

The CBPF started its new phase with the arrival of Antonio Csar Olinto, designated as head of the new CBPF/CNPq. It was within this framework of renewal that Cosmology conquered its space and came forth as a new area of the endeavors of CBPF. The history of this period is rich in debates between personalities who built the history of Physics in Brazil, but I will talk about it in another occasion. Of our interest here is only the outcome, as the head of the CBPF agreed to grant financial and institutional support to the First Brazilian School of Cosmology and Gravitation, which would later be known as Brazilian School of Cosmology and Gravitation when it went international, therefore acquiring the acronym BSCG.

This School was divided into two parts, involving basic programs that lasted a full week, and advanced seminars whose classes could be limited to one up to three sessions at most. Interestingly enough, the BSCG is structured as such, to date.

The budget of the School was very small, as it was basically funded by the CBPF. However, the enthusiasm of the students was such that turned it into a major success, contrary to the pessimistic forecast of various colleagues. To mention but one example of this important student co-participation, I recall their performance in organizing the School texts. Though the faculty had carefully prepared their class notes, we had no possibility to print them. The solution was then offered by the students themselves: they mimeographed the notes, created a strongly-yellow-colored cover and manually bound all of the texts!

This willpower on the part of the students greatly encouraged the staff, who then spent the entire School in permanent activity, thus producing a student-teacher interaction that lingered on as a hallmark and operated as a trigger for CBPFs director to convince the relevant authorities (CNPq, Capes) to provide the funds in the subsequent year for the 2nd School, much more complete and administratively more organized than the 1st .

Both the 1st and the 2nd School (held respectively in 1978 and 1979) were a means to consolidate the basic structure of Gravitational Theory for our young physicists, as well as the crucial mathematical tools and techniques for a better understanding of the General Theory of Relativity. Besides this basic endeavor, some crucial concepts of theories that are correlated with Gravitation and the General Theory of Relativity involving rudiments of the Unified Theories and some basic aspects of Relativistic Astrophysics were discussed. This may be confirmed with an overview of the course programs offered for the 2nd School.

In the 3rd and 4th Schools (held in 1982 and 1984, respectively), notions of Astrophysics presented in the previous Schools were elaborated. Furthermore, there was a focus on the study of the Theory of Elementary Particles and its last association with the so-called Standard Model of Cosmology, identified with the notion of an explosive and hot start for the Universe (known in the literature as the Hot Big Bang Hypothesis).

The Internationalization

In 1987, the 5th School of Cosmology and Gravitation could increase the knowledge base and the analysis presented in the previous Schools, thus evolving to a broader and deeper debate of the feasible potential alternatives to explain the large scale behavior of the Universe. Back then, courses based on the Standard Model were presented, as well as several talks dealing with the idea of an Eternal Universe, without beginning or end. Besides these specific approaches, the relation between Quantum Physics and Gravitation was examined in detail. Though this union is still far from being complete, the basic ideas involving quantum principles of gravitation were presented in the 5th School that were later developed in the 6th School.

The 5th School was also the first one opened to the international scientific community: researchers and students from twenty-four (24) countries were enrolled and, from that 5th edition onward, the Schools name became international and it was then renamed as the Brazilian School of Cosmology and Gravitation. The lectures presented there also reflected this internationalization.

The ideas preliminarily presented in the previous School were developed during the two weeks of the 6th School of Cosmology and Gravitation, in 1989. The courses underlined the emphasis given to quantum processes in Cosmology. That fact is a natural evolution of the previous events, reflecting the important role played, even then, by the examination of quantum processes in Cosmology. Besides these course programslasting a week eachsmall working meetings were held as parallel courses. Amongst these additional events, two were particularly important: the opening of a session of student-participant seminars, thus promoting greater interaction between them and exhibitors; the start of an extraordinary debate session where the ten presenting professors individually exposed their ideas on the main current issues of Cosmology and related areas. This experience was so satisfactory that it was integrated in the organization of subsequent Schools.

In 1991, due to financial difficulties of the countrys Science/Technology system, the periodicity of the Brazilian Schools of Cosmology and Gravitation could not be maintained. Nevertheless, in order not to hinder an entire generation of young scientists, a small meeting was held at CBPF: A Crash-Course on July 15-26, 1991, whose program was as follows: Cosmology: M. Novello Gravitation: I.D.Soares Relativist Thermodynamics: J. M. Salim Hamiltonian Formulation of Gravitation: N. Pinto Neto Quantum Theory of Fields with Curved Spaces: N. F. Svaiter This crash-course was attended by 79 student/grantees of different Brazilian universities and was an important basis for later studies and projects.

New models on the creation of the Universe

In 1993, the 7th Brazilian School of Cosmology and Gravitation was again held in two weeks. Besides presenting an overall panorama of the main conquests and unresolved issues of Cosmology today, this School enabled the continued discussion on a most formidable issue : the creation of the Universe. The main novelty was due to a general change in the scientists behavior concerning the remote past of our Universe: whereas up until recently the role of an explanation generator for all of natures ulterior processes was attributed to an inaccessible initial explosion, back then several competing proposals started to appear in search for access to the issue of creation, both the classical and the quantum ones. So, models of the Eternal Universe without singularity were discussed in this School, at various moments. There was, however, general consensus that the Universe would have been through an extremely hot period. It means that either a process of quantum tunneling or a previous classical collapsing phase should provide the conditions for a likely moment of tremendous concentration of matter/energy. Different proposals of that sort were examined in the courses and seminars of this School.

The 8th Brazilian School of Cosmology and Gravitation, held in 1995, consolidated the international nature of the School, not only for the fact that it involved professors who enjoyed high prestige in the international scientific community but also, and mostly, because of the large number of student-participants coming from other countries. In this School, special emphasis was given to quantum processes and their consequences in an expanding Universe. Not only quantum processes of matter in classical background (semi-classical approach) were examined but also different proposals for quantum treatment of the very gravitational field were proposed. The recent attempts to explain the existence and formation of major structures (galaxies, clusters etc.) were also examined and discussed either from a more observational and classical perspective or through elementary quantum processes.

Two round-tables were also organized: Loss of Information from Black Holes (coordinated by Prof. W.Israel) and Time Machines (coordinated by Prof. A.Starobinsky). Furthermore, a number of seminars on other topics of interest to Cosmology and related areas were included.

A Speaker is given the Nobel Prize

The 9th BSCG happened in 1998. Its international nature appears when we list of the countries where participating scientists came from: Brazil, Argentina, Canada, Denmark, France, Israel, Italy, Mexico, Portugal, Russia, Spain, United States, and Venezuela. In this School, we commemorated twenty years of its existence. On the occasion, Professor Yvonne Choquet-Bruhat was honored with a tribute pronounced by Prof. Werner Israel. Special emphasis was given to localized astrophysical processes, particularly e to properties of black holes. A series of lectures on CMBR was delivered by Professor G. Smoot, who was subsequently awarded the Nobel Prize, precisely for his endeavors in that area. The theory of the gravitational field and the analysis of field theories on the light cone and on geometries representing expanding universes were also presented.

The 10th School was held in July, 2002, and involved scientists from 16 countries: Brazil, Germany, Bolivia, Canada, Chile, Denmark, France, England, Ireland, Italy, Mexico, Poland, Russia, United States, and Turkey. At this moment, the BSCG consolidated its tendency to open the exam of non-conventional issues not only in Cosmology but also in related areas. A brief examination of the topics therein is enough to underline this fact. This tendency continued on in the other Meetings.

Some scientist's comments on the BSCG

In 1988, CBPFs Group of Cosmology and Gravitation intended to give a permanent role to the Schools by creating a Cosmology Center, under the Ministry of Science and Technology. At that time, several physicists (at the request of the minister) were asked for their opinions on the group, as transcribed below. Particular attention should be paid to the support I received from great Brazilian scientist Csar Lattes. Whenever Lattes came to Rio, we often talked about this possibility. On these occasions, Lattes would air his ideas, similar to Paul Diracs, on local effects of the properties of the evolution of the Universe, saying he had solid arguments to show how Physics very interactions would depend on the Universes state of evolution. Years earlier, Vitrio Canuto had presented an extensive review of Diracs ideas in the School and, in the early 1970s, my CERN collaborator P. Rotelli and I had produced an alternative to Diracs proposal on the cosmic dependence of weak interactions. Lattess ideas did not possess similar development to Diracs, and were very close to mine, that being the reason why we started to write the draft (for a yet unpublished paper) together.

Lattes used to think it was totally unnecessary to write about his support to my idea of transforming the Schools of Cosmology and Gravitation into a permanent and continuous forum entirely focused on cosmological issues. I eventually convinced him that this letter of his could be important to openly communicate his opinion.

We have reproduced the content of letters by some professors where their opinions on the School are recorded.

- YVONNE CHOQUET-BRUHAT (19/9/1988) (Professor at the University of Paris VI; Director of the Relativist Mechanics Laboratory and

Fellow of the French Academy of Sciences):

() The Brazilian Schools of Cosmology and Gravitation that you have organized since 1978 have proved extremely successful both for the advancement of science at an international level, and for the development of a remarkably good Brazilian group in these fields. Having myself attended two of these Schools, I have been able to appreciate their excellent organization, the high level course programs on the most up-to-date topics by the best specialists in the field, a fruitful experience to all by the active participation of many in the audience, from the Director of the School to the youngest colleagues. These meetings have certainly contributed to obtaining many results in the fields of Cosmology and Gravitation, which have given your group the high reputation that it enjoys internationally.

- RUBEN ALDROVANDI (29/09/1988) (So Paulo Institute of Theoretical Physics):

  Although I think you know my opinion on the CBPF Group of Cosmology and Gravitation and on the Brazilian School it has been organizing for so many years, this seems to be a good opportunity to put it down in written words. The Group is the only one worthy of this name in Brazil, as other people working on those subjects never really seem to get their act together. I have very high regards for the quality, coherence andin Brazil this is essentialendurance shown during all the difficult times the Group has been in existence. As to the School: I have been in many Schools, and most are fairly good, but have never met one that is better organized than this. (...) Such an institution would give stability to the School and, I am convinced, greatly contribute to the development of activities in the sister sciences of Cosmology and Gravitation. For the reasons given above, it is a matter of course that the CBPF Group and its School are the ideal nucleation centre for the Institute.

- EDWARD W. KOLB (23/09/1988) (Professor of Astronomy and Astrophysics at the University of Chicago and at the FERMILAB):

  (...) As you know, I had the opportunity of attending the 4th and 5th Schools as a lecturer. I cannot express the student's view, but from my perspective they were both great successes. I benefited a great deal from the lectures by the many distinguished scientists and from questions and discussions with students. CBPFs Gravitation and Cosmology Group is large and active. The people already present at CBPF could easily serve as a nucleus for a more ambitious program. An Institute with a larger scope would be beneficial to Brazilian science in two ways: It would attract to Rio the best people in the international scientific community to share recent developments in general relativity and cosmology; and it would afford the opportunity for the rest of the world to learn about the great work done in Rio by Brazilian scientists. I can think of no better use of resources available to help the development of science in Brazil. I would be happy to do anything I can to help your initiative. Good luck with your efforts.

- VITORIO CANUTO (31/10/1988) (Member of NASA, Goddard Institute for Space Studies):

  (...) In all of Latin America, Brazil is the country that, thanks to your efforts, has taken the leadership in the field of General Relativity and Cosmology, as witnessed by the success of the several Schools that you have convened in the last ten years. From both the scientific and the organizational points of views, I believe they were a remarkable success. Cosmology is about to be reborn thanks to launching the Space Telescope next year. The wealth of new data available in the near future will dramatically change the field, and the fact that your Schools have already prepared young researchers in this field represents an investment on which this Institute can confidently be built. For these reasons, I firmly believe that an Institute of Research in Cosmology and Gravitation will be an outstanding Brazilian contribution not only to the development of science in Latin America but to future generations of young scientists. As can be seen from the excerpts above, even back then the Brazilian Schools of Cosmology and Gravitation already had aninternationally recognizedtradition of providing young researchers and students with easy access, and as thorough as possible, to the current state of research in some sectors of Cosmology, Gravitation, Astrophysics, and related areas. The following passages have been taken from scientists who participated in the Schools of Cosmology and Gravitation at different times.

- BAHRAN MASSHOOM (Missouri, EUA), 1993:

  The organization of the School was excellent: a rigorous schedule of lectures combined with evening seminars. There was ample time, however, to get to know the participants and to have lengthy discussions of scientific issues of mutual interest that arose in the course of lectures and seminars. (...) On the administrative side, I can only have high praise for the professionalism and dedication of the staff combined with a pleasant human touch that added warmth to the atmosphere of the School. The quality of the School was outstanding. I was also impressed with the excellent quality of graduate students at the School.

- BERNARD JONES (Copenhagen, Denmark), 1993:

  The organization of the School was in fact one of the best I have ever encountered. In fact, it was so good I never noticed it, since everything seemed to work like clockwork and, most important, the organizing team exhibited a remarkable degree of flexibility. You, evidently, have the organization of this kind of meeting down to an art-form. I made many contacts among the young people at the School and I am currently looking into the question of partially financing a bi-lateral cooperation on the subjects of mutual interest. I have contacted our Ministry of Education and will see other relevant groups over the next couple of months. I am hopeful we will be able to invite people to spend some time here.

- VITALY MELNIKOV (Head of CSVRs Department of Fundamental Interaction and Metrology; President of the Russian Gravitational Association, Moscow, Russia), 1993:

  The scientific level of the VII Brazilian School of Cosmology and Gravitation was on a good international level. Practically all modern problems on cosmology and gravitation were discussed at the School. Lecturers were renowned scientists from Europe, USA, and Brazil. It is very nice that among lecturers were some scientists representing Russian schools in basic sciences: Prof. A.Dolgov, I.Tyutin (seminar), Gitman (seminar), and myself. It may contribute to further cooperation and interaction between Brazilian and Russian basic sciences in the field of cosmology and gravitation. There were interesting discussions on the cosmological constant problem and inflationary models, as well as discussions concerning each lecture. The fact that nearly all the Brazilian groups were represented at the School and also many scientists from Argentine, Mexico, other Latin American countries, and even some people from Europe makes this School in essence an international one. The scientific organization of the School was excellent: strict time-table, full attendance, copying of the lectures, work of secretaries, conditions to work, discussions, etc. The fact that all participants lived in one compact and nearly isolated place is very good for productive interaction between all the participants and lecturers. I should like to note that it is a very good practice that all participants had their accommodations paid for by the Organizing Committee, where the scientific merit was the only reason for choosing the attendants. It is the same practice that is used in many other renowned schools like Les Houches, in France, Erice, in Italy, etc. Especially I should like to stress the great role of Prof. Mrio Novello in the preparation and organization of the work of the School. Due to his attitude, the atmosphere was very friendly and creative. Conditions of living and meals were also good. As to suggestions for future schools I should like to point out that some topics may be represented more widely like quantum cosmology and quantum gravity and also experimental problems of gravitation. In general, I think the traditional interaction of Brazilian and Russian scientists in cosmology and gravitation should be kept and enhanced. And, of course, the best traditions of the Brazilian School of Cosmology and Gravitation, which already were present at the VII School, must be kept.

- A.DOLGOV (Theoretical Astrophyiscs Center - TAC, Copenhagen, Denmark), 1998:

  The Brazilian Schools of Cosmology and Gravitation already have a long and glorious history. They started 20 years ago and, ever since, remain as one of the leading schools on the subject, not only in Brazil but in the world. It is difficult to overstate their educational and scientific value. The level of lecturers is always first rate. The scientific programs each year contain most interesting, important, and up-to-date subjects. In parallel to the main courses of lectures, more brief scientific seminars are organized, where original works by the local and visiting physicists are presented. This makes the Schools not only educationally important but also plays an essential role in the recognition of Brazilian scientific achievements. I would also like to stress the great, excellent, and difficult work done by Professor M.Novello in organizing these Schools.

- IGOR NOVIKOV (Director, Theoretical Astrophysics Center, Copenhagen, Denmark), 1998: I am writing in connection with the great tradition of Brazilian physicists: a series of scientific meetings called the Brazilian Schools of Cosmology and Gravitation (BSCG). (...) The BSCG have taken place approximately every two years starting from 1978. In this year of 1998, the IX BSCG was held in which I had the privilege to participate as an invited lecturer. The main goals of the Schools are to provide the possibility to present and discuss the new achievements in cosmology, general theory of relativity, astrophysics, quantum field theory and in related areas. I have learned these Schools from my colleagues and from Proceedings of the Schools for many years. This year as a participant of the IX BSCG I personally observed the highest scientific and organizational level of the School. The unique format of the BSCG and very friendly working atmosphere provided many fruitful discussions both in pure science and in scientific education. It leads to a real progress in physics and is especially important and competitive at a world class level, and the list of lecturers at BSCG is a who's who of the leaders of cosmology and physics of the international level. I believe that the outstanding BSCG is the result of enormous work of the talented organizers of the School under the leadership of the Head of BSCG, Prof. M.Novello. It would be very important both for Brazilian physics and for the world physics community to continue the Brazilian Schools of Cosmology and Gravitation in the future.

- EDWARD W. KOLB (Theoretical Astrophysics, FERMILAB; The University of Chicago, EUA), 1998:

I have had the pleasure of attending two of the Brazilian Schools of Cosmology and Gravitation. In addition to an enthusiastic audience for my lectures, I learned a great deal from the other fine lectures at the Schools. The Schools were exceptionally well run and well balanced. I believe that the Schools have had many benefits for Brazilian science. Not only are the students exposed to ideas and research of leading scientists from the entire world, but scientific leaders from throughout the world are exposed to the very fine young Brazilian researchers. There are many talented young scientists who would otherwise not be easily noticed outside Brazil. Because of the contacts made during my visits to Brazil to attend the Schools, several young scientists have been invited to spend long periods visiting our group at Fermi National Accelerator Laboratory. I am sure that we benefitted from their visits, and I believe that they benefitted from visiting us as well. Nowadays it is difficult to provide continuity even to successful projects. In spite of difficulties you may face, I would like to encourage you to do whatever it takes to continue with the Brazilian Schools of Cosmology and Gravitation. The benefits of the School are quite considerable.

- J. NARLIKAR (Inter-University Centre for Astronomy and Astrophysics - IUCAA, India), 1998:

  I am writing this letter to give my impressions on the Schools of Cosmology and Gravitation conducted by your group in Brazil over the last 20 years. I recall participating in one of the schools in 1987 as a resource person. It was indeed an exhilarating experience to meet the students who were attracted not only from Brazil but also from other countries. The resource persons were also from many different countries and enjoyed international reputation. The School which I attended and lectured in certainly ????? went a long way in bringing to the student community the latest ideas in cosmology and astrophysics. Knowing that many of the students would normally miss the lectures that are routinely delivered in schools held in Europe or the United States, I think the BSCG is playing a very vital role in this field. I do hope that you will continue this activity and possibly expand upon it if your funding agency so permits. You have established a tradition which has to be continued, and I hope that it will.

- FANG LI-ZHI (University of Arizona, Tucson, USA), 1998:

  (...) Gravitational theory and cosmology are two of the most fundamental fields of physics. It could not exist without strong public support. However, given the small number of researchers in gravitation and cosmology, these fields make unexpectedly large contributions to formal and informal science education. In the current world, more and more countries recognize that the synergistic, educational, and cultural contributions of the study of cosmology and gravitation are worthy. Therefore, not only big and rich countries attach importance to these fields, but also many others. For instance, even under the current Asian financial crisis, the programs of cosmological and gravitational research in Korea, Vietnam, and Taiwan have firmly been funded by their own authorities. I had the honor to be invited as a lecturer at the BSCG in 1984. Since then I have kept in touch with colleagues of the BSCG. I would like to evaluate the BSCG to be the first rank of schools in the field. All lecturers are influential, and all lectures delivered at the BSCG are on the frontier of gravitation and cosmology research. In addition, the BSCG provides unusual opportunities for international exchange and cooperation of colleagues from Brazil and Latin America with the rest of the world. Therefore, I strongly recommend support to the BSCG School, and its activity should be regular and permanent.

- G.F.R.ELLIS (University of Cape Town, Department of Mathematics and Applied Mathematics, South Africa), 1998:

  This letter is to state that the series of scientific meetings called the Brazilian Schools of Cosmology and Gravitation (BSCG) have been a significant series of meetings, pulling together high quality lecturers from around the world, and resulting from time to time in good quality publications of significant merit. I therefore believe that continuation of these schools on a regular basis will be a very worthwhile project, and will make a significant contribution to the development of relativity and cosmology not merely to Brazil, but in the whole of Latin America. I am therefore pleased to support your request that funding for these schools should be continued.

- VLADIMIR MOSTEPANENKO (A.Friedmann Laboratory for Theoretical Physics, Moscow, Russia; Visiting Professor, UFPb, Joo Pessoa), 1998:

  Let me express my gratitude for your kind invitation to take part in the IX Brazilian School of Cosmology and Gravitation and to give the lectures there. The School of Cosmology and Gravitation has become a traditional event in Brazil. During twenty years it has gathered the most qualified lecturers on the subject from all over the world and the most promising young Brazilian researchers working in the field of cosmology and gravitation. It is a great honor to Brazil that this country considers it possible to support this field of fundamental physics research. Giving seemingly small contribution to technologies, Cosmology and Gravitation investigate and solve the most profound problems of the structure and evolution of our Universe. These problems have attracted the most prominent scientists from different countries during all the history of mankind. Now both Gravitation and Cosmology are the experimentally based exact sciences with great perspectives. I hope that the tradition of the Brazilian Schools of Cosmology and Gravitation will be prolonged giving significant contribution to education and science in Brazil.

- YVONNE CHOQUET-BRUHAT (Universit Pierre et Marie Curie, Gravitation et Cosmologie Relativistes, Paris, France), 1998:

  The Brazilian School of Cosmology and Gravitation has held regular meetings - or rather summer schools - since 1978. The list of speakers at these schools is an impressive assembly of internationally renowned names of specialists covering the broad area of General Relativity and Cosmology. I myself have been fortunate enough to participate in two of these schools. I have learned greatly from the lectures of colleagues working in fields distinct but related to mine (which is mainly mathematical problems in General Relativity). The school was also attended by a member of graduate students. The solid background as well as the advanced view points that they received there was certainly a great asset for their future. The Brazilian School of Cosmology and Gravitation has an international reputation, enhanced and perpetuated by the volumes of its proceedings. This school totally deserves to be supported.

5 Brief description

5.1 Bouncing Cosmological Models

The standard cosmological model (SCM) furnishes an accurate description of the evolution of the universe, which spans approximately 13.7 billion years. The main hypothesis on which the model is based are the following:

- Gravity is described by General Relativity;
- The universe obeys the Cosmological Principle . As a consequence, all the relevant quantities depend only on global Gaussian time;
- Above a certain scale, the matter content of the model is described by a continuous distribution of matter/energy, which is described by a perfect fluid.

In spite of its success, the SCM suffers from a series of problems such as the initial singularity, the cosmological horizon, the flatness problem, baryon asymmetry, and the nature of dark matter and dark energy. Although inflation (which for many is currently a part of the SCM) partially or totally answers some of these, it does not solve the crucial problem of the initial singularity. The existence of an initial singularity is disturbing: a singularity can be naturally considered as a source of lawlessness, because the spacetime description breaks down there, and physical laws presuppose spacetime. Regardless of the fact that several scenarios have been developed to deal with the singularity issue, the breakdown of physical laws continues to be a conundrum after almost a hundred years since the discovery of the FLRW solution (which inevitably displays a past singularity, or in the words of Friedmann, a beginning of the world). In the review we made for Physics Report, we concentrate precisely on the issue of the initial singularity. We shown that non-singular universes have been recurrently present in the scientific literature. In spite of the fact that the idea of a cosmological bounce is rather old, the first exact solutions for a bouncing geometry were obtained by Novello and Salim, and Melnikov and Orlov in the late 70s. It is legitimate to ask why these solutions did not attract the attention of the community then. In the beginning of the 80s, it was clear that the SCM was in crisis (due to the problems mentioned above, to which we may add the creation of topological defects, and the lack of a process capable of producing the initial spectrum of perturbations, necessary for structure formation). On the other hand, at that time the singularity theorems were taken as the last word about the existence of a singularity in reasonable cosmological models. The appearance of inflationary theory gave an answer to some of the issues in a relatively economical way, and opened the door for an explanation of the origin of the spectrum of primordial fluctuations. Faced with these developments, and taking into account the status of singularity theorems at that time, the issue of the initial singularity was not pressing anymore, and was temporarily abandoned in the hope that quantum gravity would properly address it. At the end of the 90s, the discovery of the acceleration of the universe brought back to the front the idea that the density of energy ρ+ 3p could be negative, which is precisely one of the conditions needed for a cosmological bounce in GR, and contributed to the revival of nonsingular universes. Bouncing models even made it to the headlines in the late 90s and early XXI century, since some models in principle embedded in string theory seemed to suggest that a bouncing geometry could also take care of the problems solved by inflation. Perhaps the main motivation for nonsingular universes is the avoidance of lawlessness, as mentioned above. Also, since we do not know how to handle infinite quantities, we would like to have at our disposal solutions that do not entail divergencies. As seen in Physics Report, this can be achieved at a classical level, and also by quantum modifications. On a historical vein, this situation calls for a parallel with the status of the classical theory of the electron by the end of the 19th century. The divergence of the field on the world line of the electron led to a deep analysis of Maxwells theory, including the acceptance of a cooperative influence of retarded and advanced fields and the related causality issues. However, this divergence is milder than that of some solutions of General Relativity, since it can be removed by the interaction of the electron with the environment. Clearly, this is not an option when the singularity is that of a cosmological model.

Another motivation for the elimination of the initial singularity is related to the Cauchy problem. In the SCM, the structure of spacetime has a natural foliation (if no closed timelike curves are present), from which a global Gaussian coordinate system can be constructed, with $g_{00} = 1$ and $g_{01} = 0$, in such a way that $ds2 = dt^2 - g_{ij}dx^i dx^j$. The existence of a global coordinate system allows a rigorous setting for the Cauchy problem of initial data. However, it is the gravitational field that diverges on a given spatial hypersurface $t = const$ (denoted by Σ) at the singularity in the SCM. Hence, the Cauchy problem cannot be well formulated on such a surface: we cannot pose on Σ the initial values for the field to evolve. There are more arguments that suggest that the singularity should be absent in an appropriate cosmological model. According to some proposals the second law of thermodynamics is to be supplemented with a limit on the entropy of a system of largest linear dimension R and proper energy E, given by $S/E \leq 2\pi R/\hbar c$. Currently this bound is known to be satisfied in several physical systems. It was shown that the bound is violated as the putative singularity is approached in the radiation-dominated FLRW model (taking as R the particle horizon size). The restriction to FLRW models was lifted later on, where it was shown, independently of the spacetime model, and under the assumptions that (1) causality and the strong energy condition (SEC) hold, (2) for a given energy density, the matter entropy is always bounded from above by the radiation entropy, that the existence of a singularity is inconsistent with the entropy bound: a violation occurs at time scales of the order of Planck's time. ¿From the point of view of quantum mechanics, we could ask if it is possible to repeat in gravitation what was done to eliminate the singularity in the classical theory of the electron. Namely, can the initial singularity be smoothed via quantum theory of gravity? The absence of the initial singularity in a quantum setting is to be expected on qualitative grounds. There exists only one quantity with dimensions of length that can be constructed from Newton constant G, the velocity of light c and $\hbar$ (namely $L_{Pl} = \sqrt{G\hbar/c^3}$.). This quantity would play in quantum gravity a role analogous to that of the energy of the ground state of the hydrogen atom (which is the only quantity with dimensions of energy that can be built with fundamental constants). As in the hydrogen atom, L_{Pl} would imply some kind of discreteness, and a spectrum bounded from below, hence avoiding the singularity. Also, since it is generally assumed that L_{Pl} sets the scale for quantum gravity effects, geometries in which curvature can become larger than L_{Pl}^{-2} or can vary very rapidly on this scale would be highly improbable. Yet another argument that suggests that quantum effects may tame a singularity is given by the Rayleigh-Jeans spectrum. According to classical physics, the spectral energy distribution of radiation in thermal equilibrium diverges like ω^3 at high frequencies, but when quantum corrections are taken into account, this classical singularity is regularized and the Planck distribution applies. We may expect that QG effects would regularize the initial singularity. As a consequence of all these arguments indicating that the initial singularity may be absent in realistic descriptions of the universe, many cosmological solutions displaying a bounce were examined in the last decades. In fact, the pattern in scientific cosmologies somehow parallels that of the cosmogonic myths in diverse civilizations, which can be classified in two broad classes. In one of them, the universe emerges in a single instant of creation (as in the Jewish-Christian and the Brazilian Carajas cosmogonies). In the second class, the universe is eternal, consisting of an infinite series of cycles (as in the cosmogonies of the Babylonians and Egyptians). We have seen that there are reasons to assume that the initial singularity is not a feature of our universe. Quite naturally, the idea of a non-singular universe has been extended to encompass cyclic cosmologies, which display phases of expansion and contraction. The first scientific account of cyclic universes is in the papers of Friedmann, Einstein, Tolman and Lemaitre and his Phoenix universe, all published in the 1930's. A long path has been trodden since those days up to recent realizations of these ideas. Some cyclic models could potentially solve the problems of the standard cosmological model, with the interesting addition that they do not need to address the issue of the initial conditions. Another motivation to consider bouncing universes comes from the recognition that a phase of accelerated contraction can solve some of the problems of the SCM in a manner similar to inflation. Let us take for instance the flatness problem. Present observations imply that the spatial curvature term, if not negligible, is at least non-dominant w.r.t. the curvature term: $r^2 = \epsilon/a^2H^2 \leq 1$, but during a phase of standard, decelerated expansion, r grows with time. So we need an impressive fine-tuning at, say, the GUT scale, to get the observed value. This problem can be solved by introducing an early phase during which the value of r, initially of order 1, decreases so much in time that its subsequent growth during FLRW evolution keeps it still below 1 today. Thus, an era of accelerated contraction may solve the flatness problem (and the other kinematical issues of the SCM). This property helps in the construction of a scenario for the creation of the initial spectrum of cosmological perturbations in non-singular models. The main goal of this review is to present some of the many non-singular solutions available in the literature, exhibit the mechanism by which they avoid the singularity, and discuss what observational consequences follow from these solutions and may be taken (hopefully) as an unmistakable evidence of a bounce. We shall not pretend to produce an exhaustive list, but we intend to include at least an explicit form for the time evolution of a representative member of each type of solution. The models examined here are restricted to those close or identical to the FLRW geometry. It will suffice for our purposes in this review to define a singularity as the region where a physical property of the matter source or the curvature blows up. In fact, since we shall be dealing almost exclusively with geometries of the Friedmann type, the singularity is always associated with the divergence of some functional of the curvature. Let us remark at this point that there are at least two different types of nonsingular universes: (a) bouncing universes (in which the scale factor attains a minimum), and (b) eternal universes, which are past infinity and ever expanding, and exist forever. Class (a) includes cyclic universes. The focus of this review are those models in class (a), although we shall review a few examples of models in class (b).

5.2 Effective Geometry in non-linear Electrodynamics

In recent years, there has been a growing interest in models that mimic in the laboratory some features of gravitation. The actual realization of these models relies on systems that are very different in nature: ordinary non-viscous fluids, superfluids, flowing and non-flowing dielectrics, non-linear electromagnetism in vacuum, and Bose-Einstein condensates. The basic feature shared by these systems is that the behavior of the fluctuations around a background solution is governed by an effective metric. More precisely, the particles associated to the perturbations do not follow geodesics of the background spacetime but of a Lorentzian geometry described by the effective metric, which depends on the background solution as pointed out some time ago by Unruh and earlier by Plebanski. It is important to notice that only some kinematical aspects of general relativity can be imitated by this method, but not its dynamical features. Although most of these works concerns sound propagation, the most fashionable results deal with non-linear Electrodynamics. This is related to the possibility of dealing with phenomena that are treatable in actual laboratory experiments. This is one of the main reasons that induce us to analyze carefully a certain number of non-equivalent non-linear electromagnetic configurations. Among these results we can quote the possibility of imitating a non-gravitational Black Hole in laboratory dealing with non-linear electrodynamics effects (see Appendix).

5.3 Non-linear field theory in flat and curved space-time

Recent works have shown the important role that Nonlinear Electrodynamics (NLED) can have in two crucial questions of Cosmology, concerning particular moments of its evolution for very large and for low-curvature regimes, that is for very condensed phase and at the period of acceleration. We present here a a toy model of a complete cosmological scenario in which the main factor responsible for the geometry is a nonlinear magnetic field which produces a FRW homogeneous and isotropic geometry. In this scenario we distinguish four distinct phases: a bouncing period, a radiation era, an acceleration era and a re-bouncing. It has already been shown that in NLED a strong magnetic field can overcome the inevitability of a singular region typical of linear Maxwell theory; on the other extreme situation, that is for very weak magnetic field it can accelerate the expansion. The present model goes one step further: after the acceleration phase the universe re-bounces and enter in a collapse era. This behavior is a manifestation of the invariance under the dual map of the scale factor $a(t) \rightarrow 1/a(t)$, a consequence of the corresponding inverse symmetry of the electromagnetic field ($F \rightarrow 1/F$, where $F \equiv F^{\mu\nu}F_{\mu\nu}$) of the NLED theory presented here. Such sequence collapse-bouncing-expansion-acceleration-re-bouncing-collapse constitutes a basic unitary element for the structure of the universe that can be repeated indefinitely yielding what we call a Cyclic Magnetic Universe (see Appendix).

Summary

In the last years there has been increasing of interest on the cosmological effects induced by Nonlinear Electrodynamics (NLED). The main reason for this is related to the drastic modification NLED provokes in the behavior of the cosmological geometry in respect to two of the most important questions of standard cosmology, that is, the initial singularity and the acceleration of the scale factor. Indeed, NLED provides worthwhile alternatives to solve these two problems in a unified way, that is without invoking different mechanisms for each one of them separately. Such economy of hypotheses is certainly welcome. The partial analysis of each one of these problems was initiated by our group in ICRA-Br. In this workk we present a new cosmological model, that unifies both descriptions.

The general form for the dynamics of the electromagnetic field, compatible with covariance and gauge conservation principles reduces to $L = L(F)$, where $F \equiv F^{\mu\nu}F_{\mu\nu}$. We do not consider here the other invariant $G \equiv F^{\mu\nu}F^{*}_{\mu\nu}$ constructed with the dual, since its practical importance disappears in cosmological framework once in our scenario the average of the electric field vanishes in a magnetic universe as we shall see in the next sections. Thus, the Lagrangian appears as a regular function that can be developed as positive or negative powers of the invariant F. Positive powers dominate the dynamics of the gravitational field in the neighborhood of its moment of extremely high curvatures. Negative powers control the other extreme, that is, in the case of very weak electromagnetic fields. In this case as it was pointed out previously it modifies the evolution of the cosmic geometry for large values of the scale factor, inducing the phenomenon of acceleration of the universe. The arguments presented make it worth considering that only the averaged magnetic field survives in a FRW spatially homogeneous and isotropic geometry. Such configuration of pure averaged magnetic field combined with the dynamic equations of General Relativity received the generic name of Magnetic Universe.

The most remarkable property of a Magnetic Universe configuration is the fact that from the energy conservation law it follows that the dependence on time of the magnetic field $H(t)$ is the same irrespective of the specific form of the Lagrangian. This property allows us to obtain the dependence of the

magnetic field on the scale factor $a(t)$, without knowing the particular form of the Lagrangian $L(F)$. Indeed, as we will show later on, from the energy-momentum conservation law it follows that $H = H_0\, a^{-2}$. This dependence is responsible for the property which states that strong magnetic fields dominates the geometry for small values of the scale factor; on the other hand, weak fields determines the evolution of the geometry for latter eras when the radius is big enough to excite these terms.

In order to combine both effects, here we will analyze a toy model. The symmetric behavior of the magnetic field in both extremes – that is for very strong and very weak regimes – allows the appearance of a repetitive configuration of the kind exhibited by an eternal cyclic universe.

Negative power of the field in the Lagrangian of the gravitational field was used in attempting to explain the acceleration of the scale factor of the universe by modification of the dynamics of the gravitational field by adding to the Einstein-Hilbert action a term that depends on negative power of the curvature, that is

$$S = \frac{M_{\rm Pl}^2}{2} \int \sqrt{-g} \left(R - \frac{\alpha^4}{R} \right) d^4x,$$

Although this Lagrangian was shown to be in disagreement with solar system observations, it started a program which introduced polynomial Lagrangian of the form

$$\sum_n c_n\, R^n$$

containing positive and negative values of n.

This modification introduced an idea that is worth to be generalized: the dynamics should be invariant with respect to the inverse symmetry transformation. In other words,if $\mathbb{X}$ represents the invariant used to construct a Lagrangian for a given field, the Action should be invariant under the map $\mathbb{X} \rightarrow 1/\mathbb{X}$. Since the Electrodynamics is the paradigm of field theory, one should start the exam of such a principle into the realm of this theory. In other words we will deal here with a new symmetry between strong and weak electromagnetic field. In a previous work, a model assuming this idea was presented and its cosmological consequences analyzed. In this model, the action for the electromagnetic field was modified by the addition of a new term, namely

$$S = \int \sqrt{-g} (-\frac{F}{4} + \frac{\gamma}{F}) d^4x.$$

This action yields an accelerated expansion phase for the evolution of the universe, and correctly describes the electric field of an isolated charge for a sufficiently small value of parameter γ. The acceleration becomes a consequence of the properties of this dynamics for the situation in which the field is weak.

In another cosmological context, in the strong regime, it has been pointed out in the literature by us, that NLED can produces a bouncing, altering another important issue in Cosmology: the singularity problem. In this article we would like to combine both effects improving the action to discuss the consequences of NLED for both, weak and strong fields.

It is a well-known fact that under certain assumptions, the standard cosmological model unavoidably leads to a singular behavior of the curvature invariants in what has been termed the Big Bang. This is a highly distressing state of affairs, because in the presence of a singularity we are obliged to abandon the rational description of Nature. It is possible that a complete quantum cosmology could describe the state of affairs in a very different and more complete way. For the time being, while such complete quantum theory is not yet known, one should attempt to explore alternatives that are allowed and that provide some sort of phenomenological consequences of a more profound theory.

It is tempting then to investigate how NLED can give origin to an unified scenario that not only accelerates the universe for weak fields (latter cosmological era) but that is also capable of avoiding an initial singularity as a consequence of its properties in the strong regime.

Scenarios that avoid an initial singularity have been intensely studied over the years. As an example of some latest realizations we can mention the pre-big-bang universe and the ekpyrotic universe. While these models are based on deep modifications on conventional physics, that are extremely difficult to be observed, the model we present here relies instead on the electromagnetic field. The new ingredient that we introduce concerns the dynamics that is rather different from that of Maxwell in distinct regimes. Specifically, the Lagrangian we will work with is given by

$$L_T = \alpha^2 F^2 - \frac{1}{4} F - \frac{\mu^2}{F} + \frac{\beta^2}{F^2}. \tag{5.3.1}$$

The dimensional constants α, β and μ are to be determined by observation. Thus the complete dynamics of electromagnetic and gravitational fields are governed by Einstein equations plus L_T.

We shall see that in Friedmann-Robertson-Walker (FRW) geometry we can distinguish four typical eras which generate a basic unity of the cosmos (BUC) that repeat indefinitely. The whole cosmological scenario is controlled by the energy density ρ and the pressure p of the magnetic field. Each era of the BUC is associated with a specific term of the Lagrangian. As we shall see the conservation of the energy-momentum tensor implies that the field dependence on the scale factor yields that the invariant F is proportional to a^{-4}. This dependence is responsible by the different dominance of each term of the Lagrangian in different phases. The first term $\alpha^2 F^2$ dominates in very early epochs allowing a bouncing to avoid the presence of a singularity. Let us call this the ***bouncing era***. The second term is the Maxwell linear action which dominates in the ***radiation era***. The inverse term μ^2/F dominates in the ***acceleration era***. Finally the last term β^2/F^2 is responsible for a ***re-bouncing***. Thus each BUC can be described in the following way:

- The bouncing era: There exists a collapsing phase that attains a minimum value for the scale factor $a_B(t)$;
- The radiation era: after the bouncing, $\rho + 3p$ changes the sign; the universe stops its acceleration and start expanding with $\ddot{a} < 0$;
- The acceleration era: when the $1/F$ factor dominates the universe enters an accelerated regime;
- The re-bouncing era: when the term $1/F^2$ dominates, the acceleration changes the sign and starts a phase in which $\ddot{a} < 0$ once more; the scale factor attains a maximum and re-bounces

The universe starts a collapsing phase entering a new bouncing era. This unity of four stages, the BUC, constitutes an eternal cyclic configuration that repeats itself indefinitely.

The plan of the work is as follows. First we review the Tolman process of average in order to conciliate the energy distribution of the electromagnetic field with a spatially isotropic geometry, presents the notion of the Magnetic Universe and its generic features concerning the dynamics of electromagnetic field generated by a Lagrangian $L = L(F)$. Then we present the conditions of bouncing and acceleration of a FRW universe in terms of properties to be satisfied by L. Later on we introduce the notion of inverse symmetry of the electromagnetic field in a cosmological context. This principle is used to complete the form of the Lagrangian that guides the combined dynamics of the unique long-range fields yielding a spatially homogeneous and isotropic nonsingular universe. We present then a complete scenario consisting of the four eras: a bouncing, an expansion with negative acceleration, an accelerated phase and a re-bouncing. Finally let us point out that although the total Lagrangian of NLED seems at first sight to induce an energy which is not strictly positive definite, this is not the case in the actual toy model. Furthermore, even in the case of a static spherically symmetric field of a charged particle, the negative contribution to the total energy - as measured in the asymptotic spatial infinity - reduces to a finite constant depending only on the free parameters of the theory. Thus, as it was done by Born and Infeld in their NLED, this constant can be ruled out by the addition of a constant term in the Lagrangian, which do not affect the dynamics of the electromagnetic field and makes the total energy positive definite.

5.4 Spinor theory of Gravity

From Einstein Equivalence Principle (EEP) it follows that universality of gravitational processes leads naturally to its identification to a metric tensor $g_{\mu\nu}$. However anyone that accepts this interpretation of the EEP should ask, before adopting the General Relativity approach the following question: giving the observational fact that any piece of matter/energy provokes a modification of the geometry in which this piece is merged, could one be led to the unique conclusion that this modification is driven by a differential equation containing derivatives up to second order of the metric tensor and by properties of the matter that represents its energy distribution? Should one be obliged to conclude that there is no other logical way to understand this fact? Is there a unique and only way that compels any sort of gravitationally interacting matter to modify space-time geometry through a direct relation between a continuous local modification of the geometry and the corresponding matter-energy content? In other words, are we contrived to accept that geometry is also a physical component of nature, requiring unequivocally a dynamical equation itself? Is this the unique way to implement the Equivalence Principle? General Relativity is a complete realization of EEP that answers **yes** to these questions. These lectures will deal with Pre-Gravity Theory, which provides a distinct and competitive way to implement EEP which answers **no** to all these questions. In Pre-Gravity the gravitational field is represented in terms of two fundamental spinor fields Ψ_E and Ψ_N. Its origins goes back to a complementary view of EEP, according to which the geometrical field is an induced quantity that depends on some intimate microscopic sub-structure. This sub-structure does not have by itself a geometric origin but instead it is a matter field. We could say that GR is based on a vision according to which space-time is to be understood as the arena of Physics (in Wheeler's words) and gravity is nothing but the consequence of a direct modification of the intrinsic geometry of such an arena. PG on the other hand, considers that the arena contains only matter and energy and the geometry is nothing but a specific way related to these real quantities or substances interacts among themselves. In this way, in Pre-gravity it has no practical sense to attribute a dynamics to the geometry. Its evolution is just a natural consequence of the dynamics of matter interacting gravitationally, as we shall see. Accepting the idea that the metric tensor is a derived quantity that is, it is not an independent dynamical variable, then we face the question: what should be the intermediate dynamical variables that represents the gravitational phenomenon? In his analysis of similar question, Feynmann argued against the possibility to identify such dynamical entity to different kinds of continuum fields like scalar, spinor and vector. Let us review this analysis. The argument against the scalar field rests on the impossibility of describe the influence of gravity in photon propagation. Accepting that the net effect of a scalar field should produce only conformally flat geometries then it follows that conformal invariance of Maxwell electrodynamics imply the absence of any direct influence of gravity on photon propagation. This was ruled out by the Sobral observation. The impossibility to identify gravity to vector field is related to the purely attractive effect of gravity. For neutrino-like field the Feynmann argument rests on the impossibility of having a $1/r$ static potential. Then he concludes that only a tensorial field $\varphi_{\mu\nu}$ could fulfill this criteria which led that the dynamical quantity of gravitational field has to be identified with the metric tensor. The Spinor Theory of Gravity provides a distinct answer and circumvent these difficulties, see Appendix.

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7 Some symbolic dates of important Cosmology moments in the 20th Century

- 1915 German physicist A. Einstein (1879-1955) creates a new theory of gravitation, the General Relativity (GR), identifying the gravitational force with the geometric structure of space-time.
- 1917 Einstein proposes the first relativist cosmologic model and introduces a new universal constant represented by the Greek letter ?? (lambda), called cosmologic constant. This model represents a finite, static universe whose total volume does not vary with time, that is, it does not admit expansion. Using GR equations with the cosmologic constant, Dutch astronomer W. De Sitter (1872-1934) establishes the second cosmologic model and shows that, contrary to Einsteins model, solutions can be built to represent an expanding universe in stationary regime, that is, with constant expanding speed. The existence of ? is enough to produce this universe, as the action of matter and energy is disregarded. So, W. De Sitters model does not have matter; it is pure geometry.
- 1922 Russian physicist A. Friedmann (1888-1925) develops a cosmologic model that represents a spatially homogeneous (same properties anywhere in the space) and isotropic (same properties in any direction in the space) universe. This universe expands from the beginning, when the volume is zero, to a maximum volume and, then, it contracts, reaching singularity again (volume equal to zero). The source of this geometry is a perfect fluid whose energy is incoherently distributed, without interaction between its parts (that is, without pressure).
- 1924 Friedmann publishes a second cosmologic model, similar to that from 1922, but with an important difference: in this new solution, the three-dimensional space structure allows the total volume of the universe to grow indefinitely.
- 1927 Belgian priest and physicist G. Lematre (1894-1966) constructs a cosmologic model representing an expanding universe, containing matter, radiation, and cosmologic constant. He associates the initial sin-

gularity of this model to the notion of primordial atom, presenting a cosmogonist hypothesis according to which the Universe would have resulted from the radioactive disintegration of an atom. Nearly thirty years later, this explosion reappears in cosmologic imagination as represented by the Big Bang scenario. American mathematician E.Kasner (1878-1955) builds a solution of Einsteins equations, without matter and without cosmologic constant, which represents a spatially homogeneous but anisotropic universe, that is, with distinct properties in different directions.

- 1929 American astronomer E. P. Hubble (1889-1953) deduces the empirical relation of the separation of galaxies from observational data, and introduces the concept of the expanding universe, perhaps the major Cosmology discovery to date. American mathematician and physicist H.P.Robertson (1903-1961) establishes a mathematic form that represents spatially homogeneous and isotropic universes like Friedmanns. This model of universe follows the Cosmologic Principle according to which all spatial points have the same physical and geometric properties.

- 1932 Einstein and de Sitter discover a cosmologic solution similar to Friedmanns, with homogeneous and isotropic space, characterized by a Euclidean geometry. The source of this universe is a perfect fluid without pressure.

- 1933 Bulgarian astronomer F. Zwicky (1898-1974) proposes the concept of dark matter, thanks to observations of local speeds in the galaxies in clusters. Zwicky and collaborators infer that there is much more matter in the Universe than that emitting visible light (stars).

- 1937 A new question appears in Cosmology: traditional thinking is shifted, that is, from how matter influences the global behavior of the Universe to how the Universe influences the very laws of Physics. British physicist P.A.M. Dirac (1902-1984) offers the hypothesis that some of the fundamental constants of Physics (Newtons constant, for instance) could depend on the cosmologic state in which the Universe is (gravitational interaction changes with cosmic evolution). According to this line of thought, in 1967 Russian-American physicist G.Gamow (1904-1968) suggests that an electrons charge could vary with cosmic time (electromagnetic interaction changes with cosmic evolution). In 1972, Brazilian physicist M.Novello redirects this analysis by arguing that the fundamental constants of Physics were not the ones to depend on cosmic time, but rather the very mechanisms of interaction. As an example to this orientation, he then suggests that, in the processes of matter disintegration via weak interaction, the violation of parity would depend on the stage of evolution of the Universe (weak interaction changes with cosmic evolution).

- 1949 Austrian mathematician K.Gdel (1906-1978) shows that the equations of General Relativity enable the generation of geometries with closedtime type of curves, that is, pathways that lead into the past. From then on, the causality structure and the notion of global cosmic time receive profound criticism which has not yet been resolved by General Relativity to date.

- 1967 Russian physicist A. Sakharov (1921-1989) proposes a model of elementary particles that can explain the matter-antimatter asymmetry of the fundamental constituents of baryonic matter (as the proton and neutron) existing in the Universe. 1941 A. MacKellar observes the first data on the existence of a sea of photons in thermodynamic equilibrium like a thermal spectrum (black body) at 2.3o Kelvin. These data were ignored for more than twenty years partly because of the World War II conjuncture and were only observed again in the 1960s by two American radio-astronomers.

- 1963 American radio-astronomers A. Penzias and R. Wilson observe the existence of a background cosmic radiation, constituted by a sea of photons in thermodynamic equilibrium like a thermal spectrum (black body) at 2.7o Kelvin, thus proving the phenomenon observed by MacKellar in 1941. Background cosmic radiation is considered an evidence of the Big Bang scenario.

- 1970 V.C. Rubin and W.K. Ford find evidence of the dark matter as they studied the rotation curve of stars in galaxies near the Milky Way.

- 1972 The first Cosmology and Gravitation Group is created in Brazil at the Brazilian Center for Physical Research, CBPF.

- 1977 In order to explain the abundance of light chemicals (hydrogen, helium etc.) and the different structure scales of the Universe, B. W. Lee and S. Weinberg, in the 1970s, and, as a complement to that, the endeavors by J.R. Bond, G. Efstathiou, J. Silk develop in the 1980s the concept of non-baryonic dark matter, that is, dark matter not composed of photons, neutrons, and electrons like ordinary matter.

- 1978 The I Brazilian School of Cosmology and Gravitation (BSCG) is held at CBPF-RJ. From then on, these meetings have been held every two years, where the research from Cosmology, Gravitation, Astrophysics, and related areas is presented. A. Penzias and R. Wilson are awarded the Nobel Prize for having discovered the cosmic background radiation.

- 1979 Brazilian physicists M. Novello and J. M. Salim develop the first cosmologic model with analytical solution that possesses bouncing, that is, there would be a previous collapse phase of the Universe where the volume would have reduced in time, reaching a minimum value, and then expanded again. The sources of this geometry are non-linear photons. In the same year, Russian physicists V. Melnikov and S. V. Orlov propose another cosmologic model with bouncing, whose sources are quantized scalar fields (spontaneous break of symmetry).

- 1981 Inflationary Universe Model The proposal of an inflationary model appears, re-updating the importance of the cosmologic constant in a brief historical period of the evolution of the Universe. Inflation of the Universe is the existence of a period of extremely accelerated geometric expansion, which would have occurred next to the singularity in Friedmanns model.

- 1982 Canadian cosmologist J.E. Peebles relates the evolution of small changes in the temperature of cosmic background radiation to the creation of structures such as galaxies and galaxy clusters, taking dark matter and initial fluctuations into account.

- 1983 J. Huchra, M. Davis, D. Latham, and J. Tonry manage to map, for the first time, the distribution of ordinary matter in large scales in the Universe.

- 1987 Gravitational Lenses Discovery of the first gigantic arcs formed by gravitational lensing. Besides proving the deviation of light by gravity, the study of this phenomenon confirms the presence of dark matter in clusters of galaxies [the term gravitational lens is given to any body of matter that can produce change to the trajectory of light passing by as a result of the gravitational force exercised by this body].

- 1990 Launching the Hubble Space Telescope One of the main objectives of this space mission was to determine the current expansion rate of the Universe, named Hubble parameter. The satellite is now used for countless cosmologic studies.

- 1998 Acceleration of the Universe Measures of luminosity and redshift of supernova star explosions suggest strong evidence that the Universe would have suffered a transition and is currently going through a phase of accelerated expansion.

- 21st Century. The observation that the Universe is in accelerated expansion has created a serious problem for the Theory of General Relativity. According to the GR, the cause of this acceleration is associated to a substance with extravagant characteristics that has conventionally been named ”dark energy”. This dark energy seems to be the dominant substance in the Universe, though ”what it is” and ”what type of energy it is” are not precisely known. Dark energy and dark matter are the observed phenomena that most directly demonstrate that the current Theories of Elementary Particles and Gravitation are either incorrect or incomplete. Cosmic observations in the 21st Century show that we should seriously consider the hypothesis that Einstein's Theory of Gravitation could be modified, which allows for the potential appearance of a new Cosmology, since every new theory of gravitation founds a new Cosmoloy.

8 Appendices

9 Non linear Electrodynamics

M. NOVELLO, J M SALIM AND S E P BERGLIAFFA

9.1 Introduction

In recent years, there has been a growing interest in models that mimic in the laboratory some features of gravitation. The actual realization of these models relies on systems that are very different in nature: ordinary non-viscous fluids, super-fluids, flowing and non-flowing dielectrics, non-linear electromagnetism in vacuum, and Bose-Einstein condensates. The basic feature shared by these systems is that the behavior of the fluctuations around a background solution is governed by an "effective metric". More precisely, the particles associated to the perturbations do not follow geodesics of the background space-time but of a Lorentzian geometry described by the effective metric, which depends on the background solution. It is important to notice that only some kinematical aspects of general relativity can be imitated by this method, but not its dynamical features.

By use of this analogy, the geometrical tools of General Relativity can be used to study some condensed matter systems. More importantly perhaps is the fact that the analogy has permitted the simulation of several configurations of the gravitational field, such as wormholes and closed space-like curves for photons, and warped spacetimes for phonons. Particular attention has been paid to analog black holes, because these would emit Hawking radiation exactly as the gravitational black holes do, and are obviously much easier to generate in the laboratory. The fact that analog black holes emit thermal radiation was shown first by Unruh in the case of dumb black holes, and it is the prospect of observing this radiation (thus testing the hypothesis that the thermal emission is independent of the physics at arbitrarily short wavelengths) that motivates the quest for a realization of analog black holes in the laboratory. Let us emphasize that the actual observation of the radiation is a difficult task from the point of view of the experiment, if only because of the extremely low temperatures involved. In the case of a quasi one-dimensional flow of a Bose-Einstein condensate for instance, the temperature of the radiation would be around 70 nK, which is comparable but lower than the temperature needed form the condensate.

We shall begin by presenting the basics of the idea of the effective geometry by studying a simple case: nonlinear electromagnetism. Later on we shall analyze another example: photons in a flowing dielectric medium. We shall see that, in analogy to the most general nonlinear electromagnetic case, the photons experience bi-refringence and bi-metricity. Then we show that is possible to build a static and spherically symmetric analog black hole, generated by a *flowing* isotropic dielectric that depends on an applied electric field. We give a specific example, in which the radius of the horizon and the temperature depend on three parameters (the zeroth order permittivity, the charge that generates the external field, and the linear susceptibility) instead of depending only on the zeroth order permittivity. As we shall show another feature of this black hole is that there is a new term in the surface gravity (and hence in the temperature of Hawking radiation), in addition to the usual term proportional to the acceleration of the fluid. This new term depends exclusively on the dielectric properties of the fluid, and it might give an opportunity to get Hawking radiation with temperature higher than that reported up to date.

9.2 The effective metric

Historically, the first example of the idea of effective metric was presented by W. Gordon in 1923. In modern language, the wave equation for the propagation of light in a moving nondispersive medium, with slowly varying refractive index n and 4-velocity u^μ:

$$\left[\partial_\alpha \partial^\alpha + (n^2 - 1)(u^\alpha \partial_\alpha)^2\right] F_{\mu\nu} = 0.$$

Taking the geometrical optics limit, the Hamilton-Jacobi equation for light rays can be written as $g^{\mu\nu} k_\mu k_\nu = 0$ where

$$g^{\mu\nu} = \eta^{\mu\nu} + (n^2 - 1) u^\mu u^\nu \tag{9.2.1}$$

is the effective metric for this problem. It must be noted that only photons in the geometric optics approximation move on geodesics of $g^{\mu\nu}$: the particles that compose the fluid couple instead to the background Minkowskian metric.

Let us study now in detail the example of nonlinear electromagnetism. We start with the action

$$S = \int \sqrt{-\gamma}\, L(F)\, d^4x, \tag{9.2.2}$$

where $F \equiv F^{\mu\nu}F_{\mu\nu}$ and L is an arbitrary function of F. Notice that γ is the determinant of the background metric, which we take in the following to be that of flat spacetime, but the same techniques can be applied when the background is curved. Varying this action w.r.t. the potential A_μ, related to the field by the expression

$$F_{\mu\nu} = A_{\mu;\nu} - A_{\nu;\mu} = A_{\mu,\nu} - A_{\nu,\mu},$$

we obtain the Euler-Lagrange equations of motion (EOM)

$$(\sqrt{-\gamma}\, L_F F^{\mu\nu})_{;\nu} = 0, \tag{9.2.3}$$

where L_F is the functional derivative $L_F \equiv \frac{\delta L}{\delta F}$. In the particular case of a linear dependence of the Lagrangian with the invariant F we recover Maxwell's equations of motion.

As mentioned in the Introduction, we want to study the behavior of perturbations of these EOM around a fixed background solution. Instead of using the traditional perturbation method, we shall use a more elegant method set out by Hadamard. In this method, the propagation of low-energy photons are studied by following the evolution of the wave front, through which the field is continuous but its first derivative is not. To be specific, let Σ be the surface of discontinuity defined by the equation

$$\Sigma(x^\mu) = constant.$$

The discontinuity of a function J through the surface Σ will be represented by $[J]_\Sigma$, and its definition is

$$[J]_\Sigma \equiv \lim_{\delta \to 0^+} \left(J|_{\Sigma+\delta} - J|_{\Sigma-\delta} \right).$$

The discontinuities of the field and its first derivative are given by

$$[F_{\mu\nu}]_\Sigma = 0, \qquad [F_{\mu\nu,\lambda}]_\Sigma = f_{\mu\nu}k_\lambda, \tag{9.2.4}$$

where the vector k_λ is nothing but the normal to the surface Σ, that is, $k_\lambda = \Sigma_{,\lambda}$.

To set the stage for the nonlinear case, let us first discuss the propagation in Maxwell's electrodynamics, for which $L_{FF} = 0$. The EOM then reduces to $F^{\mu\nu}_{;\nu} = 0$, and taking the discontinuity we get

$$f^{\mu\nu}k_\nu = 0. \tag{9.2.5}$$

The other Maxwell equation is given by $F^{*\ \ ;\nu}_{\mu\nu} = 0$ or equivalently,

$$F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = 0. \tag{9.2.6}$$

The discontinuity of this equation yields

$$f_{\mu\nu}k_\lambda + f_{\nu\lambda}k_\mu + f_{\lambda\mu}k_\nu = 0. \qquad (9.2.7)$$

Multiplying this equation by k^λ gives

$$f_{\mu\nu}k^2 + f_{\nu\lambda}k^\lambda k_\mu + f_{\lambda\mu}k^\lambda k_\nu = 0, \qquad (9.2.8)$$

where $k^2 \equiv k_\mu k_\nu \gamma^{\mu\nu}$. Using the orthogonality condition from previous equation it follows that

$$f^{\mu\nu}k^2 = 0 \qquad (9.2.9)$$

Since the tensor associated to the discontinuity cannot vanish (we are assuming that there is a true discontinuity!) we conclude that the surface of discontinuity is null w.r.t. the metric $\gamma^{\mu\nu}$. That is,

$$k_\mu k_\nu \gamma^{\mu\nu} = 0. \qquad (9.2.10)$$

It follows that $k_{\lambda;\mu}k^\lambda = 0$, and since the vector of discontinuity is a gradient,

$$k_{\mu;\lambda}k^\lambda = 0. \qquad (9.2.11)$$

This shows that the propagation of discontinuities of the electromagnetic field, in the case of Maxwell's equations (which are linear), is along the null geodesics of the Minkowski background metric.

Let us apply the same technique to the case of a nonlinear Lagrangian for the electromagnetic field, given by $L(F)$. Taking the discontinuity of the EOM, we get

$$L_F f^{\mu\nu}k_\nu + 2\eta\; L_{FF}\; F^{\mu\nu}k_\nu = 0, \qquad (9.2.12)$$

where we defined the quantity η by $F^{\alpha\beta}f_{\alpha\beta} \equiv \eta$. Note that contrary to the linear case in which the discontinuity tensor $f_{\mu\nu}$ is orthogonal to the propagation vector k^μ, here there is a complicated relation between the vector $f^{\mu\nu}k_\nu$ and quantities dependent on the background field. This is the origin of a more involved expression for the evolution of the discontinuity vector, as we shall see next. Multiplying equation (9.2.8) by $F^{\mu\nu}$ we obtain

$$\eta\; k^2 + F^{\mu\nu}f_{\nu\lambda}k^\lambda k_\mu + F^{\mu\nu}f_{\lambda\mu}k^\lambda k_\nu = 0. \qquad (9.2.13)$$

Now we substitute in this equation the term $f^{\mu\nu}k_\nu$ from Eqn.(9.2.12), and we arrive at the expression

$$\eta k^2 - 2\frac{L_{FF}}{L_F}\eta(F^{\mu\lambda}k_\mu k_\lambda - F^{\lambda\mu}k_\mu k_\lambda), \qquad (9.2.14)$$

which can be written as $g^{\mu\nu}k_\mu k_\nu = 0$, where

$$g^{\mu\nu} = L_F \gamma^{\mu\nu} - 4L_{FF}\, F^{\mu\alpha} F_\alpha{}^\nu. \tag{9.2.15}$$

We then conclude that

> The low-energy photons of a *nonlinear* theory of electrodynamics with $L = L(F)$ do not propagate on the null cones of the background metric but on the null cones of an *effective* metric, generated by the self-interaction of the electromagnetic field.

This statement is always true in case of Lagrangians depending only of the invariant F. For Lagrangians that depend also of F^*, there may be some special cases in which the propagation coincides with that in Minkowski. Another feature of the more general case $L = L(F, F^*)$ is that bi-refringence is present. That is, the two polarization states of the photon propagate in a different way. In some special cases, there is also bi-metricity (one effective metric for each state). Even more special cases (such as Born-Infeld electrodynamics) exhibit only a single metric. Some of these features are present in our next example.

9.3 Effective metric in flowing fluids with zero vorticity

Another example in which an effective metric arises naturally is that of fluid dynamics for inviscid fluids. The equations decribing this system are the continuity equation,

$$\partial_t \rho + \vec{\nabla}.(\rho \vec{v}) = 0$$

and Euler's equation,

$$\rho(\partial_t \vec{v} + (\vec{v}.\vec{\nabla})\vec{v}) = -\vec{\nabla} p - -\rho \vec{\nabla}\Phi.$$

If we assume that assuming that there is no vorticty, the velocity of the fluid can be expressed in terms of a potential:

$$\vec{v} = -\vec{\nabla}\psi.$$

If we also assume that the fluid is barotropic, that is

$$\vec{\nabla} h = \frac{1}{\rho}\vec{\nabla} p,$$

Euler eqn. reduces to

$$-\partial_t\psi + h + \frac{1}{2}(\vec{\nabla}\psi)^2 + \psi + \Phi = 0 \tag{9.3.1}$$

Linearize the EOM around some assumed background using

$$\rho = \rho_0 + \epsilon\rho_1 + O(\epsilon^2)$$

and similar developments for p and ψ_1 (the background quantities have a 0 subindex).

Keeping up to first order in ϵ, we get from the linearized EOM:

$$-\partial_t\left(\frac{\partial\rho}{\partial p}\rho_0(\partial_t\psi_1 + \vec{v}_0.\vec{\nabla}\psi_1)\right) + \vec{\nabla}.\left(\rho_0\vec{\nabla}\psi_1 - \frac{\partial\rho}{\partial p}\rho_0\ \vec{v}_0(\partial_t\psi_1 + \vec{v}_0.\vec{\nabla}\psi_1)\right) = 0$$

Introducing the velocity of sound $c_s^{-2} = \frac{\partial\rho}{\partial p}$, and the metric

$$g_{\mu\nu} = \frac{\rho_0}{c_s}\begin{pmatrix} -(c_s^2 - v_0^2) & \vdots & -v_0^j \\ \cdots & . & \cdots \\ -v_0^j & \vdots & \delta_{ij} \end{pmatrix}$$

We can write the wave equation

$$\triangle\psi_1 = 0, \tag{9.3.2}$$

where $\triangle$ is the d'Alembertian in the geometry $g_{\mu\nu}$:

$$\triangle\psi_1 = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\ g^{\mu\nu}\partial_\nu\psi_1), \tag{9.3.3}$$

The scalar field ψ_1 moves in an effective curved spacetime, in which the geometry depends on the background fluid.

Many of the notions of GR (like horizon and ergosphere) can be applied in this context. In particular, it is rather easy to generate an analog black hole in this model, and it can be shown that this analog black hole emits Hawking radiation.

9.3.1 Effective metric(s) in the presence of a dielectric

We now move to another interesting case where the effective geometry is useful to study the motion of low-energy photons. We shall analyze the propagation of such photons in a nonlinear medium. Let us define first the antisymmetric tensors $F_{\mu\nu}$ and $P_{\mu\nu}$, which are convenient to represent the electromagnetic field when material media are present. These tensors can be expressed in terms of the strengths (E, H) and the excitations (D, B) of the electric and magnetic fields as

$$\begin{aligned} F_{\mu\nu} &= v_\mu E_\nu - v_\nu E_\mu - \eta_{\mu\nu}{}^{\alpha\beta} v_\alpha B_\beta, \\ P_{\mu\nu} &= v_\mu D_\nu - v_\nu D_\mu - \eta_{\mu\nu}{}^{\alpha\beta} v_\alpha H_\beta. \end{aligned}$$

where v_μ represents the 4-velocity of an arbitrary observer (which we will take later as co-moving with the fluid). The Levi-Civita tensor introduced above is defined in such way that $\eta^{0123} = +1$ in Cartesian coordinates. Since the electric and magnetic fields are space-like vectors, the notation $E^\alpha E_\alpha \equiv -E^2, H^\alpha H_\alpha \equiv -H^2$ will be used. We will consider here media with properties determined only by the tensors $\epsilon_{\alpha\beta}$ and $\mu_{\alpha\beta}$ (*i.e.* media with null magneto-electric tensor), which relate the electromagnetic excitations to the field strengths by the constitutive laws,

$$D_\alpha = \epsilon_\alpha{}^\beta(E,H)E_\beta, \qquad B_\alpha = \mu_\alpha{}^\beta(E,H)H_\beta. \tag{9.3.4}$$

In order to get the effective metric, we shall use Hadamard's method as in the previous section. By taking the discontinuity of the field equations ${}^*F^{\mu\nu}{}_{;\nu} = 0$ and $P^{\mu\nu}{}_{;\nu} = 0$, and assuming that

$$\epsilon^{\mu\beta} = \epsilon(E)(\gamma^{\mu\beta} - v^\mu v^\beta), \tag{9.3.5}$$

and

$$\mu^{\mu\beta} = \mu_0(\gamma^{\mu\beta} - v^\mu v^\beta), \tag{9.3.6}$$

with $\mu_0 = \text{const.}$, we get the following equations:

$$\epsilon(k.e) - \frac{\epsilon'}{E}(E.e)(k.E) = 0, \tag{9.3.7}$$

$$\mu_0(k.h) = 0, \tag{9.3.8}$$

$$\epsilon(k.v)e^\mu - \frac{\epsilon'}{E}E^\alpha e_\alpha (k.v)E^\mu + \eta^{\mu\nu\alpha\beta}k_\nu v_\alpha h_\beta = 0, \tag{9.3.9}$$

$$\mu_0(k.v)h^\mu - \eta^{\mu\nu\alpha\beta}k_\nu v_\alpha e_\beta = 0, \tag{9.3.10}$$

where k^μ is the wave propagation vector, ϵ' is the derivative of ϵ w.r.t. E, and

$$[E_{\mu,\lambda}]_\Sigma = e_\mu\, k_\lambda, \qquad [H_{\mu,\lambda}]_\Sigma = h_\mu\, k_\lambda.$$

Note in particular that previous equation shows that the vectors k^μ and e^μ are not always orthogonal, as would be the case if ϵ' was zero. Substituting in the previous equation, we get

$$Z^{\mu\beta} e_\beta = 0, \tag{9.3.11}$$

where the matrix Z is given by

$$Z^{\mu\beta} = \left[k^2 + (k.v)^2(\mu_0\epsilon - 1)\right]\gamma^{\mu\beta} - \mu_0\frac{\epsilon'}{E}(k.v)^2 E^\mu E^\beta + (v.k)(v^\mu k^\beta + k^\mu v^\beta) - \left[\epsilon\mu_0(k.v) + k^2\right] v^\mu v^\beta \tag{9.3.12}$$

Non-trivial solutions can be found only for cases in which $\det\left|Z^{\mu\beta}\right| = 0$ (this condition is a generalization of the well-known Fresnel equation).

This equation can be solved by expanding e_ν as a linear combination of the four linearly independent vectors v_ν, E_ν, k_ν and $\eta_{\alpha\beta\mu\nu}v^\alpha E^\beta k^\mu$ (the particular case in which the vectors v_ν, E_ν and k_ν are coplanar will be examined below). That is,

$$e_\nu = \alpha E_\nu + \beta\eta_{\alpha\lambda\mu\nu}v^\alpha E^\lambda k^\mu + \gamma k_\nu + \delta v_\nu. \tag{9.3.13}$$

Notice that taking the discontinuity of $E^\mu{}_{,\lambda}$ we can show that $(e.v) = 0$. This restriction imposes a relation between the coefficients of Eqn.(9.3.13):

$$\delta = -\gamma(k.v)$$

With the expression given in Eqn.(9.3.13), Eqn. (9.3.11) reads

$$\alpha\left[k^2 - \left(1 - \mu_0\,(\epsilon\,E)'\right)(k.v)^2\right] - \gamma\left[\mu_0(k.v)^2\frac{1}{E}\,\epsilon'^\alpha k_\alpha\right] = 0,$$
$$\alpha E^\mu k_\mu + \gamma(1-\mu_0\epsilon)(k.v)^2 + \delta(k.v) = 0,$$
$$\alpha(k.v)E^\mu k_\mu + \gamma(k.v)k^2 + \delta\left[k^2 + \mu_0\epsilon\,(k.v)^2\right] = 0,$$
$$\beta\left[k^2 - (1-\mu_0\epsilon)(k.v)^2\right] = 0.$$

The solution of this system results in the following dispersion relations:

$$k_-^2 = (k.v)^2\left[1 - \mu_0(\epsilon\,E)'\right] + \frac{1}{\epsilon E}\,\epsilon'^\alpha E^\beta k_\alpha k_\beta, \tag{9.3.14}$$
$$k_+^2 = [1 - \mu_0\epsilon(E)](k.v)^2. \tag{9.3.15}$$

They correspond to the propagation modes

$$e_\nu^- = \rho^-\left\{\mu_0\,\epsilon(k.v)^2 E_\nu + E^\alpha k_\alpha[k_\nu - (k.v)v_\nu]\right\}, \tag{9.3.16}$$
$$e_\nu^+ = \rho^+\,\eta_{\alpha\lambda\mu\nu}v^\alpha E^\lambda k^\mu, \tag{9.3.17}$$

where ρ^- and ρ^+ are arbitrary constants. The labels "+" and "−" refer to the ordinary and extraordinary rays, respectively. Eqns. that govern the propagation of photons in the medium characterized by $\mu = \mu_0$ =const., and $\epsilon = \epsilon(E)$. They can be rewritten as $g^{\mu\nu}_{\pm} k_\mu k_\nu = 0$, where we have defined the effective geometries

$$g^{\mu\nu}_{(-)} = \gamma^{\mu\nu} - \left[1 - \mu_0 \, (\epsilon \, E)'\right] v^\mu v^\nu - \frac{1}{\epsilon E} \, \epsilon'^\mu E^\nu, \tag{9.3.18}$$

$$g^{\mu\nu}_{(+)} = \gamma^{\mu\nu} - [1 - \mu_0 \, \epsilon] v^\mu v^\nu. \tag{9.3.19}$$

The metric given above was derived previously, while the second metric very much resembles the metric derived by Gordon. The difference is that in the case under consideration, ϵ is a function of the modulus of the external electric field, while Gordon worked with a constant permeability.

We see then that in this example each polarization state has its own dispersion relation, so there is bi-refringence. There is also bi-metricity, because each type of photon moves according a different metric.

Let us discuss now a particular instance in which the vectors used as a basis in previous Eqn. are not linearly independent. If we assume that

$$E^\mu = a k^\mu + b v^\mu, \tag{9.3.20}$$

then vectors e^μ, k^μ, and v^μ are coplanar. In this case, the basis chosen is not appropriate. Notice however that if we assume that e^μ is a combination of vectors that are perpendicular to k^μ, so that $(e.k) = 0$, then $(E.e) = 0$. The converse is also true: if $(E.e) = 0$, then $(k.e) = 0$. For this particular case, in which e^μ is perpendicular to v^μ, k^μ (and consequently to E^μ), imply that

$$\left[k^2 + (k.v)^2 (\mu_0 \epsilon - 1)\right] \; e^\mu = 0$$

We see then that in the case in which $E_\mu = a k_\mu + b v_\mu$, Fresnel's equation determines that the polarization of the photons is perpendicular to the direction of propagation and to the velocity of the fluid. Moreover, the motion of these photons is governed by the metric $g^{\mu\nu}_{+}$. For instance, if the electric field, the velocity of the fluid, and the direction of propagation are all radial, then the polarization is in the plane perpendicular to the propagation, and the two polarization modes feel the same geometry.

9.4 The Analog Black Hole

We shall show in this section that the system described by the effective metrics given above can be used to produce an analog black hole. It will be convenient to rewrite at this point the inverse of the effective metric using a different notation:

$$g_{\mu\nu}^{(-)} = \gamma_{\mu\nu} - \frac{v_\mu v_\nu}{c^2}(1-f) + \frac{\xi}{1+\xi}\, l_\mu l_\nu, \tag{9.4.1}$$

where we have defined the quantities

$$f \equiv \frac{1}{c^2\mu_0\epsilon(1+\xi)}, \qquad \xi \equiv \frac{\epsilon' E}{\epsilon}, \qquad l_\mu \equiv \frac{E_\mu}{E}.$$

Note that $\epsilon = \epsilon(E)$. We have introduced here the velocity of light c, which was set to 1 before. Taking a Minkowskian background in spherical coordinates, and

$$v_\mu = (v_0, v_1, 0, 0), \qquad E_\mu = (E_0, E_1, 0, 0), \tag{9.4.2}$$

we get for the effective metric,

$$g_{00}^{(-)} = 1 - \frac{v_0^2}{c^2}\,(1-f) + \frac{\xi}{1+\xi}\, l_0^2, \tag{9.4.3}$$

$$g_{11}^{(-)} = -1 - \frac{v_1^2}{c^2}\,(1-f) + \frac{\xi}{1+\xi}\, l_1^2, \tag{9.4.4}$$

$$g_{01}^{(-)} = -\frac{v_0 v_1}{c^2}\,(1-f) + \frac{\xi}{1+\xi}\, l_0\, l_1, \tag{9.4.5}$$

and $g_{22}^{(-)}$ and $g_{33}^{(-)}$ as in Minkowski spacetime. The vectors v_μ and l_μ satisfy the constraints

$$v_0^2 - v_1^2 = c^2, \tag{9.4.6}$$

$$l_0^2 - l_1^2 = -1, \tag{9.4.7}$$

$$v_0 l_0 - v_1 l_1 = 0. \tag{9.4.8}$$

This system of equations can be solved in terms of v_1, and the result is

$$v_0^2 = c^2 + v_1^2, \tag{9.4.9}$$

$$l_0^2 = \frac{v_1^2}{c^2}, \qquad l_1^2 = \frac{c^2 + v_1^2}{c^2}. \tag{9.4.10}$$

Now we can rewrite the metric in terms of $\beta \equiv v_1/c$, a definition which coincides with the usual one for small values of v_1. The explicit expression of the metric coefficients is:

$$g_{00}^{(-)} = \frac{1 - \beta^2(c^2\mu_0\epsilon - 1)}{c^2\mu_0(\epsilon + \epsilon' E)}, \tag{9.4.11}$$

$$g_{01}^{(-)} = \beta\sqrt{1+\beta^2}\,\frac{1-c^2\mu_0\epsilon}{c^2\mu_0(\epsilon+\epsilon' E)}, \qquad (9.4.12)$$

$$g_{11}^{(-)} = \frac{\beta^2 - c^2\mu_0\epsilon(1+\beta^2)}{c^2\mu_0(\epsilon+\epsilon' E)}. \qquad (9.4.13)$$

¿From Eqn.(9.4.11) it is easily seen that, depending on the function $\epsilon(E)$, this metric has a horizon at $r = r_h$, given by the condition $g_{00}(r_h) = 0$ or, equivalently,

$$\left(c^2\mu_0\epsilon - \frac{1}{\beta^2}\right)\Bigg|_{r_h} = 1. \qquad (9.4.14)$$

The metric given above resembles the form of Schwarzschild's solution in Painlevé-Gullstrand coordinates:

$$ds^2 = \left(1-\frac{2GM}{r}\right)dt^2 \pm 2\sqrt{\frac{2GM}{r}}\,dr\,dt - dr^2 - r^2 d\Omega^2. \qquad (9.4.15)$$

With the coordinate transformation

$$dt_P = dt_S \mp \frac{\sqrt{2GM/r}}{1-\frac{2GM}{r}}\,dr, \qquad (9.4.16)$$

the line element given in above equation can be written in Schwarzschild's coordinates. The "+" sign covers the future horizon and the black hole singularity.

The effective metric looks like the metric in Eqn.(9.4.15). In fact, it can be written in Schwarzschild's coordinates, with the coordinate change

$$dt_{PG} = dt_S - \frac{g_{01}(r)}{g_{00}(r)}dr. \qquad (9.4.17)$$

Using this transformation with the metric coefficients given in Eqns.(9.4.11) and (9.4.12), we get the expression of $g_{11}^{(-)}$ in Schwarzschild coordinates:

$$g_{11}^{(-)} = -\frac{\epsilon(E)}{(1-\beta^2[c^2\mu_0\epsilon(E)-1])(\epsilon(E)+\epsilon(E)' E)}. \qquad (9.4.18)$$

Note that $g_{01}^{(-)}$ is zero in the new coordinate system, while $g_{00}^{(-)}$ is still given by Eqn.(9.4.11). Consequently, the position of the horizon does not change, and is still given by Eqn.(9.4.14).

Working in Painlevé-Gullstrand coordinates, we have shown that the metric for the "−" polarization describes a Schwarzschild black hole if Eqn.(9.4.14) has a solution. Afterwards we have rewritten the "−" metric in the more familiar Schwarzschild coordinates. Let us consider now photons with the other polarization. They "see" the metric given by Eqn.(9.3.19), whose inverse is given by:

$$g^{(+)}_{\mu\nu} = \gamma_{\mu\nu} - \frac{v_\mu v_\nu}{c^2}\left(1 - \frac{1}{c^2\mu_0\epsilon(E)}\right). \tag{9.4.19}$$

Using this equation and Eqns.(9.4.9) and (9.4.10) it is straightforward to show that

$$g^{(+)}_{00} = 1 - \left(1+\beta^2\right)\left(1 - \frac{1}{c^2\mu_0\epsilon(E)}\right), \tag{9.4.20}$$

$$g^{(+)}_{01} = -\beta\sqrt{1+\beta^2}\left(1 - \frac{1}{c^2\mu_0\epsilon(E)}\right), \tag{9.4.21}$$

$$g^{(+)}_{11} = -1 - \beta^2\left(1 - \frac{1}{c^2\mu_0\epsilon(E)}\right). \tag{9.4.22}$$

This metric also corresponds to a Schwarzschild black hole, for some $\epsilon(E)$ and β. Comparing Eqns.(9.4.11) and (9.4.20) we see that the horizon of both analog black holes is located at r_h, given by Eqn.(9.4.14).

By means of the coordinate change defined by Eqn.(9.4.17), we can write this metric in Schwarzschild's coordinates. The relevant coefficients are given by

$$g^{(+)}_{00} = \frac{1+\beta^2(1-c^2\mu_0\epsilon(E))}{c^2\mu_0\epsilon(E)}, \tag{9.4.23}$$

$$g^{(+)}_{11} = -\frac{1}{1+\beta^2(1-c^2\mu_0\epsilon(E))}. \tag{9.4.24}$$

It is important to stress then that *the horizon is located at r_h given by Eqn.(9.4.14) for photons with any polarization*. Moreover, the motion of the photons in both geometries will be qualitatively the same, as we shall show below.

9.5 An example

We have not specified up to now the functions $\epsilon(E)$ and $E(r)$ that determine the dependence of the coefficients of the effective metrics with the coordinate r. From now on we assume a linear $\epsilon(E)$, a type of behaviour which is exhibited for instance by electrorheological fluids. Specifically, we take

$$\epsilon(E) = \epsilon_0(\overline{\chi} + \chi^{(2)}E(r)), \tag{9.5.1}$$

with $\overline{\chi} = 1 + \chi^{(1)}$. The nontrivial Maxwell's equation then reads

$$\left(\sqrt{-\gamma}\,\epsilon(r)F^{01}\right)_{,1} = 0. \tag{9.5.2}$$

Taking into account that $(F^{01})^2 = \frac{E^2}{c^2}$, we get as a solution of Eqn.(9.5.2) for a point source in a flat background in spherical coordinates

$$F^{01} = \frac{-\overline{\chi} \pm \sqrt{\overline{\chi}^2 + 4\chi^{(2)} Q/\epsilon_0 r^2}}{2c\chi^{(2)}}. \tag{9.5.3}$$

Let us consider a particular combination of parameters: $\chi^{(2)} > 0$, $Q > 0$ and the "$+$" sign in front of the square root in F^{01}, in such a way that $E > 0$ for all r. To get more manageable expressions for the metric, it is convenient to define the function $\sigma(r)$:

$$E(r) \equiv \frac{\overline{\chi}}{2\chi^{(2)}}\, \sigma(r) \tag{9.5.4}$$

where

$$\sigma(r) = -1 + \frac{1}{r}\sqrt{r^2 + q} \tag{9.5.5}$$

and

$$q = \frac{4\chi^{(2)} Q}{\epsilon_0 \overline{\chi}^2}. \tag{9.5.6}$$

In terms of σ, the metrics take the form

$$ds^2_{(-)} = \frac{2 - \beta^2\, [\, \overline{\chi}\, (\sigma(r) + 2) - 2]}{2\, \overline{\chi}\, (1 + \sigma(r))}\, d\tau^2 - \frac{2 + \sigma(r)}{[2 - \beta^2\, (\overline{\chi}\, (\sigma(r) + 2) - 2)]\, (1 + \sigma(r))}\, dr^2 - r^2 d\Omega^2, \tag{9.5.7}$$

$$ds^2_{(+)} = \frac{2 - \beta^2\, [\, \overline{\chi}\, (\sigma(r) + 2) - 2]}{\overline{\chi}\, (2 + \sigma(r))}\, d\tau^2 - \frac{2}{2 + \beta^2\, [2 - \overline{\chi}\, (\sigma(r) + 2)]}\, dr^2 - r^2 d\Omega^2. \tag{9.5.8}$$

Notice that the (t, r) sectors of these metrics are related by the following expression:

$$ds^2_{(+)} = \Phi(r)\, ds^2_{(-)} \tag{9.5.9}$$

where the conformal factor Φ is given by:

$$\Phi = 2\, \frac{1 + \sigma(r)}{2 + \sigma(r)}$$

We shall study next some features of the effective black hole metrics. It is important to remark that up to this point, the velocity of the fluid v_1 is completely arbitrary; it can even be a function of the coordinate r. We shall assume in the following that v_1 is a constant. This assumption, which will be lifted later on, may seem rather restrictive but it helps to display the main features of the effective metrics in an easy way.

To study the motion of the photons in these geometries, we can use the technique of the effective potential. Standard manipulations show that in the case of a static and spherically symmetric metric, the effective potential is given by

$$V(r) = \varepsilon^2 \left(1 + \frac{1}{g_{00}(r)\, g_{11}(r)}\right) - \frac{L^2}{r^2 g_{11}(r)} \tag{9.5.10}$$

where ε is the energy and L the angular momentum of the photon.

In terms of $\sigma(r)$, and of the impact parameter $b^2 = L^2/\varepsilon^2$, the "small" effective potential $v(r) \equiv V(r)/\varepsilon^2$ for the metric Eqn.(9.5.7) in Schwarzschild coordinates can be written as follows:

$$v^{(-)}(r) = 1 - \frac{2(1+\sigma(r))^2}{2+\sigma(r)} - \frac{b^2}{r^2} \frac{(2-\beta^2\sigma(r))(1+\sigma(r))}{2+\sigma(r)} \tag{9.5.11}$$

A short calculation shows that $v^{(-)}$ is a monotonically decreasing function of β. $b = 1, 3, 5$ (starting from the lowest curve), and $\beta = 0.5$.

The effective potential for the Gordon-like metric can be obtained in the same way. From Eqns.(9.5.10) and (9.5.8) we get

$$v^{(+)}(r) = 1 - \frac{2+\sigma(r)}{2} + \frac{b^2}{2r^2}\, [2 - \beta^2\sigma(r)]. \tag{9.5.12}$$

We see that, in the case of a constant flux velocity, the shape of the effective potential for both metrics qualitatively agrees with that for photons moving on the geometry of a Schwarzschild black hole.

9.6 Surface gravity and temperature

Let us now go back to the more general case of $\beta = \beta(r)$, and calculate the "surface gravity" of our analog black hole. We present first the results for the constant permittivity case. By setting $\epsilon'(E) \equiv 0$ in the metrics Eqns.(9.3.18) and (9.3.19), we regain the example of constant index of refraction It is easy to show that the horizon of the black hole in this case is given by

$$\beta^2(r_h) = \frac{1}{\bar{\chi} - 1}. \tag{9.6.1}$$

The "surface gravity" of a spherically symmetric analog black hole in Schwarszchild coordinates is given by

$$\kappa = \frac{c^2}{2} \lim_{r \to r_h} \frac{g_{00,r}}{\sqrt{|g_{11}|\; g_{00}}}. \tag{9.6.2}$$

For the metrics Eqns.(9.3.18) and (9.3.19) with $\epsilon = \epsilon_0 \bar{\chi}$ and r_h given by Eqn.(9.6.1), the analog surface gravity is

$$\kappa = -\frac{c^2}{2} \frac{1-\bar{\chi}}{\sqrt{\bar{\chi}}} \, (\beta^2)' \Big|_{r_h}. \tag{9.6.3}$$

This equation can be rewritten in terms of the velocity of light in the medium and the refraction index, respectively given by

$$c_m^2 = \frac{1}{\mu_0 \epsilon}, \qquad n = \frac{c}{c_m}. \tag{9.6.4}$$

The result is

$$\kappa = \frac{c^2}{2} \frac{1-n^2}{n} \, (\beta^2)_{,r} \tag{9.6.5}$$

In this expression we can see the influence of the dielectric properties of the fluid (through the index of refraction of the medium) and also of its dynamics through the physical acceleration in the radial direction, given by

$$a_r|_{r_h} = \frac{c^2}{2} \, (\beta^2)' \Big|_{r_h},$$

for $\beta^2(r_h) \ll 1$. This acceleration is a quantity that must be determined solving the equations of motion of the fluid [1].

Going back the the more general case of a linear permittivity, described by the metrics given above and considering that $\beta(r_h) \ll 1$, the radius of the horizon is [2]:

$$r_h^2 = \frac{q\bar{\chi}^2}{4} \beta^4(r_h). \tag{9.6.6}$$

Using the expressions given above, the result for the surface gravity of the "$-$" black hole for $\beta(r_h) \ll 1$ is

$$\kappa^{(-)} = \frac{c^2}{\beta} \left(\frac{1}{\bar{\chi}\sqrt{q}} - \frac{1}{2}(\beta^2)' \right) \Bigg|_{r_h}. \tag{9.6.7}$$

[1] If we set $\beta \equiv 0$ in Eqns.(9.4.11)-(9.4.13), we cease to have a black hole.

[2] Notice that we cannot take the limit $q \to 0$ in this expression or in any expression in which this one has been used.

This equation differs from the surface gravity of the case of constant permittivity (Eqn.(9.6.3)) by the presence of a new term that does not depend on the acceleration of the fluid. To see where this new term comes from, we can go back to the definition of the surface gravity given in Eqn.(9.6.2), and use the fact that in the high frequency limit the velocity of light and the index of refraction in a medium of variable ϵ are still given by Eqn.(9.6.4), replacing the constant permittivity by $\epsilon = \epsilon(E)$. The result is

$$\kappa = \left(\frac{c^2}{2} \frac{1 - n^2(E)}{n(E)} \, (\beta^2)_{,r} + \frac{n(E)\epsilon(E)}{\epsilon(E) + \epsilon(E)' E} \, (c_m^2)_{,r} \right) \Bigg|_{r_h} \tag{9.6.8}$$

In this expression, the first term is the generalization of the case $\epsilon = \text{const.}$ (compare with Eqn.(9.6.5)), which mixes the acceleration of the fluid with its dielectric properties. On the other hand, the second term, which is the new term displayed in Eqn.(9.6.7), is related to the radial variation of the velocity of light in the medium. It is important to point out that the result exhibited in Eqn.(9.6.8) is parallel to that of dumb holes: Unruh found in that case that the surface gravity for constant speed of sound is proportional to the acceleration of the fluid (as in the first term of Eqn.(9.6.8)). This was generalized by Visser, who showed that for a position-dependent velocity of sound a second term appears, coming from the gradients of the speed of sound, in analogy with the second term of Eqn.(9.6.8).

It is easy to show that the these results also apply to the black hole described by the Gordon-like metric. This is not surprising though, because of the conformal relation between the two metrics, given by Eqn.(9.5.9) .

Let us remark once more that the concept of temperature, and indeed that of effective geometry is valid in this context only for low-energy photons, *i.e.* photons with wavelengths long compared to the intermolecular spacing in the fluid. For shorter wavelengths, there would be corrections to the propagation dictated by the effective metric. However, results for other systems (such as dumb black holes and Bose-Einstein condensates) suggest that the phenomenon of Hawking radiation is robust (*i.e.* independent of this "high-energy" physics). Consequently, it makes sense to talk about the temperature of the radiation in these systems.

At first sight it may seem that by choosing an appropriate material and a convenient value of the charge we could obtain a high value of the temperature of the radiation, given by

$$T \equiv \frac{\hbar}{2\pi k_B c} \, \kappa \approx 4 \times 10^{-21} \, \kappa \ \text{Ks}^2/\text{m}. \tag{9.6.9}$$

However, the equation for the surface gravity can be rewritten as [3]

$$\kappa = c^2 \left(\frac{\beta}{2r} - \beta_{,r} \right) \bigg|_{r_h} .$$

We see then that, because $\beta(r_h) \ll 1$, the new term appearing in κ is bound to be very small. In spite of this result, the emergence in the surface gravity of the term due to the variable velocity of light suggests that it may be worth to study if some media with nonlinear dependence on an external electromagnetic field can be used to generate analog black holes whose Hawking radiation could be measured in laboratory.

[3] Note that this equation depends on $\chi^{(2)}$ through the expression for r_h, Eqn.(9.6.6).

10 Einstein linearized equations of GR from Heisenberg dynamics

E.HUGUET and M. NOVELLO

10.1 Introduction

The result presented in this paper is concerned by solutions of linearized equations of Einstein's general relativity (LEGR). These solutions are constructed from a spinor which obeys a non-linear Heisenberg equation (NLHE) (1) satisfying the Inomata condition (2). As they stand these solutions may look somehow artificial, it is thus important to make our motivations clear. The present work originates from our current investigation of a new theory of gravitation recently proposed by one of us (3), and called Spinor Theory of Gravity (STG). Although the solutions presented here are completely independent of the hypothesis underlying STG let us briefly summarize its mean features (details may be found in (3)).

The STG originates mainly on the observation that Einstein theory of gravitation is based on two independent principles: the equivalence principle (EP) and the Einstein's equations for the metric tensor $g_{\mu\nu}$. The first states that the gravitation may be described as a modification of the geometry of space-time, the second comes from the natural assumption that the gravitational field should have a dynamics of its own. In STG this last assumption is relaxed. In fact, STG rely on a very different hypothesis: it postulate the existence of two fundamentals spinors Ψ_N, Ψ_E coupling universally with all forms of matter/energy. The choice of two spinors is motivated by the fact that the metric tensor has ten components. The coupling between spinors and matter/energy takes place consistently with the EP. The spinor fields - through its vector and axial currents - induce an effective metric of the form $g_{\mu\nu} = \eta_{\mu\nu} + \varphi_{\mu\nu}$ where $\eta_{\mu\nu}$ is the usual Minkowskian metric and $\varphi_{\mu\nu}$ depends on the two fundamentals spinors through the basic currents $I_\mu^{(A)} := \overline{\Psi}_A\gamma_\mu\gamma^5\Psi_A$ and $J_\mu^{(A)} := \overline{\Psi}_A\gamma_\mu\psi_A$ with $A = N, E$. It is worth to emphasize that in STG, by contrast to solutions presented hereafter, $\varphi_{\mu\nu}$ is not a perturbation of $\eta_{\mu\nu}$. By contrast to the usual Einstein theory, the induced metric $g_{\mu\nu}$ do not have a proper dynamics: the evolution of the metric is inherited from that of the fundamentals spinors. These obey non-linear Heisenberg equations of motion. Of course, as a proposal, STG has to be confronted to well established results. In particular, one must recover the weak field limit. The solutions presented hereafter were discovered in that context.

10.1.1 Basic properties of the spinor field

Let us consider a four-component spinor $\boldsymbol{\psi}$ and the two associated currents

$$J^{\mu} = \bar{\boldsymbol{\psi}}\gamma^{\mu}\boldsymbol{\psi}, \qquad I^{\mu} = \bar{\boldsymbol{\psi}}\gamma^{\mu}\gamma^{5}\boldsymbol{\psi},$$

where γ^{μ} are the usual Dirac matrices verifying $\{\gamma^{\mu},\gamma^{\nu}\} = 2\eta^{\mu\nu}$ and $\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3$. We assume that the dynamics of $\boldsymbol{\psi}$ is given by the non-linear lagrangian

$$\mathcal{L} = \frac{i}{2}\bar{\boldsymbol{\psi}}\gamma^{\mu}\partial_{\mu}\boldsymbol{\psi} - \frac{i}{2}(\partial_{\mu}\bar{\boldsymbol{\psi}})\gamma^{\mu}\boldsymbol{\psi} - s(A^2+B^2),$$

where $A := \bar{\boldsymbol{\psi}}\boldsymbol{\psi}$, $B := i\bar{\boldsymbol{\psi}}\gamma^5\boldsymbol{\psi}$ and s is a real parameter. This lagrangian leads to the NLHE for $\boldsymbol{\psi}$:

$$i\gamma^{\mu}\partial_{\mu}\boldsymbol{\psi} - 2s(A+iB\gamma^5)\boldsymbol{\psi} = 0. \tag{10.1.1}$$

A lot of simplifications in calculations with non-linear spinors can be obtained using the identity

$$\begin{aligned}(\bar{\boldsymbol{\psi}}Q\gamma_{\mu}\boldsymbol{\psi})\gamma^{\mu}\boldsymbol{\psi} = {} & (\bar{\boldsymbol{\psi}}Q\boldsymbol{\psi})\boldsymbol{\psi} \\ & - (\bar{\boldsymbol{\psi}}Q\gamma^5\boldsymbol{\psi})\gamma^5\boldsymbol{\psi},\end{aligned} \tag{10.1.2}$$

where Q equal to γ^{ν}, γ^5, $\gamma^{\nu}\gamma^5$ or the identity of the Clifford algebra. The above identity is a consequence of the Pauli-Kofink (PK) relations (3). In particular, from (10.1.2) with $Q = I$ and $Q = \gamma^5$ one obtains

$$\begin{aligned} J^2 \equiv J_{\mu}\,J^{\mu} &= A^2 + B^2, \\ I^2 \equiv I_{\mu}\,I^{\mu} &= -A^2 - B^2, \end{aligned}$$

which shows that J_{μ} is time-like, I_{μ}, space-like and that they are orthogonal to each other. In addition the Heisenberg potential appears clearly as a vector current-current coupling.

The Inomata condition

A very interesting class of solutions of NLHE are provided by spinors which satisfy the Inomata condition (2):

$$\partial_{\mu}\boldsymbol{\psi} = (aJ_{\mu} + bI_{\mu}\gamma^5)\boldsymbol{\psi},$$

where a, b are complex numbers. If ψ verify this relation then it provides a solution of the NLHE for $2s = i(a-b)$. This can be checked directly by using the above relation in (10.1.1).

Another important property of Inomata-spinors, which can be shown by an explicit calculation, is that for any combination Γ constructed with such spinor ψ and elements of the Clifford algebra the integrability condition

$$[\partial_\mu, \partial_\nu]\Gamma = 0,$$

holds iff $\mathrm{Re}(a) = \mathrm{Re}(b)$. In particular, setting $r := \mathrm{Re}(a)$ and using the results of the previous section one has

$$\partial_\mu J_\nu = 2r\, C_{\mu\nu}, \qquad \partial_\mu I_\nu = 2r\, D_{\mu\nu},$$

with

$$C_{\mu\nu} := J_\mu J_\nu + I_\nu I_\mu, \qquad D_{\mu\nu} := J_\mu I_\nu + J_\nu I_\mu.$$

A number of identities relating the tensors $C_{\mu\nu}, D_{\mu\nu}$ and the currents J_μ and I_μ can now be obtained by using the above relations and the identity (10.1.2). Those useful for our calculations are:

$$\begin{aligned}
&\eta^{\mu\nu} C_{\mu\nu} = 0, \quad \eta^{\mu\nu} D_{\mu\nu} = 0, \quad C^{\mu\nu} D_{\mu\nu} = 0,\\
&\partial_\mu C^{\mu\nu} = 0, \ \ \partial_\mu D^{\mu\nu} = 0,\\
&C^{\mu\nu} J_\nu = X J^\mu, \quad D^{\mu\nu} J_\nu = X I^\mu,\\
&C^{\mu\nu} I_\nu = -X I^\mu, \quad D^{\mu\nu} I_\nu = -X J^\mu,\\
&C^{\mu\nu} C_{\nu\sigma} = X\,(J^\mu J_\sigma - I^\mu I_\sigma),\\
&D^{\mu\nu} D_{\nu\sigma} = X\,(I^\mu I_\sigma - J^\mu J_\sigma),\\
&C^{\mu\nu} D_{\nu\sigma} = X\,(J^\mu I_\sigma - I^\mu J_\sigma),
\end{aligned}$$

where we have set $X \equiv J^2$.

Linearized Einstein equations in terms of the currents of Inomata-spinors

We are now in position to give new forms to describe the linear equation of General Relativity. Let us consider a small perturbation $\varphi_{\mu\nu}$ to the Minkowski metric $\eta_{\mu\nu}$. The metric tensor thus reads $g_{\mu\nu} = \eta_{\mu\nu} + \varphi_{\mu\nu}$. Using the results of previous sections a straightforward calculation shows that

$$\varphi_{\mu\nu} := \frac{C_{\mu\nu}}{X},$$

satisfy the equation of motion of a massless spin-2 field (LEGR). Indeed, for such field it follows that

$$\Box\varphi_{\mu\nu} - \partial_\alpha(\partial_\mu\varphi_\nu{}^\alpha + \partial_\nu\varphi_\mu{}^\alpha) + \\ + \partial_\mu\partial_\nu\varphi_\alpha{}^\alpha - \eta_{\mu\nu}(\Box\varphi_\alpha{}^\alpha - \partial_\alpha\partial_\beta\varphi^{\alpha\beta}) = 0.$$

Let us note that one can construct other combinations of the vector and the axial currents that satisfy the massless spin-2 equation like, for instance, the quantities $\Omega_{\mu\nu}$ and $\Delta_{\mu\nu}$ defined as:

$$\Omega_{\mu\nu} := 4r\,(1-\alpha)\frac{D_{\mu\nu}}{X^\alpha},$$
$$\Delta_{\mu\nu} := 4r\,\left(\frac{I_\mu I_\nu + J_\mu J_\nu}{X^\beta} - \frac{2\beta}{X^\beta}\,J_\mu J_\nu\right),$$

where α and β are arbitrary parameters. However, a straightforward calculation shows that these are pure gauge that can be transformed away by a coordinate transformation. Indeed, we have

$$\Omega_{\mu\nu} = \partial_\mu\chi_\nu + \partial_\nu\chi_\mu, \qquad \Delta_{\mu\nu} = \partial_\mu\eta_\nu + \partial_\nu\eta_\mu,$$

where

$$\chi_\mu \equiv \frac{I_\mu}{X^\alpha}, \qquad \eta_\mu \equiv \frac{J_\mu}{X^\alpha}.$$

Due to the linearity of the equation we can write the spin-2 field as the combination

$$\Phi_{\mu\nu} = a\,\frac{C_{\mu\nu}}{X} + b\,\Delta_{\mu\nu} + c\,\Omega_{\mu\nu}.$$

One could argue on the degree of generality of such decomposition. Indeed, the two vectors J_ν and I_μ, do not have enough components to describe arbitrary second order symmetric tensor. In usual GR framework this could be corrected by means of a tetrad frame, which contains the necessary number (four) vectors to describe arbitrary metrics. In the present context, this can be achieved by the introduction of a second spinor Ψ_E. This is precisely what occurs in the Spinor Theory of Gravity in which the presence of two spinors allows to describe the full gravitational field. In this vein, the general form of the spin-2 massless field is provided by

$$\begin{aligned}\Phi_{\mu\nu} &= a\,\frac{C_{\mu\nu}}{X} + b\,\Delta_{\mu\nu} + c\,\Omega_{\mu\nu} \\ &+ m\,\frac{z_{\mu\nu}}{X_E} + n\,\delta_{\mu\nu} + q\,\omega_{\mu\nu},\end{aligned}$$

where $z_{\mu\nu}, \delta_{\mu\nu}$ and $\omega_{\mu\nu}$ are constructed with the second spinor field Ψ_E in an analogous way as $C_{\mu\nu}$, $\Delta_{\mu\nu}$ and $\Omega_{\mu\nu}$ are written in terms of the spinor $\boldsymbol{\psi}$.

Conclusion

In this paper, we have shown that the linearized equations of Einstein general relativity can be understood as a consequence of a more fundamental equation of motion for spinor fields that obey the non linear Heisenberg equation satisfying the Inomata condition. In this respect, this property may be viewed as pointing to a close connection between gravitation and non-linear spinor dynamics as it was proposed by us.

11 Constructing Dirac linear fermions in terms of non linear Heisenberg spinors

M. NOVELLO

11.1 ..

There are evidences that neutrino changes from one flavor to another as observed for instance in neutrino oscillations found by the Super-Kamiokande Collaboration. This mix is understood as an evidence that the neutrino has a small mass. This has important consequences not only in local laboratory experiments but also in astrophysics and even in cosmology. In a closely related path, the possibility that not only left-handed but also right-handed neutrinos exist has recently attracted interest, receiving a new treatment in a very imaginative example presented in dealing with the possibility of neutrino superfluidity. The main idea requires the existence of an interaction between neutrinos that in the case of small energy and momentum can be described as a sort of Fermi process. If the field is the same, this interaction is nothing but an old theory of Heisenberg concerning self-interacting fermions. Recent experiments strongly support the idea that there are only three neutrino flavours. Based on this and on the possibility of mixing neutrino species, it has been argued that neutrino flavours are combinations of mass eigenstatesof mass mi through a unitary matrix. It would be interesting if we could describe all these properties as consequences of the existence of a common root for the neutrino species, e.g., if they are particular realizations of a unique structure. In this paper we will develop a model of such idea and work out a unified description of the three species of neutrinos by showing that they can be considered as having a common origin on a more fundamental nonlinear structure. Actually such property is not exclusive for neutrinos but instead is typical for any Dirac fermion (e.g., quark, electron). However as we shall see, the decomposition of the Dirac fermion in terms of non-linear structure contains three parameters (associated to the Heisenberg self-interaction constant) that separate different classes of Dirac spinors and three elements for each class that could be associated to three types of particles in each class. This form of decomposition may appear as if we were inverting the common procedure and treating the simple linear case of Dirac spinor as a particular state of a more involved self-interacting nonlinear structure. This goes in the same direction as some modern treatments in which linearity is understood as a realization of a subjacent nonlinear structure. In this vein we will examine the hypothesis that neutrinos are special states of nonlinear Heisenberg spinors.

The main outcome of the present paper is the proof of the statement that a massive or massless neutrino that satisfies Dirac equation can be described as a deformation of the Heisenberg spinor.

This finally proves the following Lemma: A free linear massive (or massless) Dirac field can be represented as a combination of Inomata spinors satisfying the non-linear Heisenberg equation.

11.1.1 From Heisenberg to Dirac: How elementar is the neutrino?

We will make a small *intermezzo* now to exemplify the interest by its own of non-linear Heisenberg dynamics. Indeed, in this section we will describe an unexpected result of the Inomata class $\mathcal{IC}$ which states that for any spinor of $\mathcal{IC}$ it is possible to construct another spinor which satisfies the linear Dirac equation. In other words, we claim that a spinor that satisfies the linear Dirac equation may be constructed in terms of a non linear structure. This is a very important and non-trivial result that merits some analysis. Although this property is not directly related to the Pre-Gravity Theory, it allows us to understand the importance of the non-linear Heisenberg structure. Besides, it points in a path to be followed in the future, for a possible unifications scheme of distinct interactions, like for instance Fermi weak forces and gravity.

Let us start by defining a plane π_H characterized by the left and right-handed Heisenberg spinor:

$$\Psi^H = \Psi^H_L + \Psi^H_R = \frac{1}{2}(1+\gamma^5)\Psi^H + \frac{1}{2}(1-\gamma^5)\Psi^H \tag{11.1.1}$$

We now show that it is possible to write the left and the righ-handed Dirac spinor as a deformation of Ψ^H in the plane π_H given by

$$\Psi^D_L = e^F\,\Psi^H_L \tag{11.1.2}$$

$$\Psi^D_R = e^G\,\Psi^H_R \tag{11.1.3}$$

What are the properties of F and G in order that Ψ^D satisfies Dirac equation? In order to answer this question we have to make some additional calculations. From eq. (13.1.43) we obtain

$$\partial_\mu \Psi_L^H = (a\, J_\mu + b\, I_\mu)\, \Psi_L^H \tag{11.1.4}$$

$$\partial_\mu \Psi_R^H = (a\, J_\mu - b\, I_\mu)\, \Psi_R^H \tag{11.1.5}$$

Now comes a miracle that permits the accomplishment of our procedure, which is the fact that the two vectors J_μ and I_μ can be written as gradients of nonlinear expressions under the form

$$\begin{aligned} J_\mu &= \partial_\mu S, \\ I_\mu &= \partial_\mu R, \end{aligned} \tag{11.1.6}$$

where S and R are given in eq. (13.1.47) and (13.1.54). ¿From these equations it follows

$$\partial_\mu \Psi_L^D = \left(\frac{\partial F}{\partial S}\, J_\mu + \frac{\partial F}{\partial R}\, I_\mu \right) \Psi_L^D + (a\, J_\mu + b\, I_\mu)\, \Psi_L^D. \tag{11.1.7}$$

$$\partial_\mu \Psi_R^D = \left(\frac{\partial G}{\partial S}\, J_\mu + \frac{\partial G}{\partial R}\, I_\mu \right) \Psi_R^D + (a\, J_\mu - b\, I_\mu)\, \Psi_R^D. \tag{11.1.8}$$

Multiplying these expressions by $i\,\gamma^\mu$ it follows that Ψ^D satisfies Dirac equation if F and G are given by:

$$F = -\,\frac{1}{2}\,(b - \bar{b})\, R + (2is - \frac{1}{2}(b - \bar{b}))S + \frac{iM}{a + \bar{a}}\, e^{-(a+\bar{a})S} \tag{11.1.9}$$

$$G = \frac{1}{2}\,(b - \bar{b})\, R + (2is - \frac{1}{2}(b - \bar{b}))S + \frac{iM}{a + \bar{a}}\, e^{-(a+\bar{a})S} \tag{11.1.10}$$

To arrive at this result it is convenient to use the formulas provided by Pauli-Kofink identities (see (13.1.28) and (11.1.1)) to obtain:

$$\begin{aligned} J_\mu\, \gamma^\mu\, \Psi_L &= (A - iB)\, \Psi_R \\ I_\mu\, \gamma^\mu\, \Psi_L &= -(A - iB)\, \Psi_R \\ I_\mu\, \gamma^\mu\, \Psi_R &= (A + iB)\, \Psi_L \\ J_\mu\, \gamma^\mu\, \Psi_R &= (A + iB)\, \Psi_L. \end{aligned} \tag{11.1.11}$$

where

$$A + iB = \frac{J^2}{A - iB}.$$

Thus, the linear Dirac field can be written in terms of the non-linear Heisenberg field by:

$$\Psi_L^D = \sqrt{\frac{J}{A - iB}} \exp\left(\frac{iM}{(a+\bar{a})\,J} + (2is - \frac{1}{2}(b - \bar{b}))S\right) \Psi_L^H \tag{11.1.12}$$

$$\Psi_R^D = \sqrt{\frac{A - iB}{J}} \exp\left(\frac{iM}{(a+\bar{a})\,J} + (2is - \frac{1}{2}(b - \bar{b}))S\right) \Psi_R^H, \tag{11.1.13}$$

where $J \equiv \sqrt{J^2}$. Using expression (13.1.47) we can simplify these expressions, once we can write

$$\exp\left(2is - \frac{1}{2}(b - \bar{b})\right)S = J^{2\sigma}$$

where we have defined

$$\sigma \equiv \frac{is - \frac{1}{4}(b - \bar{b})}{a + \bar{a}} = -\frac{i}{4}\,\frac{Im(a)}{Re(a)}.$$

Then, finally, for the Dirac spinor

$$\Psi^D = \exp\frac{iM}{(a+\bar{a})\,J}\, J^{2\sigma} \left(\sqrt{\frac{J}{A - iB}}\Psi_L^H + \sqrt{\frac{A - iB}{J}}\,\Psi_R^H\right) \tag{11.1.14}$$

or, for the mass-less neutrino

$$\Psi^D = J^{2\sigma} \left(\sqrt{\frac{J}{A - iB}}\Psi_L^H + \sqrt{\frac{A - iB}{J}}\,\Psi_R^H\right) \tag{11.1.15}$$

This ends the proof of the following

Lemma: Free linear massive (or mass-less) Dirac field can be represented as a combination of Inomata spinors satisfying the non-linear Heisenberg equation.

We must analyze carefully the domain of parameters a and b once the potentials S and R become singular in the imaginary axis and in the real axis, respectively. Thus we can distinguish different domains in the space of these two parameters. We set $a = a_0\, e^{i\varphi}$ and $b = b_0\, e^{i\theta}$. Then, the constraints on these parameters presented previously, that allows the existence of the Inomata solution, is written under the form:

$$\frac{cos\varphi}{\cos\theta} > 0, \tag{11.1.16}$$

$$\cos\varphi\,(tan\varphi - tan\theta) < 0, \tag{11.1.17}$$

once the Heisenberg constant s is positive. Let us name the following sectors:

W_1 for $0 < \varphi < \frac{\pi}{2}$; W_2 for $\frac{\pi}{2} < \varphi < \pi$; W_3 for $\pi < \varphi < \frac{3\pi}{2}$, and W_4 for $\frac{3\pi}{2} < \varphi < 2\pi$. In an analogous way we define Z_1, Z_2, Z_3 and Z_4 for similar sectors of θ. We distinguish then six domains:

$$\Omega_1 \equiv W_1 \otimes Z_1$$

$$\Omega_2 \equiv W_4 \otimes Z_1$$

$$\Omega_3 \equiv W_4 \otimes Z_4$$

$$\Omega_4 \equiv W_2 \otimes Z_2$$

$$\Omega_5 \equiv W_3 \otimes Z_2$$

$$\Omega_6 \equiv W_3 \otimes Z_3$$

The missing domains $W_1 \otimes Z_4$ and $W_2 \otimes Z_3$ are forbidden because they violate constraint (11.1.17). Thus, for the massless case, equation (11.1.14) shows that different choices of the parameters - a and b for a given value of constant s yields different spinor configurations Ψ^D. This allows us to write

$$\Psi^D = \sum_{\Omega_i} c_i \, \Gamma^{i,s} \tag{11.1.18}$$

where $\Gamma^{i,s}$ is defined by the rhs of equation (11.1.14) and we have to sum over all possible independent domains. Note furthermore that we are not obliged at this level to specify the helicity. This expression exhibits the existence of a degeneracy: for each Heisenberg theory characterized by a given value of the self-coupling s there exists six distinct class of Dirac spinors, which we could identify to three neutrinos and its corresponding anti-neutrinos. In this framework we can understand the change of flavor of massless neutrinos. Besides, changing the value of s allows the decomposition not only of neutrinos but also of others fields in terms of fundamental Heisenberg spinors. This is the end of the Intermezzo.

12 Cosmological effects of non linear Electrodynamics

M NOVELLO, ALINE N. ARAUJO and J M SALIM

Recent works have shown the important role that Nonlinear Electrodynamics (NLED) can have in two crucial questions of Cosmology, concerning particular moments of its evolution for very large and for low-curvature regimes, that is for very condensed phase and at the period of acceleration. We present here a a toy model of a complete cosmological scenario in which the main factor responsible for the geometry is a nonlinear magnetic field which produces a FRW homogeneous and isotropic geometry. In this scenario we distinguish four distinct phases: a bouncing period, a radiation era, an acceleration era and a re-bouncing. It has already been shown that in NLED a strong magnetic field can overcome the inevitability of a singular region typical of linear Maxwell theory; on the other extreme situation, that is for very weak magnetic field it can accelerate the expansion. The present model goes one step further: after the acceleration phase the universe rebounces and enter in a collapse era. This behavior is a manifestation of the invariance under the dual map of the scale factor $a(t) \rightarrow 1/a(t)$, a consequence of the corresponding inverse symmetry of the electromagnetic field ($F \rightarrow 1/F$, where $F \equiv F^{\mu\nu}F_{\mu\nu}$) of the NLED theory presented here. Such sequence collapse-bouncing-expansion-acceleration-re-bouncing-collapse constitutes a basic unitary element for the structure of the universe that can be repeated indefinitely yielding what we call a Cyclic Magnetic Universe.

12.1 Introduction

In the last years there has been increasing of interest on the cosmological effects induced by Nonlinear Electrodynamics (NLED). The main reason for this is related to the drastic modification NLED provokes in the behavior of the cosmological geometry in respect to two of the most important questions of standard cosmology, that is, the initial singularity and the acceleration of the scale factor. Indeed, NLED provides worthwhile alternatives to solve these two problems in a unified way, that is without invoking different mechanisms for each one of them separately. Such economy of hypotheses is certainly welcome. The partial analysis of each one of these problems was initiated in previous paper. Here we will present a description of a new cosmological model.

The most general form for the dynamics of the electromagnetic field, compatible with covariance and gauge conservation principles reduces to $L = L(F)$, where $F \equiv F^{\mu\nu}F_{\mu\nu}$. We do not consider here the other invariant $G \equiv F^{\mu\nu}F^{*}_{\mu\nu}$ constructed with the dual since in our scenario the average of the electric field vanishes in a magnetic universe as we shall see in the next sections. Thus, the Lagrangian appears as a regular function that can be developed as positive or negative powers of the invariant F. Positive powers dominate the dynamics of the gravitational field in the neighborhood of its moment of extremely high curvatures. Negative powers control the other extreme, that is, in the case of very weak electromagnetic fields. In this case as it was pointed out previously it modifies the evolution of the cosmic geometry for large values of the scale factor, inducing the phenomenon of acceleration of the universe. The arguments presented by Lemoine make it worth considering that only the averaged magnetic field survives in a FRW spatially homogeneous and isotropic geometry. Such configuration of pure averaged magnetic field combined with the dynamic equations of General Relativity received the generic name of Magnetic Universe.

The most remarkable property of a Magnetic Universe configuration is the fact that from the energy conservation law it follows that the dependence on time of the magnetic field $H(t)$ is the same irrespective of the specific form of the Lagrangian. This property allows us to obtain the dependence of the magnetic field on the scale factor, without knowing the particular form of the Lagrangian $L(F)$. Indeed, as we will show later on, from the energy-momentum conservation law it follows that $H = H_0\, a^{-2}$. This dependence is responsible for the property which states that strong magnetic fields dominates the geometry for small values of the scale factor; on the other hand, weak fields determines the evolution of the geometry for latter eras when the radius is big enough to excite these terms.

In order to combine both effects, here we will analyze a toy model. The symmetric behavior of the magnetic field in both extremes – that is for very strong and very weak regimes – allows the appearance of a repetitive configuration of the kind exhibited by an eternal cyclic universe.

Negative power of the field in the Lagrangian of the gravitational field was used by Carroll and others in an attempt to explain the acceleration of the scale factor of the universe by modification of the dynamics of the gravitational field by adding to the Einstein-Hilbert action a term that depends on negative power of the curvature, that is

$$S = \frac{M_{\rm Pl}^2}{2}\int \sqrt{-g}\left(R - \frac{\alpha^4}{R}\right) d^4x,$$

This modification showed not to be a good candidate to describe a local gravitational field. However, as a by-product of such proposal, one could envisage the possibility to deal with a new symmetry between strong and weak fields. In a paper by Novello et al, a model assuming this idea was presented and its cosmological consequences analyzed. In this model, the action for the electromagnetic field was modified by the addition of a new term, namely

$$S = \int \sqrt{-g} \left(-\frac{F}{4} + \frac{\gamma}{F} \right) d^4x. \tag{12.1.1}$$

This action yields an accelerated expansion phase for the evolution of the universe, and correctly describes the electric field of an isolated charge for a sufficiently small value of parameter γ. The acceleration becomes a consequence of the properties of this dynamics for the situation in which the field is weak.

In another cosmological context, in the strong regime, it has been pointed out in the literature that NLED can produces a bouncing, altering another important issue in Cosmology: the singularity problem. In this article we would like to combine both effects improving the action given in Eqn.(12.1.1) to discuss the consequences of NLED for both, weak and strong fields.

It is a well-known fact that under certain assumptions, the standard cosmological model unavoidably leads to a singular behavior of the curvature invariants in what has been termed the Big Bang. This is a highly distressing state of affairs, because in the presence of a singularity we are obliged to abandon the rational description of Nature. It may happen that a complete quantum cosmology could describe the state of affairs in a very different and more complete way. For the time being, while such complete quantum theory is not yet known, one should attempt to explore alternatives that are allowed and that provide some sort of phenomenological consequences of a more profound theory.

It is tempting then to investigate how NLED can give origin to an unified scenario that not only accelerates the universe for weak fields (latter cosmological era, for latter times) but that is also capable of avoiding an initial singularity as a consequence of its properties in the strong regime.

Scenarios that avoid an initial singularity have been intensely studied over the years. As an example of some latest realizations we can mention the pre-big-bang universe and the ekpyrotic universe. While these models are based on deep modifications on conventional physics (assuming the important role of new entities as scalar fields, string theory or branes) the model we present here relies instead on the electromagnetic field. The new ingredient that we introduce concerns the dynamics that is rather different from that of Maxwell in distinct regimes. Specifically, the Lagrangian we will work with is given by

$$L_T = \alpha^2 F^2 - \frac{1}{4} F - \frac{\mu^2}{F} + \frac{\beta^2}{F^2}. \tag{12.1.2}$$

The dimensional constants α, β and μ are to be determined by observation. Thus the complete dynamics of electromagnetic and gravitational fields are governed by Einstein equations plus L_T.

We shall see that in Friedmann-Robertson-Walker (FRW) geometry we can distinguish four typical eras which generate a basic unity – which we will call ***tetraktys*** – that repeat indefinitely[1]. The whole cosmological scenario is controlled by the energy density ρ and the pressure p of the magnetic field. Each era of the tetraktys is associated with a specific term of the Lagrangian. As we shall see the conservation of the energy-momentum tensor implies that the field dependence on the scale factor yields that the invariant F is proportional to a^{-4}. This dependence is responsible by the different dominance of each term of the Lagrangian in different phases. The first term $\alpha^2 F^2$ dominates in very early epochs allowing a bouncing to avoid the presence of a singularity. Let us call this the ***bouncing era***. The second term is the Maxwell linear action which dominates in the ***radiation era***. The inverse term μ^2/F dominates in the ***acceleration era***. Finally the last term β^2/F^2 is responsible for a ***re-bouncing***. Thus the tetraktys universe can be described in the following way:

- The bouncing era: There exists a collapsing phase that attains a minimum value for the scale factor $a_B(t)$;
- The radiation era: after the bouncing, $\rho + 3p$ changes the sign; the universe stops its acceleration and start expanding with $\ddot{a} < 0$;
- The acceleration era: when the $1/F$ factor dominates the universe enters an accelerated regime;
- The re-bouncing era: when the term $1/F^2$ dominates the acceleration changes the sign and starts a phase in which $\ddot{a} < 0$ once more; the scale factor attains a maximum and re-bounces starting a new collapsing phase and entering a bouncing era once more.

This unity of four stages, the tetraktys, constitutes an eternal cyclic configuration that repeats itself indefinitely.

The plan of the article is as follows. In section II we review the Tolman process of average in order to conciliate the energy distribution of the electromagnetic field with a spatially isotropic geometry. Section III presents the notion of the Magnetic Universe and its generic features concerning the dynamics of electromagnetic field generated by a Lagrangian $L = L(F)$. Section

[1]This term was taken from Pithagoras who represented the unity of the world constituted by four basic elements by a geometrical figure called tetratkys.

IV presents the conditions of bouncing and acceleration of a FRW universe in terms of properties to be satisfied by L. In section V we introduce the notion of inverse symmetry of the combined electromagnetic and gravitational fields in a cosmological context. This principle is used to complete the form of the Lagrangian that guides the combined dynamics of the unique long-range fields yielding a spatially homogeneous and isotropic nonsingular universe. In sections VI and VII we present a complete scenario consisting of the four eras: a bouncing, an expansion with negative acceleration, an accelerated phase and a re-bouncing. We end with some comments on the form of the scale factor and future developments. In appendix we present the compatibility of our Lagrangian with standard Coulomb law and the modifications induced on causal properties of nonlinear electrodynamics.

12.2 The average procedure and the fluid representation

The effects of a nonlinear electromagnetic theory in a cosmological setting have been studied in several articles.

Given a generic gauge-independent Lagrangian $L = L(F)$, written in terms of the invariant $F \equiv F_{\mu\nu}F^{\mu\nu}$ it follows that the associated energy-momentum tensor, defined by

$$T_{\mu\nu} = \frac{2}{\sqrt{-\gamma}} \frac{\delta L \sqrt{-\gamma}}{\delta \gamma^{\mu\nu}}, \tag{12.2.1}$$

reduces to

$$T_{\mu\nu} = -4\, L_F\, F_\mu{}^\alpha\, F_{\alpha\nu} - L\, g_{\mu\nu}. \tag{12.2.2}$$

In the standard cosmological scenario the metric structure of space-time is provided by the FLRW geometry. For compatibility with the cosmological framework, that is, in order that an electromagnetic field can generates a homogeneous and isotropic geometry an average procedure must be used. We define the volumetric spatial average of a quantity X at the time t by

$$\overline{X} \equiv \lim_{V \to V_0} \frac{1}{V} \int X \sqrt{-g}\; d^3x, \tag{12.2.3}$$

where $V = \int \sqrt{-g}\; d^3x$ and V_0 is a sufficiently large time-dependent three-volume. In this notation, for the electromagnetic field to act as a source for the FLRW model we need to impose that

$$\overline{E}_i = 0, \quad \overline{H}_i = 0, \quad \overline{E_i H_j} = 0, \tag{12.2.4}$$

$$\overline{E_i E_j} = -\frac{1}{3} E^2 g_{ij}, \quad \overline{H_i H_j} = -\frac{1}{3} H^2 g_{ij}. \tag{12.2.5}$$

With these conditions, the energy-momentum tensor of the EM field associated to $L = L(F)$ can be written as that of a perfect fluid,

$$T_{\mu\nu} = (\rho + p)v_\mu v_\nu - p\, g_{\mu\nu}, \tag{12.2.6}$$

where

$$\rho = -L - 4L_F E^2,$$
$$p = L - \frac{4}{3}\left(2H^2 - E^2\right) L_F, \tag{12.2.7}$$

where $L_F \equiv dL/dF$.

12.3 Magnetic universe

A particularly interesting case occurs when only the average of the magnetic part does not vanishes and $E^2 = 0$. Such situation has been investigated in the cosmological framework yielding what has been called *magnetic universe.* This should be a real possibility in the case of cosmology, since in the early universe the electric field is screened by the charged primordial plasma, while the magnetic field lines are frozen. In spite of this fact, some attention was devoted to the mathematically interesting case in which $E^2 = \sigma^2 H^2 \neq 0$.

An interesting feature of such magnetic universe comes from the fact that it can be associated with a four-component non-interacting perfect fluid. Let us give a brief proof of the statement that in the cosmological context the energy-content that follows from this theory can be described in terms of a perfect fluid. We work with the standard form of the FLRW geometry in Gaussian coordinates provided by (we limit the present analysis to the Euclidean section)

$$ds^2 = dt^2 - a(t)^2 \left(dr^2 + r^2 d\Omega^2\right). \tag{12.3.1}$$

The expansion factor, θ defined as the divergence of the fluid velocity reduces, in the present case, to the derivative of logarithm of the scale factor

$$\theta \equiv v^\mu_{;\mu} = 3\,\frac{\dot{a}}{a} \tag{12.3.2}$$

The conservation of the energy-momentum tensor projected in the direction of the co-moving velocity $v^\mu = \delta^\mu_0$ yields

$$\dot{\rho} + (\rho + p)\theta = 0 \tag{12.3.3}$$

Using Lagrangian L_T in the case of the magnetic universe yields for the density of energy and pressure given in equations (12.2.7):

$$\rho = -\,\alpha^2\, F^2 + \frac{1}{4}\, F + \frac{\mu^2}{F} - \frac{\beta^2}{F^2} \tag{12.3.4}$$

$$p = -\,\frac{5\alpha^2}{3}\, F^2 + \frac{1}{12}\, F - \frac{7\mu^2}{3}\,\frac{1}{F} + \frac{11\beta^2}{3}\,\frac{1}{F^2} \tag{12.3.5}$$

Substituting these values in the conservation law, it follows

$$L_F \left[(H^2)^{\cdot} + 4\, H^2\, \frac{\dot{a}}{a} \right] = 0. \tag{12.3.6}$$

where $L_F \equiv \partial L / \partial F$.

The important result that follows from this equation is that the dependence on the specific form of the Lagrangian appears as a multiplicative factor. This property shows that any Lagrangian $L(F)$ yields the same dependence of the field on the scale factor irrespective of the particular form of the Lagrangian. Indeed, equation (12.3.6) yields

$$H = H_0\, a^{-2}. \tag{12.3.7}$$

This property implies that for each power F^k it is possible to associate a specific fluid configuration with density of energy ρ_k and pressure p_k in such a way that the corresponding equation of state is given by

$$p_k = \left(\frac{4k}{3} - 1 \right) \rho_k. \tag{12.3.8}$$

We restrict our analysis in the present paper to the theory provided by a toy-model described by the Lagrangian

$$\begin{aligned} L_T &= L_1 + L_2 + L_3 + L_4 \\ &= \alpha^2\, F^2 - \frac{1}{4}\, F - \frac{\mu^2}{F} + \frac{\beta^2}{F^2} \end{aligned} \tag{12.3.9}$$

where α, β, μ are parameters characterizing a concrete specific model. For latter use we present the corresponding many-fluid component associated to Lagrangian L_T. We set for the total density and pressure $\rho_T = \sum \rho_i$ and

$p_T = \sum p_i$ where

$$\begin{aligned}
\rho_1 &= -\alpha^2 F^2 \ , \ p_1 = \frac{5}{3}\rho_1 \\
\rho_2 &= \frac{1}{4} F \ , \ p_2 = \frac{1}{3}\rho_2 \\
\rho_3 &= \frac{\mu^2}{F} \ , \ p_3 = -\frac{7}{3}\rho_3 \\
\rho_4 &= -\frac{\beta^2}{F^2} \ , \ p_4 = -\frac{11}{3}\rho_4.
\end{aligned} \tag{12.3.10}$$

Or, using the dependence of the field on the scale factor equation (12.3.7),

$$\begin{aligned}
\rho_1 &= -4\alpha^2 H_0^4 \frac{1}{a^8} \\
\rho_2 &= \frac{H_0}{2} \frac{1}{a^4} \\
\rho_3 &= \frac{\mu^2}{2H_0^2} a^4 \\
\rho_4 &= -\frac{\beta^2}{4H_0^4} a^8.
\end{aligned} \tag{12.3.11}$$

Let us point out a remarkable property of the combined system of this NLED generated by L_T and Friedman equations of cosmological evolution. A simple look into the above expressions for the values of the density of energy exhibits what could be a possible difficulty of this system in two extreme situations, that is, when F^2 and $1/F^2$ terms dominate, since if the radius of the universe can attain arbitrary small and/or arbitrary big values, then one should face the question regarding the positivity of its energy content. However, as we shall show in the next sections, the combined system of equations of the cosmic metric and the magnetic field described by General Relativity and NLED, are such that a beautiful conspiracy occurs in such a way that the negative contributions for the energy density that came from terms L_1 and L_4 never overcomes the positive terms that come from L_2 and L_3. Before arriving at the undesirable values where the density of energy could attain negative values, the universe bounces (for very large values of the field) and re-bounces (in the other extreme, that is, for very small values) to precisely avoid this difficulty. This occurs at the limit value $\rho_B = \rho_{RB} = 0$, as follows from equation

$$\rho = \frac{\theta^2}{3}. \tag{12.3.12}$$

We emphasize that this is not an extra condition imposed by hand but a direct consequence of the dynamics described by L_T. Indeed, at early stages of the expansion phase the dynamics is controlled by the approximation Lagrangian $L_T \approx L_{1,2} = L_1 + L_2$. Then

$$\rho = \frac{F}{4}\,(1 - 4\alpha^2\,F).$$

Using the conservation law (12.3.3) we conclude that the density of energy will be always positive since there exists a minimum value of the scale factor given by $a^4_{mim} = 8\alpha^2 H_o^2$. A similar conspiracy occurs in the other extreme where we approximate $L_T \approx L_{2,3} = L_2 + L_3$, which shows that the density remains positive definite, since $a(t)$ remains bound, attaining a maximum in the moment the universe makes a re-bounce. These extrema occurs precisely at the points where the total density vanishes. Let us now turn to the generic conditions needed for the universe to have a bounce and a phase of accelerated expansion.

12.4 Conditions for bouncing and acceleration

12.4.1 Acceleration

¿From Einstein's equations, the acceleration of the universe is related to its matter content by

$$3\frac{\ddot{a}}{a} = -\frac{1}{2}(\rho + 3p). \tag{12.4.1}$$

In order to have an accelerated universe, matter must satisfy the constraint $(\rho + 3p) < 0$. In terms of the quantities defined in Eqn. (12.2.7),

$$\rho + 3p = 2(L - 4H^2 L_F). \tag{12.4.2}$$

Hence the constraint $(\rho + 3p) < 0$ translates into

$$L_F > \frac{L}{4H^2}. \tag{12.4.3}$$

It follows that any nonlinear electromagnetic theory that satisfies this inequality yields accelerated expansion. In our present model it follows that terms L_2 and L_4 produce negative acceleration and L_1 and L_3 yield inflationary regimes ($\ddot{a} > 0$).

For latter uses we write the value of $\rho + 3p$ for the case of Lagrangian L_T :

$$\rho + 3p = -6\alpha^2 F^2 + \frac{F}{2} - \frac{6\mu^2}{F} + \frac{10\beta^2}{F^2}.$$

12.4.2 Bouncing

In order to analyze the conditions for a bouncing it is convenient to re-write the equation of acceleration using explicitly the expansion factor Θ, which is called the Raychaudhuri equation:

$$\dot{\theta} + \frac{1}{3}\theta^2 = \frac{1}{2}(\rho + 3p) \tag{12.4.4}$$

Thus besides condition (12.4.3) for the existence of an acceleration a bounce needs further restrictions on $a(t)$. Indeed, the existence of a minimum (or a maximum) for the scale factor implies that at the bouncing point B the inequality $(\rho_B + 3p_B) < 0$ (or, respectively, $(\rho_B + 3p_B) > 0$) must be satisfied. Note that at any extremum (maximum or minimum) of the scale factor the density of energy vanishes. This is a direct consequence of the first integral of Friedman equation which, in the Euclidean case, reduces to equation (12.3.12).

12.5 Duality on the Magnetic Universe as a consequence of the inverse symmetry

The cosmological scenario that is presented here deals with a cyclic FRW geometry which has a symmetric behavior for small and big values of the scale factor. This scenario is possible because the behavior of its energy content at high energy is the same as it has in its weak regime. This is precisely the case of the magnetic universe that we are dealing with here. To obtain a perfect symmetric configuration for our model we will impose a new dynamical principle:

- **The inverse symmetry principle:**

  The NLED theory should be invariant under the inverse map

$$F \rightarrow \tilde{F} = \frac{4\mu^2}{F}.$$

This restricts the number of free parameters from three to two, once a direct application of this principle implies that $\beta^2 = 16\alpha^2\mu^4$. This symmetry induces a corresponding one for the geometry. Indeed, the cosmological dynamics is invariant under the associated dual map

$$a(t) \to \tilde{a}(t) = \frac{H_0}{\sqrt{\mu}} \frac{1}{a} \tag{12.5.1}$$

It is precisely this invariance that is at the origin of the cyclic property of this cosmological scenario.

Let us point out that the above map is a conformal transformation. Indeed, in conformal time, the geometry takes the form

$$ds^2 = a(\eta)^2 \left(d\eta^2 - dr^2 - r^2 d\Omega^2 \right). \tag{12.5.2}$$

Thus making the conformal map

$$\tilde{g}_{\mu\nu} = \Omega^2 \, g_{\mu\nu}$$

where $\Omega = \lambda / a^2$, and $\lambda \equiv H_0 / \sqrt{\mu}$. Note that although the Lagrangian L_T is not invariant under a conformal transformation, the average procedure used to make compatible the dynamical system of the electomagnetic field and the Friedman equation is invariant. Indeed, we have

$$\tilde{F} = \tilde{g}^{\mu\nu} \tilde{g}^{\alpha\beta} \, F_{\alpha\mu} F_{\beta\nu} = \frac{4\mu^2}{F}.$$

12.6 A complete scenario

There is no doubt that electromagnetic radiation described by a maxwellian distribution has driven the cosmic geometry for a period. Now we would like to analyze the modifications introduced by the non linear terms in the cosmic scenario. The simplest way to do this is to combine the previous lagrangian with the dependence of the magnetic field on the scale factor. We set

$$L_T = \alpha^2 \, F^2 - \frac{1}{4} \, F - \frac{\mu^2}{F} + \frac{\beta^2}{F^2} \tag{12.6.1}$$

where β is related to the other parameters α and μ by the inverse symmetry principle, as displayed above.

12.7 Potential

It will be more direct to examine the effects of the magnetic universe controlled by the above lagrangian if we undertake a qualitative analysis using an analogy with classical mechanics. Friedman's equation reduces to the set

$$\dot{a}^2 + V(a) = 0 \tag{12.7.1}$$

where

$$V(a) = \frac{A}{a^6} - \frac{B}{a^2} - Ca^6 + Da^{10} \tag{12.7.2}$$

is a potential that restricts the motion of the localization $a(t)$ of the "particle". The constants in V are given by

$$A = \frac{4\alpha^2 H_0^4}{3}, \quad B = \frac{H_0^2}{6}, \quad C = \frac{\mu^2}{6H_0^2}, \quad D = \frac{4\alpha^2\mu^4}{3H_0^4},$$

and are positive.

We can then synthesize the properties of the magnetic universe the dynamics of which is given by L_T. We recognize that the dependence of the field as $H = H_0/a^2$ implies the existence of four distinct epochs, which we will analyze now.

The derivative dL/dF has three zeros in points a, b, c. In these points $\rho + p$ vanishes. In the case of pure magnetic universe the value of F is always positive. We distinguish the following eras:

12.8 The four eras of the Magnetic Universe

The dynamics of the universe with matter density given by Eqn.(12.11.2) can be obtained qualitatively from the analysis of Einstein's equations We distinguish four distinct periods according to the dominance of each term of the energy density. The early regime (driven by the F^2 term); the radiation era (where the equation of state $p = 1/3\rho$ controls the expansion); the third accelerated evolution (where the 1/F term is the most important one) and finally the last era where the $1/F^2$ dominates and in which the expansion stops, the universe re-bounces and enters in a collapse era.

12.9 Bouncing era

In the strong field limit the value of the scalar of curvature is small and the volume of the universe attains its minimum, the density of energy and the pressure are dominated by the terms coming from the quadratic lagrangian F^2 and is approximated by the forms

$$\begin{aligned}\rho &\approx \frac{H^2}{2}\,(1 - 8\alpha^2 H^2) \\ p &\approx \frac{H^2}{6}\,(1 - 40\alpha^2 H^2)\end{aligned} \tag{12.9.1}$$

Using the dependence $H = H_o/a^2$, leads to

$$\dot{a}^2 = \frac{kH_o^2}{6\,a^2}\left(1 - \frac{8\alpha^2 H_o^2}{a^4}\right) - \epsilon. \tag{12.9.2}$$

We remind the reader that we limit our analysis here to the Euclidean section ($\epsilon = 0$). As long as the right-hand side of equation (12.9.2) must not be negative it follows that, the scale-factor $a(t)$ cannot be arbitrarily small. Indeed, a solution of (12.9.2) is given as

$$a^2 = H_o\sqrt{\frac{2}{3}\,(t^2 + 12\,\alpha^2)}. \tag{12.9.3}$$

The linear case can be achieved by setting $\alpha = 0$. The average strength of the magnetic field H evolves in time as

$$H^2 = \frac{3}{2}\,\frac{1}{t^2 + 12\,\alpha^2}. \tag{12.9.4}$$

Note that at $t = 0$ the radius of the universe attains a minimum value at the bounce:

$$a_B^2 = H_o\,\sqrt{8\,\alpha^2}. \tag{12.9.5}$$

Therefore, the actual value of a_B depends on H_o, which - for given α, μ turns out to be the sole free parameter of the model. The energy density ρ reaches its maximum for the value $\rho_B = 1/64\alpha^2$ at the instant $t = t_B$, where

$$t_B = \sqrt{12\,\alpha^2}. \tag{12.9.6}$$

For smaller values of t the energy density decreases, vanishing at $t = 0$, while the pressure becomes negative. Only for very small times $t < \sqrt{4\alpha^2/k}$ the non-linear effects are relevant for cosmological solution of the normalized scale-factor. Indeed, solution (12.9.3) fits the standard expression of the Maxwell case at the limit of large times.

12.10 Radiation era

The standard, Maxwellian term dominates in the intermediary regime. Due to the dependence on a^{-2} of the field, this phase is defined by $H^2 >> H^4$ yielding the approximation

$$\rho \approx \frac{H^2}{2}$$
$$p \approx \frac{H^2}{6} \tag{12.10.1}$$

This is the phase dominated by the linear regime of the electromagnetic field. Its properties are the same as described in the standard cosmological model.

12.11 The accelerated era: weak field drives the cosmological geometry

When the universe becomes larger, negative powers of F dominates and the distribution of energy becomes typical of an accelerated universe, that is:

$$\rho \approx \frac{1}{2}\frac{\mu^8}{H^2}$$
$$p \approx \frac{-7}{6}\frac{\mu^8}{H^2} \tag{12.11.1}$$

In the intermediate regime between the radiation and the acceleration regime the energy content is described by the combined form

$$\rho = \frac{H^2}{2} + \frac{\mu^2}{2}\frac{1}{H^2},$$

or, in terms of the scale factor,

$$\rho = \frac{H_0^2}{2}\frac{1}{a^4} + \frac{\mu^2}{2H_0^2}\,a^4. \tag{12.11.2}$$

For small a it is the ordinary radiation term that dominates. The $1/F$ term takes over only after $a = \sqrt{H_0}/\mu$, and would grows without bound afterwards. In fact, the curvature scalar is

$$R = T^{\mu}_{\mu} = \rho - 3p = \frac{4\mu^2}{H_0^2}\,a^4,$$

showing that one could expect a curvature singularity in the future of the universe for which $a \to \infty$. We shall see, however that the presence of the term $1/F^2$ changes this behavior.

Using this matter density in Eqn.(12.4.1) gives

$$3\frac{\ddot{a}}{a} + \frac{H_0^2}{2}\frac{1}{a^4} - \frac{3}{2}\frac{\mu^8}{H_0^2}\,a^4 = 0.$$

To get a regime of accelerated expansion, we must have

$$\frac{H_0^2}{a^4} - 3\frac{\mu^8}{H_0^2}\,a^4 < 0,$$

which implies that the universe will accelerate for $a > a_c$, with

$$a_c = \left(\frac{H_0^4}{3\mu^8}\right)^{1/8}.$$

12.12 Re-Bouncing

For very big values of the scale factor the density of energy can be approximated by

$$\rho \approx \frac{\mu^2}{F} - \frac{\beta^2}{F^2} \tag{12.12.1}$$

and we pass from an accelerated regime to a phase in which the acceleration is negative. When the field attains the value $F_{RB} = 16\alpha^2\mu^2$ the universe changes its expansion to a collapse. The scale factor attains its maximum value

$$a_{max}^4 \approx \frac{H_0^2}{8\alpha^2\mu^2}.$$

12.13 Positivity of the density of energy

The total density of energy of the tetraktys universe is always positive definite (see equation 12.3.12). In the bouncing and in the re-bouncing eras it takes the value $\rho_B = \rho_{RB} = 0$. At these points the density is an extremum. Actually, both points are minimum of the density. This is a direct consequence of equations (12.3.3) and (12.3.12). Indeed, derivative of (12.3.3) at the bouncing and at the re-bouncing yields

$$\ddot{\rho}_B = \frac{3}{2}\, p_B^2 > 0.$$

Thus there must exists another extremum of ρ which should be a maximum. This is indeed the case since there exists a value on the domain of the evolution of the universe between the two minima such that

$$\rho_c + p_c = 0.$$

At this point we have

$$\ddot{\rho} + \dot{p}_c\,\theta_c = 0$$

showing that at this point c the density takes its maximum value.

12.14 The behavior of the scale factor

Let us pause for a while and describe the form of the scale factor as function of time in the four regimes. To simplify such description let us separate in three parts:

Phase A: Bouncing-Radiation

Phase B: Radiation-Acceleration

Phase C: Acceleration-Rebouncing

characterized respectivelly by the dynamics controlled by: $L_A = L_1 + L_2; L_B = L_2 + L_3; L_C = L_3 + L_4$. It is straightforward to obtain an analytical expression for each one of these periods. We obtain for phase A :

$$a(t)_{BR} = \sqrt{H_0}\,\left(\frac{2}{3}\,t^2 + 12\,\alpha^2\right)^{\frac{1}{4}} \tag{12.14.1}$$

The inverse symmetric phase C is given by

$$a(t)_{AR} = Constant\,\left((t-t_c)^2 + \frac{8\alpha^2\mu^4 H_0^2}{\mu^2}\right)^{-\frac{1}{4}} \tag{12.14.2}$$

For the case of phase B it is convenient to use an auxiliary coordinate Ψ and write A specific form is provided by

$$t = \frac{\sqrt{3}}{2\sqrt{\mu}}\,F(arcos\Psi, \frac{\sqrt{2}}{2})$$

$$\Psi = \frac{1-na^4}{1+na^4} \tag{12.14.3}$$

where $n \equiv \mu/H_0^2$, and F is a first kind elliptic function.

12.15 Some general comments

Although we have analyzed a simplified toy model it displays many regular properties that should be worth of further investigation. In particular, it provides a spatially homogeneous and isotropic FRW geometry which has no Big Bang and no Big Rip. It describes correctly the radiation era and allows for an accelerated phase without introducing any extra source.

The particular form of NLED described here is based on a new principle that states an intimate relation between strong and weak field configurations.

This inverse-symmetry principle reduces the number of arbitrary parameters of the theory and allows for the regular properties of the cosmical model. The universe is a cyclic one, having its main characteristics synthesized in the following steps:

- Step 1: The universe contains a collapsing phase in which the scalar factor attains a minimum value $a_B(t)$;
- Step 2: after the bouncing the universe expands with $\ddot{a} < 0$;
- Step 3: when the $1/F$ factor dominates the universe enters an accelerated regime;
- Step 4: when $1/F^2$ dominates the acceleration changes the sign and starts a phase in which $\ddot{a} < 0$ once more, the scale factor attains a maximum and re-bounces starting a new collapsing phase;
- Step 5: the universe repeats the same behavior passing steps 1, 2, 3 and 4 again and again, indefinitely.

The particular form of the dynamics of the magnetic field is dictated by the inverse principle, which states that the behavior of the field is invariant under the mapping $F \rightarrow \tilde{F} = \frac{4\mu^2}{F}$. This reflects on the symmetric behavior of the geometry by the dual map $a \rightarrow \tilde{a} = \frac{constant}{a}$.

13 Spinor theory of Gravity

M.NOVELLO

13.1 Introduction to STG

¿From Einstein Equivalence Principle (EEP) it follows that universality of gravitational processes leads naturally to its identification to a metric tensor $g_{\mu\nu}$. However anyone that accepts this interpretation of the EEP should ask, before adopting the General Relativity approach the following question: giving the observational fact that any piece of matter/energy provokes a modification of the geometry in which this piece is merged, could one be led to the unique conclusion that this modification is driven by a differential equation containing derivatives up to second order of the metric tensor and by properties of the matter that represents its energy distribution? Should one be obliged to conclude that there is no other logical way to understand this fact? Is there a unique and only way that compels any sort of gravitationally interacting matter to modify space-time geometry through a direct relation between a continuous local modification of the geometry and the corresponding matter-energy content? In other words, are we contrived to accept that geometry is also a physical component of nature, requiring unequivocally a dynamical equation itself? Is this the unique way to implement the Equivalence Principle? General Relativity is a complete realization of EEP that answers **yes** to these questions. These lectures will deal with Pre-Gravity Theory, which provides a distinct and competitive way to implement EEP which answers **no** to all these questions.

In the Spinor Theory of Gravity (STG) the gravitational field is represented in terms of two fundamental spinor fields Ψ_E and Ψ_N. Its origins goes back to a complementary view of EEP, according to which the geometrical field is an induced quantity that depends on some intimate microscopic sub-structure. This sub-structure does not have by itself a geometric origin by instead is a matter field.

We could say that GR is based on a vision according to which space-time is to be understood as the arena of Physics (in Wheeler's words) and gravity is nothing but the consequence of a direct modification of the intrinsic geometry of such an arena. PG on the other hand, considers that the arena contains only matter and energy and the geometry is nothing but a specific way related to these real quantities or substances interacts among themselves.

In this way, in STG it has no practical sense to attribute a dynamics to the geometry. Its evolution is just a natural consequence of the dynamics of matter interacting gravitationally, as we shall see.

Accepting the idea that the metric tensor is a derived quantity that is, it is not an independent dynamical variable, then we face the question: what should be the intermediate dynamical variables that represents the gravitational phenomenon? In his analysis of similar question, Feynmann argued against the possibility to identify such dynamical entity to different kinds of continuum fields like scalar, spinor and vector. Let us review this analysis. The argument against the scalar field rests on the impossibility of describe the influence of gravity in photon propagation. Accepting that the net effect of a scalar field should produce only conformally flat geometries then it follows that conformal invariance of Maxwell electrodynamics imply the absence of any direct influence of gravity on photon propagation. This was ruled out by the Sobral observation. The impossibility to identify gravity to vector field is related to the purely attractive effect of gravity. For neutrino-like field the Feynmann argument rests on the impossibility of having a $1/r$ static potential. Then he concludes that only a tensorial field $\varphi_{\mu\nu}$ could fulfill this criteria which led that the dynamical quantity of gravitational field has to be identified with the metric tensor. The Spinor Theory of Gravity provides a distinct answer. We shall see that Feynmann critics against spinor field is surmounted if we consider two spinor fields. In this case we do obtain the required $1/r$ static potential. We will be led then to adopt a two-spinor field to be the fundamental quantity to describe gravitational processes. The metric tensor, needed to fulfill the equivalence Principle is treated as an induced quantity. Let us turn to the analysis of this theory. This is the theory we will analyze. Before this, just a few words on history.

13.1.1 Pre-history

In this session we would like to spend some time in order to clarify the status of the Pre-Gravity Theory (PGT) with respect to other theories that deal with similar objects (spinors) or with alternative dynamics of the gravitational field.

They can be separated into two classes. The first class contain theories that deal with the idea that the intermediate of the gravitational field, the hypothetical graviton, should be a composite particle. This idea goes back to the original papers of de Broglie which still in the first half of the twentieth century tried to develop this in what he called ”théorie de la fusion”.

Using the property of addition of spin, de Broglie introduced the idea of particles having states between a minimum value $\frac{1}{2}$ to a maximum value S. The higher states being a certain cooperative fusion of lower ones. He succeeded in obtaining the equations of motion of spin 1 and 2 in the weak field approximation. This approach defines a specific dynamics for each component of spin. In the traditional view, de Broglie intends to reproduce the individual dynamics of the fields in terms of the dynamics of the basic stuffs. This approach, if it could be pushed beyond its linearized formulation stands in the tradition of physics that provides one dynamic field for each basic interaction: electrodynamics and gravity being the paradigmatic ones, the unique that contains a classical interpretation. We shall see in these lectures that de Broglie's approach has no point of contact with STG which takes the radical point of view arguing that there is no such thing as an independent dynamics which controls the gravitational forces.

The second class contains theories that consider the metric as a sub-product of more fundamental entities. In this class we find, for instance, what is called Spinor gravity and the vierbein formulation. These theories provides a dynamics for the metric, but it appears in terms of more rich structures. In the case of vierbein one takes a set of four independent four-vectors E^a_μ which is a local vector for arbitrary coordinate transformation and a Lorentz vector under local Lorentz rotation. In general the dynamics associated to this structure needs the presence of a non-symmetric connection. A typical theory makes use of Cartan geometries instead of the more symmetrical Riemannian of GR. Another category deals with spinors that are at the basis of the riemannian structure like for instance taking the elements of a Clifford algebra – the generalization of the constant γ_μ of Dirac — as the basic elements of the theory. Both class accepts the idea that gravity must have its own dynamics. They differ in the way such dynamics is constructed and in the structure of the basic elements of the gravitational field: either the metric tensor itself or some sort of larger structure.

13.1.2 Historical comment

In a series of papers Heisenberg examined a proposal regarding a complete quantum theory of fields and elementary particles. Such a huge and ambitious program did not fulfill his initial expectation. It is not our intention here to discuss this program. For our purpose, it is important only to retain the original non linear equation of motion which Heisenberg postulated for the constituents of the fundamental material blocks of all existing matter. The modern point of view has developed in a very different direction and it is sufficient to take a look at the book of Particle Data Properties and the description of our actual knowledge of the elementary particle properties to realize how far from Heisenberg dream the theory has gone.

So much for the historical context. What we would like to retain from Heisenberg's approach reduces exclusively to his suggestion of a non linear equation of motion for a spinor field. We will use this equation for both our fundamental spinors, once as we will now see, it is the simplest non-linear dynamics that can be constructed in a covariant way.

13.1.3 Introducing some ideas of STG

There is no doubt that the activity in the field of experimental gravitation has increased largely in the last decades. New space measurements and astronomical discoveries, including those of cosmological origin are mainly responsible for this. At the basis of any theory of gravity compatible with such observations, one has the Einstein Equivalence Principle (EEP) which can be described as three conditions:

a. The weak equivalence principle is valid (that is, all bodies fall precisely the same way in a gravitational field);

b. The outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed;

c. The outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed;

¿From the validity of this EEP one infers that "the gravitation must be a curved space-time phenomenon". This was implemented by Einstein by assuming that the curvature of the space-time is related to the stress-energy-momentum tensor of matter in space-time and by postulating a specific form for such an equation. Taken together, the EEP and Einstein's equation constitute the basis of a successful program of a theory of gravity.

Is this the unique way to deal with the universality of gravitational processes? Is the only way to implement the EEP? Recently we proposed a new look into this old question by arguing that it is possible to treat the metric of space-time - that in General Relativity (GR) describes the gravitational interaction - as an effective geometry, that is, the metric acting on matter is not an independent field and as such does not posses its own dynamics. Instead, it inherits one from the dynamics of two fundamental spinor fields Ψ_E and Ψ_N which are responsible for the gravitational interaction and from which an effective geometry appears.

The nonlinear character of gravity should be present already at the most basic level of these fundamental structures. It seems natural to describe this nonlinearity in terms of the invariants constructed with the spinor fields. The simplest way to build a concrete model is to use the standard form of a contraction of the currents of these fields, e.g. $J_\mu J^\mu$, to construct the Lagrangian of the theory. We assume that these two fields (which are half-integer representations of the Poincaré group) interact universally with all other forms of matter and energy. As a consequence, this process can be viewed as nothing but a change of the metric of the space-time. In other words, the influence of these spinor fields on matter/energy is completely equivalent to a modification of the background geometry into an effective Riemannian geometry $g_{\mu\nu}$. In this aspect this theory agrees with the idea of General Relativity theory which states that the Equivalence Principle implies a change on the geometry of space-time as a consequence of the gravitational interaction. However, the similarities between the Spinor Theory of Gravity and General Relativity stop here.

To summarize, let us stress the main steps of this program.

a. There exist two fundamental spinor fields – which we will name Ψ_E and Ψ_N;

b. The interaction of Ψ_E and Ψ_N is described by Fermi Lagrangian;

c. The fields Ψ and Ψ_N interact universally with all forms of matter and energy;

d. As a consequence of this coupling with matter, the universal interaction produces an effective metric;

e. The dynamics of the effective metric is already contained in the dynamics of Ψ_E and Ψ_N : the metric does not have a dynamics of its own, but inherits its evolution through its relation with the fundamental spinors.

We understand that the need of two spinors is a fundamental one. Indeed, a four-component Dirac spinor Ψ has 8 degrees of freedom. Thus, we have $2x8$ quantities at our disposal to generate the 10 independent components of the metric tensor $g_{\mu\nu}$. Once in the STG only the vector and axial currents appear, we have the liberty to make a local Lorentz rotation in the spinors, which eliminates 6 superfluous conditions.

In a previous paper we presented a particular example of the effective metric in the case of a compact spherically static object, like a star and have shown that it is astonishingly similar to the Schwarzschild solution of GR.

Before entering the analysis of these questions let us briefly comment our motivation. As we shall see, the present proposal and the theory of General Relativity have a common underlying idea: the characterization of gravitational forces as nothing but the effect on matter and energy of a modification of the geometry of space-time. This major property of General Relativity remains unchanged. The main difference concerns the dynamics that this geometry obeys. In GR the dynamics of the gravitational field depends on the curvature invariants; in the Spinor Theory of Gravity such a specific dynamics simply does not exist: the geometry evolves in space-time according to the dynamics of the spinors Ψ_E and Ψ_N. The metric is not a field of its own, it does not have an independent reality but is just a consequence of the universal coupling of matter with the fundamental spinors. The motivation of walking down only half of Einstein's path to General Relativity is to avoid certain known problems that still plague this theory, including its difficult passage to the quantum world and the questions put into evidence by astrophysics involving many discoveries such as the acceleration of the universe, the problems requiring dark matter and dark energy. It seems worthwhile to quote here : "Dark energy appears to be the dominant component of the physical Universe, yet there is no persuasive theoretical explanation for its existence or magnitude. The acceleration of the Universe is, along with dark matter, the observed phenomenon that most directly demonstrates that our theories of fundamental particles and gravity are either incorrect or incomplete. Recent observations in Cosmology are responsible for an unexpected attitude: to take seriously the possibility of modifying Einstein' s theory of gravity". Pre-Gravity, a spinorial theory of gravity presents the possibility of a way out of these difficulties. The reason, which will be explained later on, can be understood from the fact that in the STG there is no direct relationship between the acceleration of the scale factor of the universe and the matter/energy distribution, contrary to the case of GR, in which the Friedman equation that controls the dynamics of the universe relates the matter-energy content to the geometry through the evolution of the scale factor $a(t)$:

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p).$$

It follows from this equation that if the universe is accelerating, then something very unusual must occur, like, for instance, a very negative pressure term dominating the evolution. As we shall see, nothing similar happens in STG, since the way in which matter influences the dynamics of the geometry does not take such form.

In the first subsection we present the mathematical background used in the paper and in particular the very important Pauli-Kofinki identity. These relations allow us to obtain a set of products of currents which will be very useful to simplify our calculations. In section II we recall the definition of the effective metric and some of its properties and compare with the field theory formulation of General Relativity. In section III we present the dynamics, separated in two parts: i) the kinetic part, which tells us how particles move in a given gravitational field; and ii) the influence of matter on the formation of the gravitational field. We shall see that in what concerns the first part, the Spinor Theory of Gravity is completely identical to General Relativity. They differ in the second part, once in STG there is no independent dynamics for the geometry. In the field theory formulation of General Relativity as it was described in the fifties by Gupta, Feynman and others, and more recently Grischuck et al, the gravitational field can be described alternatively either as the metric of space-time – as in Einstein's original version – or as a field $\varphi_{\mu\nu}$ in an arbitrary unobservable background geometry, which is chosen to be Minkowski. We shall see that by universally coupling the spinor fields to all forms of matter and energy, a metric structure appears, in a similar way to the field theoretical description of GR. The main distinction between these two approaches concerns the status of this metric. In General Relativity it has a dynamics provided by a Lagrangian constructed in terms of the curvature invariants. In our proposal, this is not the case. The metric is an effective way to describe gravity and it appears because of the universal form of the coupling of matter/energy of any form and the fundamental spinors. Before entering in the analysis of the new theory let us make some comments concerning others metric representation to describe dynamics in field theory.

We will exhibit two examples of such theoretical framework, one provided by a scalar field and another one given by non-linear Electrodynamics.

Non-linear Scalar field

In recent years a lot of speculative theories concerning non-linear scalar field have been proposed. The main motivation to consider seriously such suggestions is related to the so-called dark-energy problem, that we have mentioned above. To simplify our analysis let us limit ourselves to consider scalar field models whose Lagrangian have a non-canonical form

$$L = F(W)\,, \tag{13.1.1}$$

where $W := \partial_\mu \varphi\, \partial^\mu \varphi$. The corresponding energy-momentum tensor is given by

$$T_{\mu\nu} = -L\, g_{\mu\nu} + 2L_W\, \nabla_\mu \varphi\, \nabla_\nu \varphi\,, \tag{13.1.2}$$

where $L_W := \partial L / \partial W$.

The study of the behavior of discontinuities of the equations of motion around a fixed background solution (which we will call scalarons) can be made either using the traditional perturbation method (the eikonal approximation) or the more elegant formalism synthesized in the work of Hadamard. In this method, the propagation of high-energy scalarons is studied by following the evolution of the wave front, through which the field is continuous but its first derivative is not. To be specific, let σ be the surface of discontinuity defined by the equation

$$\sigma(x^\mu) = const\,.$$

The discontinuity of a function J through the surface σ will be represented by $[J]_\sigma$, and its definition is

$$[J]_\sigma := \lim_{\delta \to 0^+} \left(J|_{\sigma+\delta} - J|_{\sigma-\delta} \right).$$

The discontinuities of the field and its first derivative are given by

$$[\varphi]_\sigma = 0\,, \qquad [\nabla_\mu \varphi]_\sigma = 0\,,$$

$$[\nabla_\mu \nabla_\nu \varphi]_\sigma = \chi\, k_\mu\, k_\nu\,.$$

where the vector k_μ is the normal to the surface of discontinuity. Using these values in the equation of motion for the field φ,

$$\nabla_\mu \left(L_W \nabla^\mu \varphi\right) = 0\,, \tag{13.1.3}$$

we obtain

$$k_\mu\, k_\nu\, g^{\mu\nu}_{eff} = 0\,, \tag{13.1.4}$$

where the effective metric is given by

$$g^{\mu\nu}_{eff} = L_W\, g^{\mu\nu} + 2L_{WW}\, \nabla^\mu \varphi\, \nabla^\nu \varphi \tag{13.1.5}$$

and $g^{\mu\nu}$ is the background metric. Only in the case of a linear theory $L = W$, the metric that controls the propagation of the discontinuities of the field coincides with the background metric. Therefore, the propagation of discontinuities of the scalar field, which we called scalaron, follows null curves in an effective metric that is not universal, but instead depends on the field configuration. We should emphasize that this property is quite generic for any nonlinear field theory. In terms of the background geometry we can re-write the equation of propagation as

$$k_\mu k_\nu g^{\mu\nu} = -\frac{2L_{WW}}{L_W}\, \left(k_\mu \nabla^\mu \varphi\right)^2\,. \tag{13.1.6}$$

This means that in the background geometry the scalaron behaves as time-like particles in cases in which $L_W L_{WW} < 0$, and it behaves as tachyons in the cases in which $L_W L_{WW} > 0$.

13.1.4 Geometrization in non-linear field theory

The question presented in the previous section and that will appear many times in the present lectures can be set in the following convenient way: given that the description of the effects of a gravitational field in the motion of matter can be described by a modification of the geometry of space-time does this implies necessarily that the gravitational field must be described by an equation involving derivatives up to second order of the metric tensor? Should this modification of the geometry as experienced by matter in its kinematically behavior be just a simplified and compact formal way to describe such dynamics? It is certainly true that it is a formal benefit if instead of a force field we can interpret the motion of a body in a given gravitational field as nothing but as a free motion in a specific curved geometry, identifying the gravitational phenomenon as a modification of the geometric structure of space-time. Is this just a formal simplification and no more than this? We shall deal in these lectures with two opposite answers. One, provided by

General Relativity and other given by Pre-Gravity. Before going into these alternative proposals and in order to understand more deeply the status of each one, let us analyze a minor correlated question that appears in a certain class of non-linear field theories. We shall see that there exists some theoretical framework in which the modification of the background geometry describes a given dynamics but it does not allow to treat the whole interaction process through the modification of the associated geometry.

The Effective Metric of Nonlinear Electrodynamics

Historically, the first example of the idea of effective metric was presented in 1923 by W. Gordon. In modern language, the wave equation for the propagation of light in a moving non-dispersive medium, with slowly varying refractive index n and 4-velocity u^μ:

$$\left[\partial_\alpha \partial^\alpha + (n^2-1)(u^\alpha \partial_\alpha)^2\right] F_{\mu\nu} = 0.$$

Taking the geometrical optics limit, the Hamilton-Jacobi equation for light rays can be written as $g^{\mu\nu}k_\mu k_\nu = 0$, where

$$g^{\mu\nu} = \eta^{\mu\nu} + (n^2-1)u^\mu u^\nu \tag{13.1.7}$$

is the effective metric for this problem. It must be noted that only photons in the geometric optics approximation move on geodesics of $g^{\mu\nu}$: the particles that compose the fluid couple instead to the background Minkowskian metric.

Let us study now in detail the example of nonlinear electromagnetism. We start with the action

$$S = \int \sqrt{-\gamma}\, L(F)\, d^4x, \tag{13.1.8}$$

where $F \equiv F^{\mu\nu}F_{\mu\nu}$ [1] Varying this action w.r.t. the potential W_μ, related to the field by the expression

$$F_{\mu\nu} = W_{\mu;\nu} - W_{\nu;\mu} = W_{\mu,\nu} - W_{\nu,\mu},$$

we obtain the Euler-Lagrange equations of motion (EOM)

$$(L_F F^{\mu\nu})_{;\nu} = 0, \tag{13.1.9}$$

where L_F is the derivative $L_F \equiv \partial L/\partial F$. In the particular case of a linear dependence of the Lagrangian with the invariant F we recover Maxwell's

[1] We could have considered $L = L(F, G)$ instead, where $G \equiv F^*_{\mu\nu}F^{\mu\nu}$. This case is studied in detail, and L is an arbitrary function of F. Notice that γ is the determinant of the background metric.

equations of motion.

In the same framework as in the previous case of the non-linear scalar field, let us study the behavior of perturbations of these EOM around a fixed background solution. Instead of using the traditional perturbation method, we shall use the method set out by Hadamard as above. In this method, the propagation of low-energy photons are studied by following the evolution of the wave front, through which the field is continuous but its first derivative is not. To be specific, let Σ be the surface of discontinuity defined by the equation

$$\Sigma(x^{\mu}) = constant.$$

The discontinuities of the field and its first derivative are given by

$$[F_{\mu\nu}]_{\Sigma} = 0, \qquad [F_{\mu\nu,\lambda}]_{\Sigma} = f_{\mu\nu}k_{\lambda}, \tag{13.1.10}$$

where the vector k_{λ} is nothing but the normal to the surface Σ, that is, $k_{\lambda} = \Sigma_{,\lambda}$, and $f_{\mu\nu}$ represents the discontinuity of the field. To set the stage for the nonlinear case, let us first discuss the propagation in Maxwell's electrodynamics, for which $L_{FF} = 0$. The EOM then reduces to $F^{\mu\nu}_{;\nu} = 0$, and taking the discontinuity we get

$$f^{\mu\nu}k_{\nu} = 0. \tag{13.1.11}$$

The other Maxwell equation is given by $F^{*\ ;\nu}_{\mu\nu} = 0$ or equivalently,

$$F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = 0. \tag{13.1.12}$$

The discontinuity of this equation yields

$$f_{\mu\nu}k_{\lambda} + f_{\nu\lambda}k_{\mu} + f_{\lambda\mu}k_{\nu} = 0. \tag{13.1.13}$$

Multiplying this equation by k^{λ} gives

$$f_{\mu\nu}k^2 + f_{\nu\lambda}k^{\lambda}k_{\mu} + f_{\lambda\mu}k^{\lambda}k_{\nu} = 0, \tag{13.1.14}$$

where $k^2 \equiv k_{\mu}k_{\nu}\gamma^{\mu\nu}$. Using the orthogonality condition from Eqn.(9.2.5) it follows that

$$f^{\mu\nu}k^2 = 0 \tag{13.1.15}$$

Since the tensor associated to the discontinuity cannot vanish (we are assuming that there is a true discontinuity!) we conclude that the surface of discontinuity is null w.r.t. the metric $\gamma^{\mu\nu}$. That is,

$$k_{\mu}k_{\nu}\gamma^{\mu\nu} = 0. \tag{13.1.16}$$

It follows that $k_{\lambda;\mu}k^{\lambda} = 0$, and since the vector of discontinuity is a gradient,

$$k_{\mu;\lambda}k^{\lambda} = 0. \tag{13.1.17}$$

This shows that the propagation of discontinuities of the electromagnetic field, in the case of Maxwell's equations (which are linear), is along the null geodesics of the Minkowski background metric.

Let us apply the same technique to the case of a nonlinear Lagrangian for the electromagnetic field, given by $L(F)$. Taking the discontinuity of the EOM, Eqn.(9.2.3), we get

$$L_F f^{\mu\nu}k_\nu + 2\eta\ L_{FF}\ F^{\mu\nu}k_\nu = 0, \tag{13.1.18}$$

where we defined the quantity η by $F^{\alpha\beta}f_{\alpha\beta} \equiv \eta$. Note that contrary to the linear case in which the discontinuity tensor $f_{\mu\nu}$ is orthogonal to the propagation vector k^μ, here there is a complicated relation between the vector $f^{\mu\nu}k_\nu$ and quantities dependent on the background field. This is the origin of a more involved expression for the evolution of the discontinuity vector, as we shall see next. Multiplying equation (9.2.8) by $F^{\mu\nu}$ we obtain

$$\eta\ k^2 + F^{\mu\nu}f_{\nu\lambda}k^\lambda k_\mu + F^{\mu\nu}f_{\lambda\mu}k^\lambda k_\nu = 0. \tag{13.1.19}$$

Now we substitute in this equation the term $f^{\mu\nu}k_\nu$ from Eqn.(9.2.12), and we arrive at the expression

$$L_F\ \eta k^2 - 2\, L_{FF}\eta(F^{\mu\lambda}k_\mu k_\lambda - F^{\lambda\mu}k_\mu k_\lambda), \tag{13.1.20}$$

which can be written as $g^{\mu\nu}k_\mu k_\nu = 0$, where

$$g^{\mu\nu} = L_F\ \gamma^{\mu\nu} - 4\, L_{FF}\ F^{\mu\alpha}F_\alpha{}^\nu. \tag{13.1.21}$$

We then conclude that the high-energy photons of a *nonlinear* theory of electrodynamics with $L = L(F)$ do not propagate on the null cones of the background metric but on the null cones of an *effective* metric, generated by the self-interaction of the electromagnetic field. This statement is always true in case of Lagrangians depending only of the invariant F. For Lagrangians that depend also of F^*, an analogous effective geometry appears.

After these two exercises on the description of the kinematics of the "quanta" of the fields we are led to make two comments. First of all, we note that the modification of the background geometry is a powerful tool in non-linear field theories. The second comment concern the restriction of this modification. Indeed, the effective metric is important only to the limited analysis of the propagation of the discontinuities. The dynamics of the field is not related to the properties of the effective metric, in general. Only in the particular case of Born-Infeld Electrodynamics it is possible to use a functional of the metric - actually, the determinant of the effective metric tensor – to act as the Lagrangian of the field. Thus the complete Einstein geometrized scheme cannot be applied in this case, although a limited form of it is certainly not only possible but very useful. What we have learned with these two examples can be summarized: there are properties of non-linear field theories that can be usefully described by a change of the metric properties of space-time. In certain theories - like Electrodynamics - this is a very limited procedure due to the fact that the theory is not universal. In others, like gravity, it can be applied to all kind of ponderable substances and to all forms of non-gravitational energies. Does this implies necessarily that it applies to the gravitational field itself? In order to answer this question one has to answer a preliminary one: what is the gravitational field? In the General relativity it is identified with the geometry and consequently its dynamics must be provided by products of derivatives of the metric tensor. We shall see that in Pre-Gravity the answer is completely different.

Conservation laws

After accepting to describe the behavior of any matter or energy in a given gravitational field by a modification of the Minkowski metric to a Riemannian geometry one one usually remarks that Bianchi identity gives a "natural" support of Einstein path for the choice of the dynamics of the gravitational field. Indeed, in any Riemannian geometry the metric tensor satisfies identically the divergence-less of tensor $G_{\mu\nu}$ that is

$$G^{\mu\nu}; \nu = 0, \tag{13.1.22}$$

where

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu}.$$

The uses of the minimal coupling principle to couple any form of matter and non-gravitational energy with gravity yields the conservation of the energy-momentum tensor

$$T^{\mu\nu}; \nu = 0. \tag{13.1.23}$$

These two properties are not correlated. General relativity makes the hypothesis that the dynamics of the gravitational field is such that these two identities become just a single one by setting

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = -\kappa \, T_{\mu\nu}. \tag{13.1.24}$$

It is clear that (13.1.24) makes both divergence-less equations to be co-related. However, this is just a sufficient condition, not a necessary one. Indeed, the validity of (13.1.22) and (13.1.23) do not imply the validity of (13.1.24).

13.1.5 Dirac Spinors and the Clifford algebra

In these lectures we will deal with two fields Ψ_E and Ψ_N that are four-components Dirac spinors. We use capital symbols to represent the vector and axial currents constructed with Ψ_E and lower case to represent the corresponding terms of the spinor Ψ_N, namely,

$$J^\mu \equiv \overline{\Psi}_E \gamma^\mu \Psi_E$$

$$I^\mu \equiv \overline{\Psi}_E \gamma^\mu \gamma^5 \Psi_E.$$

$$j^\mu \equiv \overline{\Psi}_N \gamma^\mu \Psi_N$$

$$i^\mu \equiv \overline{\Psi}_N \gamma^\mu \gamma^5 \Psi_N.$$

We use the convention and definitions by Elbaz. For completeness we recall:

$$\overline{\Psi} \equiv \Psi^+ \gamma^0.$$

The Clifford algebra is the algebra of the Dirac γ matrix defined by its basic property

$$\gamma_\mu \, \gamma_\nu + \gamma_\nu \, \gamma_\mu = 2 g_{\mu\nu} \, I. \qquad (13.1.25)$$

In the case of Minkowski background $g_{\mu\nu} = \eta_{\mu\nu}$ we use the convention provided by the form:

$$\widetilde{\gamma}^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}$$

$$\widetilde{\gamma}_k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}.$$

that, from now on, we write 1 instead of I to represent the identity of the Clifford algebra. The γ_5 anti-commute with all γ_μ and is given by

$$\gamma_5 = \frac{i}{4!} \eta^{\alpha\beta\mu\nu} \gamma_\alpha \gamma_\beta \gamma_\mu \gamma_\nu = i \gamma_0 \gamma_1 \gamma_2 \gamma_3,$$

where the second equality is valid in a Euclidean coordinate system in the Minkowski background. Tensor $\eta^{\alpha\beta\mu\nu}$ is given in terms of the completely anti-symmetric Levi-Civita symbol as

$$\eta^{\alpha\beta\mu\nu} = \frac{-1}{\sqrt{-g}} \epsilon^{\alpha\beta\mu\nu}$$

and $g = det g_{\mu\nu}$. The γ^5 is Hermitian and the others γ_μ obey the Hermiticity relation

$$\gamma_\mu^+ = \gamma^0 \gamma_\mu \gamma^0.$$

The Pauli matrices satisfy the condition

$$\sigma_i \, \sigma_j = i \, \epsilon_{ijk} \, \sigma_k + \delta_{ij}.$$

We set

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Any spinor can be decomposed into its left and right parts through the identity

$$\Psi = \Psi_L + \Psi_R = \frac{1}{2}(1 + \gamma^5)\Psi + \frac{1}{2}(1 - \gamma^5)\Psi \qquad (13.1.26)$$

Then

$$\overline{\Psi}_L \, \Psi_L = 0,$$

and

$$\overline{\Psi}_R \, \Psi_R = 0.$$

13.1.6 Pauli-Kofink identity

The properties needed to analyze non-linear spinors are contained in the Pauli-Kofink (PK) relation. These are identities that establish a set of relations concerning elements of the four-dimensional Clifford algebra. The main property states that, for any element Q of this algebra, the PK relation ensures the validity of the identity:

$$(\overline{\Psi} Q \gamma_\lambda \Psi)\gamma^\lambda \Psi = (\overline{\Psi} Q \Psi)\Psi - (\overline{\Psi} Q \gamma_5 \Psi)\gamma_5 \Psi. \qquad (13.1.27)$$

for Q equal to I , γ^μ, γ_5 and $\gamma^\mu \gamma_5$, respectively, where I is the identity of the Clifford algebra. As a consequence of this relation we obtain two extremely important facts:

- The norm of the currents J_μ and I_μ have the same value and opposite sign.
- Vectors J_μ and I_μ are orthogonal.

Thus J_μ is a time-like vector and I_μ is space-like.

Pauli-Kofink formula implies some identities which will be used later on to simplify our calculations:

$$\begin{aligned} J_\mu\, \gamma^\mu\, \Psi &\equiv (A + iB\gamma^5)\, \Psi \\ I_\mu\, \gamma^\mu\, \gamma^5\, \Psi &\equiv -(A + iB\gamma^5)\, \Psi \\ I_\mu\, \gamma^\mu\, \Psi &\equiv (A + iB\gamma^5)\, \gamma^5\Psi \\ J_\mu\, \gamma^\mu\, \gamma^5\, \Psi &\equiv -(A + iB\gamma^5)\, \gamma^5\Psi, \end{aligned} \tag{13.1.28}$$

where $A \equiv \overline{\Psi}\,\Psi$ and $B \equiv i\overline{\Psi}\,\gamma^5\Psi$. Note that both quantities A and B are real.

13.1.7 Dirac dynamics

In these lectures we analyze two dynamics for the spinor fields: one, linear and one non-linear. For the linear case we take Dirac theory:

$$i\gamma^\mu \partial_\mu\, \Psi - \mu\, \Psi = 0 \tag{13.1.29}$$

where $M = \frac{\hbar}{c}\, \mu$ is the mass. The corresponding Lagrangian is

$$L = \hbar\, c\, \left(\frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi - \frac{i}{2} \partial_\mu \bar{\Psi} \gamma^\mu \Psi - \mu\, \bar{\Psi}\, \Psi \right) \tag{13.1.30}$$

Note that on-mass-shell Dirac Lagrangian vanishes:

$$L(oms) = 0.$$

¿From the decomposition in a right Ψ_R and left-handed Ψ_L helicity it follows that the mass-term mix both helicities:

$$i\gamma^\mu \partial_\mu\, \Psi_L - M\, \Psi_R = 0 \tag{13.1.31}$$

$$i\gamma^\mu \partial_\mu\, \Psi_R - M\, \Psi_L = 0 \tag{13.1.32}$$

13.1.8 Heisenberg dynamics

Let Ψ be a fundamental four-component spinor field. The dynamics of Ψ is given by the Heisenberg self-interaction Lagrangian (we use from now on the conventional units were $\hbar = c = 1$) :

$$L = \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi - \frac{i}{2} \partial_\mu \bar{\Psi} \gamma^\mu \Psi - V(\Psi). \tag{13.1.33}$$

The potential V is constructed with the two scalars that can be formed with Ψ, that is A and B. We will only consider the Heisenberg potential that is

$$V = s\left(A^2 + B^2\right) \tag{13.1.34}$$

where s is a real parameter of dimension $(length)^2$ yielding the equation of motion

$$i\gamma^\mu \partial_\mu \Psi^H - 2s\left(A + iB\gamma^5\right)\Psi^H = 0 \tag{13.1.35}$$

Correspondingly we have

$$i\partial_\mu \overline{\Psi}^H \gamma^\mu + 2s\,\overline{\Psi}^H \left(A + iB\gamma^5\right) = 0 \tag{13.1.36}$$

The Heisenberg potential V_H can be written in an equivalent and more suggestive form in terms of the associated currents J_μ and I_μ. As a direct consequence of Pauli-Kofinki identities, Heisenberg potential V is nothing but the norm of the four-vector current J^μ, that is

$$A^2 + B^2 = J^\mu J_\mu.$$

Note that on-mass-shell, Heisenberg Lagrangian takes the value of its potential:

$$L(oms) = V_H.$$

Gauge invariance

The dynamics displayed by both Dirac and Heisenberg equations of motion are invariant under the map

$$\widetilde{\Psi} = S\,\Psi, \tag{13.1.37}$$

where S is a unitary matrix[2]. From Noether theorem this imply that the current J_μ is conserved. When the transformation S is space-time dependent one has to introduce a modification on the derivative as much the same as it occurs for tensors in arbitrary coordinate transformation when a covariant derivative is defined. We shall deal with this spinor covariant derivative latter on.

Chiral invariance

Chiral transformation is defined by the map

$$\Psi' = \gamma_5\,\Psi.$$

[2]We treat in detail the case in which matrix S satisfies the condition $S^{-1}S_{,\mu} = c_\mu\, I$.

Dirac equation is invariant under this map only for massless neutrino equation. On the other hand, Heisenberg equation is invariant under chirality. Indeed, we have, for the conjugate spinor:

$$\overline{\Psi}' = -\overline{\Psi}\gamma_5,$$

which implies

$$A' = -A$$
$$B' = -B$$

consequently the Lagrangian remains the same

$$L' = L$$

Although the constant s is not a "mass" it provokes the similar mixing of Heisenberg spinors Ψ_L and Ψ_R. Indeed, we have

$$i\gamma^\mu \partial_\mu \Psi_L - 2s\,(A - iB)\,\Psi_R = 0 \tag{13.1.38}$$
$$i\gamma^\mu \partial_\mu \Psi_R - 2s\,(A + iB)\,\Psi_L = 0 \tag{13.1.39}$$

Plane wave solution of Heisenberg equation

Although Heisenberg equation is non-linear it admits a solution as a plane wave. Actually, any non-linear equation admits such particular type of solution as noticed by M. Born (citar non-linear optics). We set

$$\Psi = e^{ik_\alpha x^\alpha}\,\Psi^o \tag{13.1.40}$$

where Ψ^o is a constant spinor written in terms of two-components spinors:

$$\Psi^o = \begin{pmatrix} \varphi \\ \eta \end{pmatrix}.$$

It is convenient to write Heisenberg equation in the form

$$i\gamma^\mu \partial_\mu \Psi - 2s\,(A + iB\gamma^5)\,\Psi = 0 \tag{13.1.41}$$

The above decomposition implies that the two-components spinors are not completely independent. They must satisfy the constraint

$$\eta = \left(\frac{\sigma_i k^i - 2isB_o}{k_0 - 2sA_o} \right) \varphi. \tag{13.1.42}$$

Compatibility requires the "on-mass" condition

$$k_\mu k^\mu = 4s^2\,(A_o^2 - B_o^2).$$

13.1.9 The Inomata solution of Heisenberg dynamics

In ... a very interesting class of solutions of Heisenberg equation was set out by Inomata. The interest on this class rests on the fact that it allows one directly to deal with the derivatives of the spinor field allowing, consequently, to obtain derivatives of the associated metric tensor. Let us present briefly this class of Heisenberg spinors.

Inomata starts his analysis by the recognition that one can construct a subclass of solution of Heisenberg dynamics by imposing a more restrictive condition given by

$$\partial_\mu \Psi = \left(a\, J_\mu + b\, I_\mu \gamma^5 \right)\, \Psi \tag{13.1.43}$$

where a and b are complex numbers of dimensionality $(lenght)^2$. A Ψ that satisfies such Inomata condition will be called I-spinor. It is immediate to prove that if Ψ satisfies condition(13.1.43) it satisfies automatically Heisenberg equation of motion. This is a rather strong condition that deals with simple derivatives instead of the scalar structure obtained by the contraction with γ_μ typical of Dirac or Heisenberg operators. Thus prior of anything one has to examine its compatibility concerning all quantities that one can construct with such spinors. It is a remarkable result that in order that the restrictive condition eq. (13.1.43) to be integrable the constants α and β must satisfy a unique constraint given by $Re(a) - Re(b) = 0$.

Indeed, we have

$$[\partial_\mu, \partial_\nu]\, \Psi = \left(a\, \partial_{[\mu}\, J_{\nu]} + b\, \partial_{[\mu}\, I_{\nu]}\, \gamma^5 \right)\, \Psi.$$

Now, the derivative of the currents yields

$$\partial_\mu J_\nu - \partial_\nu J_\mu = (a + \bar{a})[J_\mu,\, J_\nu] + (b + \bar{b})\, [I_\mu,\, I_\nu],$$

and

$$\partial_\mu I_\nu - \partial_\nu I_\mu = (a + \bar{a} - b - \bar{b})[J_\mu\, I_\nu - I_\mu\, J_\nu].$$

Thus the condition of integrability is given by

$$Re(a) = Re(b). \tag{13.1.44}$$

Note that a and b are related to Heisenberg constant by $2s = i\,(a - b)$.

It is a rather long and tedious work to show that any combination X constructed with Ψ and for all elements of the Clifford algebra is such that the compatibility condition $[\partial_\mu, \partial_\nu]X = 0$ is automatically fulfilled under the unique condition (13.1.44). Let us now turn to some remarkable properties of I-spinors.

Lemma. The current four-vector J^{μ} is irrotational. The same is valid for the axial-current I^{μ}.

The proof that the vector J^{μ} is the gradient of a certain scalar quantity is a simple direct consequence of its definition in terms of H-spinors. However there is a further property that is worth of mention: this scalar is nothing but the norm J^2 of the current. Indeed, using equation (13.1.43), we have

$$\partial_{\mu} J_{\nu} = (a + \bar{a}) J_{\mu} J_{\nu} + (b + \bar{b}) I_{\mu} I_{\nu} \tag{13.1.45}$$

This expression shows that the derivative of the four-vector current is symmetric. Multiplying eq. (13.1.45) by J^{μ} and using the properties established before it follows then

$$J_{\mu} = \partial_{\mu} S \tag{13.1.46}$$

in which the scalar S is written in terms of the norm J^2:

$$S = \frac{1}{a + \bar{a}} \, ln \sqrt{J^2}. \tag{13.1.47}$$

Note that $S = const.$ defines a hypersurface in space-time such that the current J_{μ} is not only geodesic but orthogonal to S. It follows

$$\partial_{\mu} S \, \partial^{\nu} S \, \eta^{\mu\nu} = e^{2(a+\bar{a})S},$$

or, defining the conformal metric

$$g^{c}_{\mu\nu} \equiv e^{2(a+\bar{a})S} \, \eta_{\mu\nu}$$

we write

$$\partial_{\mu} S \, \partial^{\nu} S \, g_{c}^{\mu\nu} = 1, \tag{13.1.48}$$

showing that S is an eikonal in the conformal space.

Lemma. The two four-vectors J_{μ} and I_{μ} constitutes a basis for vectors constructed by the derivative ∂_{μ} operating on functionals of Ψ.

Proof. It is enough to show that this assertion is true for the scalars A and B. Indeed, we have:

$$\partial_{\mu} A = (a + \bar{a}) \, A \, J_{\mu} + (b - \bar{b}) \, iB \, I_{\mu}. \tag{13.1.49}$$

and

$$\partial_{\mu} B = (a + \bar{a}) \, B \, J_{\mu} + (b - \bar{b}) \, iA I_{\mu}. \tag{13.1.50}$$

It then follows that the vector I_{μ} is a gradient too. Indeed,

$$\partial_{\mu} I_{\nu} = (a + \bar{a}) J_{\mu} I_{\nu} + (b + \bar{b}) J_{\nu} I_{\mu}. \tag{13.1.51}$$

$$I_\mu = \partial_\mu R \tag{13.1.52}$$

in which the scalar R is:

$$R = \frac{1}{b - \bar{b}} \, ln \left(\frac{A - iB}{\sqrt{J^2}} \right) \tag{13.1.53}$$

or,

$$R = \frac{i}{b - \bar{b}} \, \arccos \frac{B}{\sqrt{J^2}} \tag{13.1.54}$$

13.1.10 Another form of constrained solution

Besides the formal expression that we exhibited in the previous section, there is another one that deals with the derivative of the field Ψ but which yields very different consequences. Indeed, a solution of Heisenberg dynamics is obtained if one sets instead of equation (13.1.43):

$$\partial_\mu \Psi = -i \, g_F \, \left(J_\mu + \frac{(1+\beta)}{2} \, I_\mu \, + (\frac{(1+\beta)}{2} J_\mu + \beta \, I_\mu) \, \gamma^5 \right) \, \Psi \tag{13.1.55}$$

A direct calculation, using Pauli-Kofink relations shows that such a Ψ satisfies identically Heisenberg dynamics. Indeed, we have

$$i \gamma^\mu \, \partial_\mu \Psi = g_F \, \left(A + iB\gamma^5 \right) \left(1 + \frac{1+\beta}{2} \gamma^5 - \frac{1+\beta}{2} \gamma^5 - \beta \right) \, \Psi \tag{13.1.56}$$

or

$$i \gamma^\mu \, \partial_\mu \Psi = g_F \, (1 - \beta) \left(A + iB\gamma^5 \right) \Psi \tag{13.1.57}$$

Thus we should identify $g_F \, (1 - \beta) = 2s$. Now comes, however a very distinct property: such a solution yields that both associated currents are constants, that is:

$$\partial_\mu J_\nu = 0; \tag{13.1.58}$$

$$\partial_\mu I_\nu = 0; \tag{13.1.59}$$

This can be understood on the basis of the previous Inomata ansatz if we note that for this case the constant a and b become pure imaginary numbers. Such second form of Inomata solution will play an important role when we will consider later on basic equations of the fundamental spinors of pre-gravity theory.

13.1.11 Internal connection

Sometimes it is useful to treat spinor equation of motion in a non-euclidean system of coordinates. In order to deal with the covariance of the theory we have to deal with the concept of internal connection. In the case of an arbitrary Riemannian geometry (of which the Minkowski metric is a particular case) Fock and Ivanenko displayed the main properties needed to obtain such covariant description. This means, exchanging the simple derivative for a covariant one defined by

$$\nabla_\mu \Psi = \partial_\mu \Psi - i\Gamma_\mu \Psi. \tag{13.1.60}$$

The quantity Γ_μ is called the internal connection and acts in the same way as Christoffel symbols for tensors allowing the definition of a derivative which generates tensor quantities that transform co-variantly under arbitrary coordinate transformations. In order to arrive a specific form of this connection in terms of the γ_μ and its derivatives, Fock and Ivanenko make the hypothesis that the covariant derivative of γ_μ vanishes. This is the counter-part in the spinor world of the tensorial condition of vanishing covariant derivative of the metric tensor. In the case of this Riemann hypothesis one arrives at the class of geometries called Riemannian manifolds. In the case of the spinor structure one arrives at the Fock-Ivanenko class. We note that the Fock-Ivanenko condition is much less restrictive and it implies the Riemannian one. Indeed, from the defined relation of the anti-commutation of the γ_μ and the Fock-Ivanenko condition it follows directly the vanishing of the co-variant derivative of the metric tensor. On the other hand, the vanishing of the metric tensor does not requires the vanishing of the γ_μ. Assuming the Fock-Ivanenko condition one obtains the internal connection as

$$\Gamma^0_\mu = \frac{1}{8}\left[\gamma^\alpha \gamma_{\mu,\alpha} - \gamma_{\mu,\alpha}\gamma^\alpha + \Gamma^\epsilon_{\mu\nu}\left(\gamma_\epsilon \gamma^\nu - \gamma^\nu \gamma_\epsilon\right)\right]. \tag{13.1.61}$$

The index 0 in Γ_μ is just a reminder that we are dealing with a Minkowski background in an arbitrary system of coordinates. We can globally annihilate such connection by moving to an Euclidean constant coordinate system.

13.1.12 Generalized internal connection

The expression of the internal connection as displayed by Fock and Ivanenko was obtained by assuming that the covariant derivative of all γ_μ vanish. This is a direct consequence of relation (13.1.25). Indeed, $\nabla_\mu \gamma_\nu = 0$ implies that the metric is Riemannian: $\nabla_\mu g_{\alpha\beta} = 0$. However, although the condition of vanishing covariant derivatives of γ_μ is enough to guarantee the Riemannian structure of the geometry, it is not necessary. Novello examined a case in which the dynamics of the Clifford structure is driven by the condition of the commutator:

$$\nabla_\mu \gamma_\nu = [U_\mu , \gamma_\nu], \tag{13.1.62}$$

where U_μ is an arbitrary element of the Clifford algebra.

Indeed, from the relation (13.1.25) and using the above expression with $U_\mu = A_\mu + B_\mu \gamma_5$, we have for arbitrary vectors A_μ and B_μ :

$$\nabla_\mu \gamma_\nu = [A_\mu + B_\mu \gamma_5, \gamma_\nu]. \tag{13.1.63}$$

Thus

$$\nabla_\mu g_{\alpha\beta} = [U_\mu, \gamma_\alpha]\, \gamma_\beta + \gamma_\alpha [U_\mu, \gamma_\beta] + [U_\mu, \gamma_\beta]\, \gamma_\alpha + \gamma_\beta\, [U_\mu, \gamma_\alpha]$$

Using the property that γ_5 anti-commutes with all γ_ν, it follows that $\nabla_\mu g_{\alpha\beta} = 0$. This holds for arbitrary vectors A_μ and B_μ.

We shall see that the internal connection obtained in this way provides an equivalent way to describe the non linear structure of Heisenberg spinors for a convenient choice of U_μ. Thus the generalized internal connection takes the form

$$\Gamma_\mu = \Gamma^0_\mu - iU_\mu. \tag{13.1.64}$$

13.1.13 Geometrical description of Heisenberg dynamics

We now show that it is possible to understand the self-coupling of equation (13.1.35) in terms of a modification of the internal connection. In so doing, we are preparing our analysis for the universal gravitational interaction of the non-linear spinor theory. Let us use the form (13.1.64) and set

$$\Gamma^1_\mu = -i \left(a_0 J_\mu + b_0 I_\mu \gamma^5 \right) \tag{13.1.65}$$

in which, for simplicity we use an Euclidean coordinate system in which the Fock-Ivanenko standard part of the connection vanishes. Thus we have

$$\nabla^1_\mu \Psi = \partial_\mu \Psi - \left(a_0 J_\mu + b_0 I_\mu \gamma^5 \right) \Psi \tag{13.1.66}$$

Then we can re-write Heisenberg equation in the form

$$i\gamma^\mu \, \nabla^1_\mu \Psi = 0 \tag{13.1.67}$$

once

$$i\gamma^\mu \, \nabla^1_\mu \Psi = i\gamma^\mu \partial_\mu \Psi - i\gamma^\mu \left(a_0 J_\mu + b_0 I_\mu \gamma^5 \right) \Psi \tag{13.1.68}$$

Indeed, using identities (13.1.28) we have

$$i\gamma^\mu \, \nabla^1_\mu \Psi = i\gamma^\mu \partial_\mu \Psi - i(a_0 - b_0) \left(A + iB\gamma^5 \right) \Psi \tag{13.1.69}$$

and identifying $2s = i(a_0 - b_0)$.

Let us note that we could in an equivalent way choose a modified form and instead of (13.1.65) use

$$\Gamma_\mu = -i\left(aJ_\mu + bI_\mu\right)\left(I + \gamma^5\right) \tag{13.1.70}$$

In this case the Lagrangian of the fundamental spinor takes the form

$$\begin{aligned} L &= \frac{i}{2}\bar{\Psi}\gamma^\mu\nabla_\mu\Psi - \frac{i}{2}\nabla_\mu\bar{\Psi}\gamma^\mu\Psi \\ &= \frac{i}{2}\bar{\Psi}\gamma^\mu\partial_\mu\Psi + \frac{1}{2}\bar{\Psi}\gamma^\mu\Gamma_\mu\Psi + h.c. \end{aligned} \tag{13.1.71}$$

Substituting the form (13.1.70) in this Lagrangian we obtain

$$L = \frac{i}{2}\bar{\Psi}\gamma^\mu\partial_\mu\Psi - \frac{i}{2}\partial_\mu\bar{\Psi}\gamma^\mu\Psi - \frac{i}{2}\left[(a-\bar{a}) - (b-\bar{b}\right] J_\mu J^\mu. \tag{13.1.72}$$

This is precisely the expression of Heisenberg Lagrangian (13.1.33) and (13.1.34) which led us to the identification

$$s = \frac{i}{2}\left[(a-\bar{a}) - (b-\bar{b})\right] = \frac{i}{2}(a-\bar{a})(1-\beta). \tag{13.1.73}$$

We shall define

$$g_F \equiv i\hbar\, c\,(a-\bar{a}).$$

This proves that Heisenberg self-interaction can be interpreted in a geometrical way, as a modification of the internal connection. Then, we can re-write Heisenberg equation in the compact form

$$i\gamma^\mu\,\nabla_\mu\Psi = 0 \tag{13.1.74}$$

where we used the connection provided by eq (13.1.70).

13.1.14 Geometrical realization of the interaction of Ψ_E and Ψ_N

We shall see in the next lectures that gravitational processes deal with two fundamental fields which we will call Ψ_E and Ψ_N. These spinors interact with each other and with all forms of matter. In the present section we show how the coupling of Ψ_E and Ψ_N are described in the same geometrical framework as in the self-interaction case presented above. This analysis is based on the hypothesis that these two fields are indistinguishable, as far as gravitational processes are concerned. Thus the internal connection is given by the generalization (see (13.1.70)):

$$U_\mu = \left[a\ (J_\mu + j_\mu) + b\ (I_\mu + i_\mu)\right] \left(I + \gamma^5\right) \tag{13.1.75}$$

We note that we are using the same value for constants a and b assuming that the theory is symmetrical under the exchange of Ψ_E and Ψ_N. Following the same procedure as in the precedent case one sets for the Lagrangian the form

$$\begin{aligned} L &= \frac{i}{2}\bar{\Psi}_E\gamma^\mu\nabla_\mu\Psi_E + \frac{i}{2}\bar{\Psi}_N\gamma^\mu\nabla_\mu\Psi_N + h.c. \\ &= \frac{i}{2}\bar{\Psi}_E\gamma^\mu\partial_\mu\Psi_E + \frac{1}{2}\,\bar{\Psi}_E\gamma^\mu\Gamma_\mu\Psi_E + \frac{i}{2}\bar{\Psi}_N\gamma^\mu\partial_\mu\Psi_N + \frac{1}{2}\,\bar{\Psi}_E\gamma^\mu\Gamma_\mu\Psi_N \end{aligned} \tag{13.1.76}$$

The interaction term assumes the form

$$L_{int} = -sJ^2 - sj^2 - g_F\left(J^\mu j_\mu + \beta\, I^\mu i_\mu + \frac{1+\beta}{2}\, I^\mu j_\mu + \frac{1+\beta}{2}\, J^\mu i_\mu\right) \tag{13.1.77}$$

where $s = \frac{g_F}{2}\,(1-\beta)$. The first two terms represents Heisenberg self-interactions of both fields; the others concern the interaction between Ψ_E and Ψ_N. We have already commented on the property that this interaction reduces to the Fermi Lagrangian.

This expression can be written in a compact form using two vectors Σ_μ and Π_μ defined as

$$\Sigma_\mu \equiv J_\mu + j_\mu + I_\mu + i_\mu \tag{13.1.78}$$

and

$$\Pi_\mu \equiv J_\mu + j_\mu + \beta\ (I_\mu + i_\mu)\,. \tag{13.1.79}$$

Then it follows immediately

$$L_{int} = -\,\frac{g_F}{2}\,\Sigma_\mu\Pi^\mu.$$

Let us note that we can use definitions (13.1.78) and (13.1.79) and re-write the equation of motion in a compact form. Indeed, we have

$$\begin{aligned} J_\mu + j^\mu + \frac{(1+\beta)}{2}\,(I_\mu + i^\mu) &= \frac{1}{2}\,(\Sigma_\mu + \Pi_\mu) \\ \frac{(1+\beta)}{2}\,(J_\mu + j^\mu) + \beta(I_\mu + i^\mu) &= \frac{1}{2}\,(\beta\,\Sigma_\mu + \Pi_\mu) \end{aligned} \tag{13.1.80}$$

Then, in the absence of matter we have, for both fields,

$$i\gamma^\mu\,\partial_\mu\Psi - \frac{g_F}{2}(\Sigma_\mu + \Pi_\mu)\gamma^\mu\,\Psi - \frac{g_F}{2}(\beta\,\Sigma_\mu + \Pi_\mu)\gamma^\mu\gamma^5\,\Psi = 0. \tag{13.1.81}$$

13.1.15 Numerology

Some dimensional quantities that will be used later on will be displayed in a compact way in this section.

For the field we set

$$[\Psi] = L^{-3/2};$$

and consequently, for the current

$$[J_\mu] = L^{-3};$$

Besides, we recall

$$[\hbar] = M\,L^2\,T^{-1};$$
$$[\hbar/c] = M\,L;$$
$$[s] = L^2;$$
$$[g_F] = [\hbar\,c\,s] = M\,L^5\,T^{-2};$$
$$[g_N] = M^{-1}\,L^3\,T^{-2};$$

The interaction of Ψ_E and Ψ_N presented in the previous section is similar to the Lagrangian of weak interaction processes in Fermi treatment. The Fermi constant g_F appears for dimensionality reasons. The presence of such constant in the realm of gravitational world – which is the true goal of our analysis in these Notes — may seem very unusual . However, an interesting remark attributed to W. Pauli makes this identification less strange. It is generally argued that, as far as gravity is concerned, the quantity $10^{-33}cm$ is an important one. This number appears very naturally by simple dimensional analysis and in certain scientific communities this length is associated to the appearance of quantum gravitational processes. Its expression contains three ingredients: the relativistic quantity c (the light velocity), the Heisenberg constant $\hbar$ and a typical gravity representative provided by Newton 's constant g_N, yielding the Planck-Newton constant:

$$L_{PN} = \sqrt{\frac{\hbar\, g_N}{c^3}}.$$

A similar quantity cannot be constructed with the other known long range field (electrodynamics), but it can be defined for the weak interaction. In this case we have only to exchange g_N by the Fermi constant, yielding the definition of what we call the Planck-Fermi length:

$$L_{PF} = \sqrt{\frac{g_F}{\hbar c}}.$$

Now Pauli remarks that this quantity is equal to $10^{-16}cm$, the square-root of the Planck-Newton value. It is clear that such a coincidence depends on the units used. The original argument, which in a sense was re-taken by Dicke in 1957 deals with the so-called "natural system of units" for the high energy physics community, that is for $\hbar = c = 1$ and by taking a specific unit of mass (the electron mass in Dicke's choice).

13.1.16 Field theory of gravity and General Relativity

Half a century has already elapsed since the idea of dealing with the content of General Relativity in terms of a field theory propagating in a non-observable Minkowski background was presented by Gupta, Feynman and others. In recent times this approach has been revised and commented. The field theoretical approach goes back to the fact that Einstein dynamics of the curvature of the Riemannian metric of space-time can be obtained as a sort of iterative process, starting from a linear theory of a symmetric second order tensor $\varphi_{\mu\nu}$ and by an infinite sequence of self-interacting process leading to a geometrical description.

13.1.17 Short review of Gupta-Feynman field theory presentation of General Relativity

For a weak field let us set the linear approximation

$$g_{\mu\nu} \approx \eta_{\mu\nu} + \epsilon\, \varphi_{\mu\nu}$$

where ϵ is a small parameter, such that we can neglect terms of higher order on it. It follows that the inverse contra-variant expression is provided, at the same order, by

$$g^{\mu\nu} \approx \eta^{\mu\nu} - \epsilon\, \varphi^{\mu\nu}.$$

In the linear regime Einstein's equations takes the form

$$G^{L}_{\mu\nu} = -\kappa\, T^{M}_{\mu\nu}. \tag{13.1.82}$$

where the linear differential operator is

$$G^{L}_{\mu\nu} \equiv \partial_\alpha\, \partial^\alpha\, \varphi_{\mu\nu} - \partial_\epsilon \partial_{(\mu} \varphi^{\epsilon}_{\nu)} + \partial_\mu \partial_\nu \varphi - \eta_{\mu\nu}(\partial_\alpha\, \partial^\alpha\, \varphi - \partial_\alpha \partial_\beta \varphi^{\alpha\beta}). \tag{13.1.83}$$

Gupta remarked that this equation should not be correct: the lhs is identically divergence-free and the rhs is not, once it does not contains the amount of energy present under gravitational form. In order to conciliate this, one must add a missing term to the energy-momentum tensor of matter that represents the contribution coming from the gravitational field:

$$rhs \approx T^{M}_{\mu\nu} + T^{1}_{\mu\nu}.$$

The added term is of order $0(\epsilon^2)$. This term comes from a term of order three in the Lagrangian. Thus, one has to add to the Lagrangian a term too, in order to correct the added one:

$$rhs \approx T^M_{\mu\nu} + T^1_{\mu\nu} + T^2_{\mu\nu}.$$

This process continues to infinity, once each time we add a new term, another term must be introduced for higher order of correction. An unexpected result then comes: this series admits a sum. Indeed, the result can be written in a compact form if one uses the geometrical language of Riemann manifold in the following way. We define a Riemannian geometry in terms of the metric of the background $\eta_{\mu\nu}$ as follows

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \varphi_{\mu\nu} \tag{13.1.84}$$

Note that this is not an approximation formula as above, but instead an exact one. The inverse metric $(g_{\mu\nu})^{-1} \equiv g^{\mu\nu}$ is defined by $g_{\nu\mu} g^{\mu\alpha} = \delta^\alpha_\nu$. Other definitions were also used, for instance,

$$\sqrt{-g}\, g^{\mu\nu} \equiv \sqrt{-\gamma}\, (\gamma^{\mu\nu} + \varphi^{\mu\nu})$$

where $\gamma_{\mu\nu}$ is the background Minkowski metric, written in an arbitrary system of coordinates. Using these definitions and after a rather long and terrible calculation the above series is reduced to the final geometrical form presented in General Relativity:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = -\kappa\, T_{\mu\nu}. \tag{13.1.85}$$

For each particular choice of relationship between the field $\varphi_{\mu\nu}$ and the metric, a distinct field theory representation of General Relativity appears.

Although these theories can be named " field theories" they contain the same metric content of GR, disguised in a non geometrical form. The framework of the Spinor Theory of Gravity is totally different. It is important to emphasize that we will not present a dynamics for the metric in the sense of such field theories. Instead, the geometry is to be understood as an effective one, in the sense that it is the way gravity appears for all forms of matter and energy: its evolution is given by the fundamental spinor fields Ψ_E and Ψ_N.

We learn from these field theories of gravity the way to couple the tensor field $\varphi_{\mu\nu}$ with matter in order to guarantee that the net effect of such an interaction is to produce the modification of the metric structure. This idea will guide us when coupling the two fundamental spinors with all forms of matter and energy in order to arrive at the same interpretation of the identification of the gravitational field with the metric of the space-time.

13.1.18 A new implementation of the Equivalence principle: universal coupling of Ψ_E and Ψ_N with matter

¿From the previous section, we understand that the strategy of the PreGravity theory is to treat the interaction of the spinors fields in terms of a modification of an internal connection. Now we face the question: how does matter of any form and any kind of energy interact with these two fields? Following this strategy we make a major hypothesis (which substitutes the corresponding hypothesis made by Einstein on the dynamics of $g_{\mu\nu}$) that the fundamental fields Ψ_E and Ψ_N interact universally with all forms of matter/energy through the modification of the internal connection Γ_μ. Let us review briefly the way GR describes this coupling and compare it with our procedure for the STG.

13.1.19 Kinematics: The behavior of matter in a given gravitational field

The coupling of matter to gravity is provided by the identification of the gravitational field with the geometry. This means that we have to modify the matter Lagrangian in the Minkowski background by changing $\eta_{\mu\nu}$ to $g_{\mu\nu} \equiv \eta_{\mu\nu} + \varphi_{\mu\nu}$. This part of the action – which answers the question of how gravity acts on matter – has precisely the same structure as in General Relativity. Indeed, let us consider that in the Special Theory of Relativity the dynamics of a certain matter distribution is provided by a Lagrangian L_m. General Relativity describes its interaction with gravity using the Equivalence Principle, also known as the minimal coupling principle. This requires the substitution of all terms in the action S_0 in which the Minkowski metric $\gamma_{\mu\nu}$ appears by the induced metric $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$. As an example consider a scalar field Φ such that L_0 be the associated Lagrangian – neglecting gravitational forces – given by

$$S_0 = \int \sqrt{-\gamma}\, L_0 = \int \sqrt{-\gamma}\, B^{\mu\nu}\, \gamma_{\mu\nu}. \tag{13.1.86}$$

For a specific example, we set

$$S_0 = \int \sqrt{-\gamma}\, B^{\mu\nu}\, \gamma_{\mu\nu} = \int \sqrt{-\gamma}\, \partial^\mu\Phi\, \partial^\nu\Phi\, \gamma_{\mu\nu}, \tag{13.1.87}$$

where $\gamma \equiv \det \gamma_{\mu\nu}$. In this case $B^{\mu\nu}$ can be written in terms of the energy-momentum tensor defined as

$$T_{\mu\nu} = \frac{2}{\sqrt{-\gamma}}\, \frac{\delta}{\delta\gamma^{\mu\nu}} \left(\sqrt{-\gamma} L\right). \tag{13.1.88}$$

Indeed, a direct calculation yields

$$T^{\mu\nu} = \partial_\alpha \Phi \partial_\beta \Phi \, \gamma^{\alpha\mu} \gamma^{\beta\nu} - \frac{1}{2} \partial_\lambda \Phi \partial_\sigma \Phi \, \gamma^{\lambda\sigma} \gamma^{\mu\nu} \tag{13.1.89}$$

immediately implying

$$B^{\mu\nu} = T^{\mu\nu} - \frac{1}{2} T \gamma^{\mu\nu}, \tag{13.1.90}$$

where $T \equiv T^{\mu\nu} \gamma_{\mu\nu}$. The corresponding action, including the gravitational interaction, is obtained by replacing $\gamma_{\mu\nu}$ and its inverse $\gamma^{\mu\nu}$ with the corresponding $g_{\mu\nu} = \gamma_{\mu\nu} + \varphi_{\mu\nu}$ which yields

$$S = \int \sqrt{-\gamma} \, \omega \, \partial^\mu \Phi \, \partial^\nu \Phi \, g_{\mu\nu}, \tag{13.1.91}$$

where we have used the standard definition such that $\omega \equiv \sqrt{-g} / \sqrt{-\gamma}$, and $g = det \, g_{\mu\nu}$. In this case

$$B^{\mu\nu} = \omega \, [T^{\mu\nu} - \frac{1}{2} T g^{\mu\nu}]. \tag{13.1.92}$$

This procedure can be generalized in such a way that for any kind of matter interacting with the gravitational field, the action is provided by the golden rule of General Relativity, namely

$$S = \int \sqrt{-\gamma} \, \omega \, L_M = \int \sqrt{-g} \, L_M \tag{13.1.93}$$

where the corresponding energy-momentum tensor is given by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \left(\sqrt{-g} L \right). \tag{13.1.94}$$

It follows that this quantity is divergence-free, in the induced metric $g_{\mu\nu}$, that is, $T^{\mu\nu}{}_{;\nu} = 0$. The equation of evolution of the scalar field follows from this property. This procedure is generalized for any kind of matter and non-gravitational energy. We can thus state the important point concerning the kinematics of gravity in the following statements:

In Pre-Gravity Theory the interaction of matter with gravity is precisely the same as in GR. The way matter couples with the fundamental fields Ψ_E and Ψ_N guarantees that, kinematically, the behavior of any kind of matter (and energy) in PGT coincides with the response given by GR, that is: free particles follow geodesics in a prescribed geometry, due to gravitational interaction.

13.1.20 The induced metric

In order to construct an effective metric with product of currents we must define the tensor field in the way it was proposed in the Gupta-Feynmann approach of General Relativity. In the same way as in the field theoretical prescription of General Relativity, each choice provides a distinct form of representation with the same physical content. We will use the simplest combination guided by the properties displayed in (13.1.78) and (13.1.79), that is

$$\varphi_{\mu\nu} = -\, g_F\, g_m \,\frac{\Sigma_\mu\, \Pi_\nu + \Sigma_\nu\, \Pi_\mu}{\sqrt{X}} \tag{13.1.95}$$

where parameter g_m has the dimensionality as the inverse of energy (the field $\varphi_{\mu\nu}$ is dimensionless) and X is given by

$$X \equiv \Sigma_\mu\, \Pi^\mu.$$

Let us make another comment here concerning the number of independent components of the field. Four-dimensional Dirac spinor has 8 real components. Thus, the two spinors that we are dealing here contains 16 components. This number however is not the number of independent components contained in the field $\varphi_{\mu\nu}$.

The reason is the following. The induced metric deals only with the currents associated to the two fundamental spinors fields. The dynamics of these fields are invariant under a Lorentz rotation, which is characterized by 6 numbers. It then follows that one has to substract 6 from the total 16, which leaves the necessary 10 components to define an arbitrary symmetric second-order tensor.

13.1.21 Generating the gravitational field

Let us now turn to the influence of matter on the gravitational field. The dynamics of the gravitational field is completely distinct in these two theories. In General Relativity, the metric obeys a dynamics generated by the Hilbert-Einstein Lagrangian

$$S_{HE} = \frac{1}{k_e} \int \sqrt{-g}\, R d^4x.$$

Nothing similar occurs in the Spinorial Theory of Gravity. The metric does not have a specific dynamics, but instead obeys the evolution dictated by its relationship with the dynamics of the fundamental spinors. The dynamics presented contains the following terms:

$$L = L(\Psi_E) + L(\Psi_N) + L_{int}(\Psi_E, \Psi_N) + L_{mat}. \tag{13.1.96}$$

We make our analysis on the equation for any spinor Ψ, say Ψ_E. The corresponding equation for the other field Ψ_N is obtained similarly by substituting Ψ_E by Ψ_N. We have:

$$i\gamma^\mu \partial_\mu \Psi - g_F\, \gamma_\mu \left(C^\mu + D^\mu \gamma^5 \right) \Psi = 0 \tag{13.1.97}$$

We write this equation in the equivalent compact form:

$$i\gamma^\mu \partial_\mu \Psi - g_F\, \mathcal{H}\, \Psi = 0, \tag{13.1.98}$$

That is

$$\mathcal{H} \equiv \gamma_\mu\, C^\mu + \gamma_\mu\, \gamma^5\, D^\mu. \tag{13.1.99}$$

Let us write:

$$\mathcal{H} = \mathcal{H}_s + \mathcal{H}_o + \mathcal{H}_m, \tag{13.1.100}$$

which, respectively, represents the self-interaction $\mathcal{H}_s$, the interaction with the other spinor $\mathcal{H}_o$ and the influence of matter $\mathcal{H}_m$. Thus, the quantities C^μ and D^μ are separated in three parts, according to their origin in the process of interaction. Let us analyze each part separately:

13.1.22 Self-interaction

Heisenberg dynamics is represented by:

$$\mathcal{H}_s\, \Psi = (1 - \beta)(A + iB\, \gamma^5)\, \Psi \tag{13.1.101}$$

which implies

$$\begin{aligned} C^\mu_s &\equiv J^\mu + \frac{1+\beta}{2}\, I^\mu \\ D^\mu_s &\equiv \frac{1+\beta}{2}\, J^\mu + \beta I^\mu \end{aligned} \tag{13.1.102}$$

This term, which contains only quantities constructed with the spinor Ψ itself, is given by the quartic Heisenberg Lagrangian, the simplest non-linear covariant term which can be constructed with a spinor field. The Lagrangian is provided by eq. (13.1.33)

$$L_s = \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi - \frac{i}{2} \partial_\mu \bar{\Psi} \gamma^\mu \Psi - V(\Psi). \tag{13.1.103}$$

where Heisenberg potential is

$$V = \frac{1-\beta}{2}\, g_F\, \left(A^2 + B^2 \right). \tag{13.1.104}$$

Note that Pauli-Kofink identity implies that

$$A^2 + B^2 = J_\mu J^\mu.$$

It is immediate that in the case $\beta = 1$, the self-interacting Heisenberg term vanishes.

13.1.23 Interaction with the other fundamental spinor Ψ_N

We have:

$$\begin{aligned} \mathcal{H}_o \Psi &= \gamma_\mu \left(j^\mu + \frac{(1+\beta)}{2}\, i^\mu \right) \Psi \\ &+ \gamma_\mu \gamma^5 \left(\frac{(1+\beta)}{2}\, j^\mu + \beta\, i^\mu \right) \Psi \end{aligned} \tag{13.1.105}$$

The interacting Lagrangian is provided by

$$\begin{aligned} L_o &= -g_F \{ J_\mu j^\mu + \beta I^\mu i_\mu \} \\ &+ \frac{g_F}{2} (1+\beta)(J^\mu i_\mu + I^\mu j_\mu). \end{aligned} \tag{13.1.106}$$

In the case $\beta = 1$ the interaction assumes the reduced form

$$L_F = -g_F\, \overline{\Psi} \gamma^\mu (1+\gamma^5) \Psi \;\; \overline{\Psi} \gamma_\mu (1+\gamma^5) \Psi. \tag{13.1.107}$$

In the case of the interaction of the fundamental spinors, the vectors C^μ, D^μ are given by

$$\begin{aligned} C^\mu_o &\equiv j^\mu + \frac{1+\beta}{2}\, i^\mu \\ D^\mu_o &\equiv \frac{1+\beta}{2}\, j^\mu + \beta i^\mu \end{aligned} \tag{13.1.108}$$

13.1.24 The effect of matter in the generation of gravity

This term is provided by (13.1.93) inspired by the Equivalence Principle that states that the matter interacts only through the effective metric $g_{\mu\nu}$. Variation of spinor Ψ in equation (13.1.93) yields

$$\begin{aligned} \delta S &= -\frac{1}{2} \int \sqrt{-g}\, T^{\mu\nu}\, \delta g_{\mu\nu} \\ &= -\frac{1}{2} \int \sqrt{-g}\, T^{\mu\nu}\, \delta \varphi_{\mu\nu} \\ &= g_F\, g_m \int \sqrt{-g}\, T^{\mu\nu}\, \delta \left(\frac{\Sigma_\mu \Pi_\nu}{\sqrt{X}} \right) \end{aligned} \tag{13.1.109}$$

where we used (13.1.95). Note that the product $g_F\, g_m$ has dimensionality L^3 as it should. Then we can write

$$\delta S = g_F\, g_m\; (I_1 + I_2 + I_3)$$

where

$$\begin{aligned} I_1 &= \int \sqrt{-g}\, T^{\mu\nu}\, \Sigma_\mu\, \Pi_\nu\, \delta \frac{1}{\sqrt{X}} \\ I_2 &= \int \sqrt{-g}\, T^{\mu\nu}\, \frac{1}{\sqrt{X}}\, \Sigma_\mu \delta \Pi_\nu \\ I_3 &= \int \sqrt{-g}\, T^{\mu\nu}\, \frac{1}{\sqrt{X}}\, \Pi_\nu \delta \Sigma_\mu \end{aligned} \tag{13.1.110}$$

Let us evaluate each term separately. We have

$$\delta \frac{1}{\sqrt{X}} = -\frac{1}{2\, X^{\frac{3}{2}}} \delta \bar{\Psi} \left(\Pi^\mu\, \gamma^\mu\, (1+\gamma^5) + \Sigma^\mu\, \gamma^\mu (1+\beta\gamma^5) \right) \Psi. \tag{13.1.111}$$

To simplify our notation let us define the following quantities:

$$\Phi = \frac{1}{X^{3/2}}\, T^{\mu\nu} \Sigma_\mu \Pi_\nu.$$

$$\xi^\mu = \Pi^\mu + \Sigma^\mu,$$
$$\eta^\mu = \Pi^\mu + \beta\, \Sigma^\mu.$$

Then, we can write

$$I_1 = -\frac{1}{2} \int \omega\, \sqrt{-\gamma}\, \delta \bar{\Psi}\, \Phi\, \left(\xi^\mu \gamma_\mu + \eta^\mu \gamma_\mu \gamma^5 \right)\, \Psi \tag{13.1.112}$$

Now, let us evaluate I_2. Defining

$$E^\mu = \frac{1}{\sqrt{X}}\, T^{\mu\nu} \Sigma_\nu,$$

we have:

$$\begin{aligned} I_2 &= \int \sqrt{-g}\, \frac{1}{\sqrt{X}}\, T^{\mu\nu}\, \Sigma_\mu \delta \Pi_\nu \\ &= \int \sqrt{-\gamma} \omega\, E^\mu\, \delta \Pi_\nu \\ &= \int \sqrt{-\gamma} \omega\, \delta \bar{\Psi}\, E^\mu\, \left(\gamma_\mu + \beta \gamma_\mu \gamma^5 \right)\, \Psi. \end{aligned} \tag{13.1.113}$$

For the remaining integral and defining

$$H^\mu = \frac{1}{\sqrt{X}}\, T^{\mu\nu}\Pi_\nu,$$

we find

$$\begin{aligned} I_3 &= \int \sqrt{-g}\,\frac{1}{\sqrt{X}}\, T^{\mu\nu}\ \Pi_\mu \delta\Sigma_\nu \\ &= \int \sqrt{-\gamma}\omega\ H^\mu\ \delta\Sigma_\nu \\ &= \int \sqrt{-\gamma}\omega\ \delta\bar{\Psi}\ H^\mu\ \left(\gamma_\mu + \gamma_\mu\gamma^5\right)\ \Psi. \end{aligned} \tag{13.1.114}$$

Finally, collecting all these three terms and using eq. (13.1.97) we obtain

$$\begin{aligned} C^\mu_m &\equiv g_m\omega\ \left(-\frac{1}{2}\,\Phi\,\xi^\mu + E^\mu + H^\mu\right) \\ D^\mu_m &\equiv g_m\omega\ \left(-\frac{1}{2}\,\Phi\,\eta^\mu + \beta E^\mu + H^\mu\right) \end{aligned} \tag{13.1.115}$$

In the STG this is how matter generates gravitational fields.

The most important task now is to analyze the consequences of this theory.

For later use it is useful to separate this matter influence into three parts using the notation of equation (13.1.100):

$$\mathcal{H}_m = \mathcal{T}_s + \mathcal{T}_o + \mathcal{T}_m \tag{13.1.116}$$

where

$$\begin{aligned} \mathcal{T}_s &= -\frac{g_m}{2}\,\omega\,\Phi\,(1-\beta)\,(A + iB\,\gamma^5) \\ &= -\frac{g_m\omega\Phi}{2}\,\mathcal{H}_s \end{aligned} \tag{13.1.117}$$

$$\begin{aligned} \mathcal{T}_o &= -\frac{g_m}{2}\,\omega\,\Phi\,j^\mu\left(\gamma_\mu + \frac{(1+\beta)}{2}\,\gamma_\mu\gamma^5\right) \\ &- \frac{g_m}{2}\,\omega\,\Phi\,i^\mu\left(\frac{(1+\beta)}{2}\,\gamma_\mu + \beta\,\gamma_\mu\gamma^5\right) \\ &= -\frac{g_m}{2}\,\omega\,\Phi\,\mathcal{H}_o, \end{aligned} \tag{13.1.118}$$

$$\begin{aligned} \mathcal{T}_m &= \frac{g_m}{4}\,\omega\,\gamma_\mu\ (E^\mu + H^\mu) \\ &+ \frac{g_m}{4}\,\omega\,\gamma_\mu\gamma^5\ (\beta\,E^\mu + H^\mu)\,. \end{aligned} \tag{13.1.119}$$

The origin of these terms is very similar to the other expression. Indeed, $\mathcal{T}_s$ is proportional to $\mathcal{H}_s$; the term $\mathcal{T}_o$ is proportional to $\mathcal{H}_o$. This suggests treating the third term in such a way that it can be reduced to a combination of both terms. We postpone this analysis to another place.

Let us now turn to some specific examples of solutions of the fundamental equations of Ψ_E and Ψ_N in some special situations: the gravitational field of a compact static configuration and the case of an expanding spatially homogeneous and isotropic universe.

13.1.25 Gravitational field of a compact object

In order to compare the response of both theories, Genral Relativity and Pre-Gravity, we will review very briefly what is the procedure, in the realm of General Relativity, to obtain the gravitational field of a star. Einstein's theory in the absence of matter is provided by the non-linear equations envolving the contracted curvature tensor:

$$R_{\mu\nu} = 0, \tag{13.1.120}$$

defined as the trace of the Riemannian curvature $R_{\mu\nu} = R_{\alpha\mu\beta\nu} g^{\alpha\beta}$, where:

$$R^{\mu}{}_{\epsilon\alpha\beta} = \Gamma^{\mu}{}_{\epsilon\alpha,\beta} - \Gamma^{\mu}{}_{\epsilon\beta,\alpha} + \Gamma^{\mu}_{\beta\sigma}\Gamma^{\sigma}_{\epsilon\alpha} - \Gamma^{\mu}{}_{\alpha\sigma}\Gamma^{\sigma}_{\beta\epsilon} \tag{13.1.121}$$

The connection $\Gamma^{\mu}{}_{\epsilon\alpha}$ is identified with Christoffel symbol, that is:

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu} \right) \tag{13.1.122}$$

One chooses a parametrization for the coordinates and write the expected metric in the form

$$ds^2 = A(r)\, dt^2 + B(r)\, dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\varphi^2. \tag{13.1.123}$$

Then, equations (13.1.120) reduces to the set:

The response of GR to the question " what is the gravitational field of a star?" is then given by the geometry found by Schwarzshild:

$$ds^2 = (1 - \frac{r_H}{r}) dT^2 - (1 - \frac{r_H}{r})^{-1} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\varphi^2. \tag{13.1.124}$$

Let us turn now to our Pre-Gravity theory and try to answer the same question, that is " what is the gravitational field of a star? We start by choosing a parametrization to represent the background Minkowski geometry in a spherical coordinate system

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\varphi^2. \tag{13.1.125}$$

In consequence, the γ_μ's are given in terms of the constant $\widetilde{\gamma}_\mu$ as follows:

$$\begin{aligned} \gamma_0 &= \widetilde{\gamma}_0 \\ \gamma_1 &= \widetilde{\gamma}_1 \\ \gamma_2 &= r\,\widetilde{\gamma}_2 \\ \gamma_3 &= r\, sin\theta\, \widetilde{\gamma}_3. \end{aligned}$$

In the absence of matter and energy, the effective metric can be obtained by a direct solution of the equation of the fundamental spinors. The equations of motion in this case are

$$
\begin{aligned}
i\gamma^{\mu}\,\partial_{\mu}\Psi_{E} \;&+\; \gamma^{\mu}\,\Gamma^{(0)}_{\mu}\Psi_{E} \\
&-\; g_{F}\,\gamma^{\mu}\left(J_{\mu}+j_{\mu}+\frac{1+\beta}{2}(I_{\mu}+i_{\mu})\right)\Psi_{E} \\
&-\; g_{F}\left(\beta(I_{\mu}+i_{\mu})+\frac{1+\beta}{2}\,(J_{\mu}+j_{\mu})\right)\gamma^{\mu}\gamma^{5}\,\Psi_{E} = 0
\end{aligned}
\qquad (13.1.126)
$$

$$
\begin{aligned}
i\gamma^{\mu}\,\partial_{\mu}\Psi_{N} \;&+\; \gamma^{\mu}\,\Gamma^{(0)}_{\mu}\Psi_{N} \\
&-\; g_{F}\,\gamma^{\mu}\left(J_{\mu}+j_{\mu}+\frac{1+\beta}{2}(I_{\mu}+i_{\mu})\right)\Psi_{N} \\
&-\; g_{F}\left(\beta(I_{\mu}+i_{\mu})+\frac{1+\beta}{2}\,(J_{\mu}+j_{\mu})\right)\gamma^{\mu}\gamma^{5}\,\Psi_{N} = 0
\end{aligned}
\qquad (13.1.127)
$$

This is a highly non linear system that must be solved in order to obtain the effective metric. We succeeded in finding a solution in the case of a spherically symmetric and static configuration. Using the background Minkowski metric in the form (13.1.125) we obtain the unique non identically background FI connection:

$$
\begin{aligned}
\Gamma^{(0)}_{2} &= \frac{1}{2}\,\widetilde{\gamma}_{1}\,\widetilde{\gamma}_{2} \\
\Gamma^{(0)}_{3} &= \frac{1}{2}\,sin\theta\,\widetilde{\gamma}_{1}\,\widetilde{\gamma}_{3} + \frac{1}{2}\,cos\theta\,\widetilde{\gamma}_{2}\,\widetilde{\gamma}_{3}
\end{aligned}
$$

We will look for a solution of the form

$$\Psi_{E} = f(r)\,e^{i\varepsilon lnr}\,e^{ih(\theta)}\,\Psi^{0}_{E} \qquad (13.1.128)$$

$$\Psi_{N} = g(r)\,e^{i\tau lnr}\,e^{il(\theta}\,\Psi^{0}_{N} \qquad (13.1.129)$$

where ε and τ are constants; Ψ^{0}_{E} and Ψ^{0}_{N} are constant spinors. The Heisenberg equation of motion is solved if $h(\theta)$ and $l(\theta)$ are proportional to $ln\sqrt{sin\theta}$. Moreover, $f(r)$ and $g(r)$ obey the equations

$$\frac{1}{f^{3}}\,\frac{df}{dr} = constant, \qquad (13.1.130)$$

$$\frac{1}{g^{3}}\,\frac{dg}{dr} = constant. \qquad (13.1.131)$$

We then have

$$\Psi_E = \frac{1}{\sqrt{r}} e^{i\varepsilon lnr} e^{iln\sqrt{sin\theta}} \Psi^0_E, \tag{13.1.132}$$

$$\Psi_N = \frac{1}{\sqrt{r}} e^{i\tau lnr} e^{iln\sqrt{sin\theta}} \Psi^0_N. \tag{13.1.133}$$

The dependence on the angle θ disappears in both (vector and axial) currents. The $r^{-\frac{1}{2}}$ term depends on the fact that the Heisenberg potential is of quartic order. Any other dependence should yield a different functional dependence for the effective metric. As we shall see next, this form is crucial in order to obtain the good behavior of the metric in the newtonian limit.

We set

$$\Psi^0_E = \begin{pmatrix} \varphi^0 \\ \eta^0. \end{pmatrix} \tag{13.1.134}$$

To solve the equation of motion, the constant spinor Ψ^0_E (correspondingly Ψ^0_N) must satisfy a set of equations. We set

$$\varphi^0 = (c_1 + c_2\,\sigma_1)\,\eta^0 \tag{13.1.135}$$

We look for a solution such that

$$\sigma_1\,\eta^0 = \epsilon\,\eta^0. \tag{13.1.136}$$

where $\epsilon^2 = 1$. We will choose $\epsilon = 1$ for spinor Ψ_E and $\epsilon = -1$ for Ψ_N. That is,

$$\sigma_1\,\varphi^0 = \varphi^0,$$

and where c_1 and c_2 are pure imaginary numbers. Then,

$$\varphi^0 = i\,R\,\eta^0,$$

where $R = |c_1| + |c_2|$ is a real number. Note that all currents from the expression of Ψ_E and Ψ_N are of the form a^μ/r for different constant vectors a^μ. After a rather long and tedious calculation we obtain the final expressions of these currents constructed with our solution. It is precisely these currents that provide the effective metric, namely:

$$\begin{aligned} J_o &= \frac{p}{r} \\ I_o &= \frac{q}{r} \\ J_1 &= \frac{m}{r} \\ I_1 &= \frac{n}{r} \end{aligned}$$

and analogous formulas for the other spinor:

$$
\begin{aligned}
j_o &= \frac{p'}{r} \\
i_o &= \frac{q'}{r} \\
j_1 &= \frac{m'}{r} \\
i_1 &= \frac{n'}{r}
\end{aligned}
$$

where

$$
\begin{aligned}
p &= [c_1\bar{c}_1 + c_2\bar{c}_2 + 1]\eta^+\eta + [c_1\bar{c}_2 + c_2\bar{c}_1]\eta^+\sigma_1\eta \\
q &= -[c_1 + \bar{c}_1]\eta^+\eta - [c_2 + \bar{c}_2]\eta^+\sigma_1\eta \\
m &= [c_2 + \bar{c}_2]\eta^+\eta + [c_1 + \bar{c}_1]\eta^+\sigma_1\eta \\
n &= [c_1\bar{c}_2 + c_2\bar{c}_1]\eta^+\eta + [c_1\bar{c}_1 + c_2\bar{c}_2 + 1]\eta^+\sigma_1\eta
\end{aligned}
$$

Similar formulas holds for the corresponding quantities constructed with Ψ_N involving p', q', m', n'. Analogously we set

$$\Psi_N^0 = \begin{pmatrix} \chi^0 \\ \zeta^0 \end{pmatrix} \tag{13.1.137}$$

and:

$$\chi^0 = (d_1 + d_2\,\sigma_1)\,\zeta^0 \tag{13.1.138}$$

where

$$\sigma_1\,\zeta^0 = -\,\zeta^0,$$

and

$$\chi^0 = i\,S\,\zeta^0$$

where S is a real number.

Since the constants c_1, c_2, d_1 and d_2 are purely imaginary numbers it follows that $m = q = m' = q' = 0$. This follows from the identities concerning the vector and axial current, thats is, $J^\mu J_\mu = -\,I^\mu I_\mu$. Consistency imposes the conditions

$$\varepsilon = 1 - \frac{(1+\beta)}{2}\,\left(1 + g_F(n+n')\right), \tag{13.1.139}$$

$$R = \frac{-1}{2g_F\,(p + p' + \beta\,(n+n'))} \tag{13.1.140}$$

and analogous expressions for the quantities related to spinor Ψ_N^0.

By symmetry, the components (2) and (3) of the currents $J_\mu, I_\mu, j_\mu, i_\mu$, must vanish. This is possible if the constant spinors satisfy:

$$\begin{aligned} \eta_0^+ \sigma_2 \eta_0 &= 0 \\ \eta_0^+ \sigma_3 \eta_0 &= 0 \end{aligned} \tag{13.1.141}$$

These equations are identically satisfied by condition (13.1.136). Indeed, in this case we have

$$\Psi_N^0 = \begin{pmatrix} z \\ \epsilon\, z \end{pmatrix} \tag{13.1.142}$$

where $z = m + i\, n$. Then,

$$\eta_0^+ \sigma_1 \eta_0 = 2\epsilon\, |z|^2$$

and $\eta_0^+ \sigma_2 \eta_0 = 0$ and $\eta_0^+ \sigma_3 \eta_0 = 0$.

The same happens for the other tensor. There remains two arbitrary conditions to be fixed: $\eta_0^+ \, \eta_0$ and $\zeta_0^+ \zeta_0$. Different choices yield different solutions for the spinor fields and consequently distinct configurations for the observable metric. We will fix them by conditions on the induced metric.

The induced metric

¿From the above solution of the spinor fields we can evaluate the currents and the effective geometry that acts on all forms of matter and energy. From its dependence on r and θ we have that all currents depend only on $1/r$. Using the expression of the effective metric in terms of the spinorial fields, a direct calculation gives:

$$\begin{aligned} \varphi_{00} &= -2 g_F g_m \frac{(p + p^{'})^2}{r^2} \frac{1}{\sqrt{X}} \\ \varphi_{11} &= -2 g_F g_m \frac{(n + n^{'})^2}{r^2} \frac{\beta}{\sqrt{X}} \\ \varphi_{01} &= - g_F g_m \frac{(n + n^{'})(p + p^{'})}{r^2} \frac{1 + \beta}{\sqrt{X}} \end{aligned} \tag{13.1.143}$$

Then, for the induced geometry

$$\begin{aligned} ds^2 &= (1 - \frac{r_H}{r}) dt^2 + 2 \frac{N}{r} dr dt \\ &- (1 + \frac{Q}{r}) dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\varphi^2, \end{aligned} \tag{13.1.144}$$

where:

$$r_H = 2g_F g_m \frac{1}{\sqrt{Z}}(p+p')^2,$$
$$Q = 2g_F g_m \frac{1}{\sqrt{Z}}\beta\,(n+n')^2,$$
$$N = -\frac{g_F\lambda}{2}\frac{1}{\sqrt{Z}}\,(p+p')(n+n')\,(1+\beta).$$

The constant Z is defined in terms of the norm of the currents as $Z = X r^2$. In order to compare this geometry with the corresponding solution in General Relativity, we make a coordinate transformation to eliminate the crossing term $drdt$. Setting

$$dt = dT + \frac{N}{r-r_H}\,dr,$$

we obtain

$$\begin{aligned} ds^2 &= (1-\frac{r_H}{r})dT^2 \\ &- \left(1-\frac{r_H}{r}\right)^{-1}\left(1-\frac{r_H-Q}{r}-\frac{Qr_H-N^2}{r^2}\right)dr^2 \\ &- r^2d\theta^2 - r^2 sin^2\theta d\varphi^2. \end{aligned} \tag{13.1.145}$$

At this point we remark that in the case of General Relativity, Birkhoff's theorem forbids the existence of more than one arbitrary constant in the Schwzarschild solution. In the present case of the Spinor Gravity theory, this theorem does not apply. Thus we can understand the fact that this solution contains one additional arbitrary constant. Observations impose that for small values of r_H/r the factors g_{00} and g_{11} must be in the first order respectively $g_{00} = 1 - r_H/r$ and $g_{11} = -1 - r_H/r$. This fact imply that the the constants $\eta_0^+\,\eta_0$ and $\zeta_0^+\,\zeta_0$ must be chosen such that $r_H = Q$. This fixes one constant. The other constant is provided, as in the similar procedure in GR, by the newtonian limit for $r \to \infty$, in terms of the Newton constant and the mass of the compact object that is, $r_H = 2g_N M/c^2$. Thus, the final form of the effective metric is given by

$$\begin{aligned} ds^2 &= (1-\frac{r_H}{r})dT^2 \\ &- \left(1-\frac{r_H}{r}\right)^{-1}\left[1+\sigma^2\left(\frac{r_H}{r}\right)^2\right]dr^2 \\ &- r^2d\theta^2 - r^2 sin^2\theta d\varphi^2, \end{aligned} \tag{13.1.146}$$

where

$$\sigma^2 \equiv \frac{(\beta-1)^2}{4\beta}.$$

It is a remarkable consequence of the above solution that in the case in which the self-interaction of the fundamental spinors vanishes and only the interaction between Ψ and Ψ_N occurs, that is, for $\beta = 1$ the four-geometry is precisely the same as the Schwarzschild solution in GR. On the other hand, if $\beta \neq 1$ the difference between both theories appears already in the order $(r_H/r)^2$. Indeed for General Relativity we have

$$- g_{11} = 1 + \frac{r_H}{r} + (\frac{r_H}{r})^2$$

and for the Spinor Theory we obtain

$$- g_{11} = 1 + \frac{r_H}{r} + (\frac{r_H}{r})^2 \, (1 + \sigma^2). \tag{13.1.147}$$

The parameter β should be fixed by observation.

Linearized Einstein equation as a consequence of Heisenberg dynamics

In this section we will analyze how it is possible to generate a dynamics of spin-2 field from the self-interaction of a Heisenberg spinor. Let us define

$$\Phi_{\mu\nu} \equiv \frac{c_{\mu\nu}}{X} \tag{13.1.148}$$

where $X \equiv J_\mu \, J^\mu$ and $c_{\mu\nu} \equiv J_\mu \, J_\nu + I_\mu \, I_\nu$. We will show that such field constructed in terms of the currents of a Heisenberg spinor satisfies equation (13.1.83). We have:

$$\Phi = \Phi_{\mu\nu} \, \eta^{\mu\nu} = 0. \tag{13.1.149}$$

Using the Inomata prescription we obtain:

$$\partial^\nu \Phi_{\mu\nu} = -2 J_\mu \tag{13.1.150}$$

$$\partial^\mu \partial^\nu \Phi_{\mu\nu} = 0. \tag{13.1.151}$$

$$\partial^\alpha \partial_\mu \Phi_{\alpha\nu} = -2 c_{\mu\nu} \tag{13.1.152}$$

$$\partial^\alpha \partial_\alpha \Phi_{\mu\nu} = -4 c_{\mu\nu} \tag{13.1.153}$$

Collecting all these terms and using the expression (13.1.83) for $G^L_{\mu\nu}$ it follows that indeed $\Phi = c_{\mu\nu}/X$ satisfies the linearized Einstein equation

$$G^L_{\mu\nu} = 0. \tag{13.1.154}$$

Let us make a remark concerning the linearity of the operator $G^L_{\mu\nu}$. From equations (13.1.45, 13.1.51) we have

$$\partial_\mu J_\nu = w \, c_{\mu\nu},$$

$$\partial_\mu I_\nu = w\, d_{\mu\nu},$$

where $w \equiv Re(a)$. Thus the quantities $c_{\mu\nu}$ and $d_{\mu\nu}$ or any linear combinations of them satisfy identically the mass-less spin-2 equation (13.1.83) once they can be associated to coordinate transformations of the Minkowski metric. Note however that the expression $\Phi_{\mu\nu} \equiv c_{\mu\nu}/X$ is not of this kind and consequently generates a non-trivial spin-2 field.

The following comment will simplify our search for spin-2 fields constructed in terms of Heisenberg spinors. All possible terms that are trivial spin-2 fields – call them, generically $\Theta_{\mu\nu}$ - can be written in terms of arbitrary fields ξ_μ in the form

$$\Theta_{\mu\nu} \equiv \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \tag{13.1.155}$$

Thus, it should be interesting to know how to construct this kind of tensor $\Theta_{\mu\nu}$ from Heisenberg currents. There are two possible tensors constructed with currents J_μ and I_μ which we now analyze separately. Let us consider first the axial-current and set

$$\xi_\mu \equiv \frac{I_\mu}{X^a}.$$

It follows that

$$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 2(1-a)\,\frac{d_{\mu\nu}}{X^a} \tag{13.1.156}$$

Thus, when $a \neq 1$ the tensor constructed with $d_{\mu\nu}$ divided by any power a distinct of 1 of X is a trivial spin-2 tensor. In an analogous way we can construct with the current vector the quantity

$$\eta_\mu \equiv \frac{J_\mu}{X^a}.$$

We have

$$\partial_\mu \eta_\nu + \partial_\nu \eta_\mu = \frac{1}{X^a}\,\left(I_\mu I_\nu + (1-2a)\,J_\mu J_\mu\right) \tag{13.1.157}$$

The unique case in which this is a trivial $\Theta_{\mu\nu}$ tensor occurs for $a = 0$ which is the case that we have presented previously, that is, $c_{\mu\nu}$.

Let us now consider the field defined in terms of quantities Σ_μ and Π_μ as in equations (13.1.78) and (13.1.79). For the case of a single spinor we have

$$\Sigma_\mu \equiv J_\mu + I_\mu \tag{13.1.158}$$

and

$$\Pi_\mu \equiv J_\mu + \beta\, I_\mu. \tag{13.1.159}$$

We define the tensor

$$E_{\mu\nu} \equiv \Sigma_{(\mu}\,\Pi_{\nu)} = \Sigma_\mu\,\Pi_\nu + \Sigma_\nu\,\Pi_\mu. \tag{13.1.160}$$

Then,

$$E_{\mu\nu} = (1+\beta)\, d_{\mu\nu} + 2\beta\, I_\mu\, I_\nu + 2\, J_\mu\, J_\nu \tag{13.1.161}$$

Let us define the quantity

$$\Phi_{\mu\nu} \equiv \frac{E_{\mu\nu}}{\sqrt{X}}.$$

Using the results obtained above it is straightforward to obtain the equation of motion that such tensor satisfies:

$$G^L_{\mu\nu} = \frac{w^2}{2(1-\beta)^2}\, \Phi^2 \left(-\frac{1+\beta}{1-\beta} \Phi_{\mu\nu} - \frac{1}{2}\, \Phi\, \eta_{\mu\nu} \right) + \frac{w^2\,(1+\beta)}{(1-\beta)^3}\, \Phi \Sigma_\mu \Sigma_\nu \tag{13.1.162}$$

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