On Bose-condensation of photons in relativistic plasma
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Gregory Vereshchagin and Mikalai Prakapenka

ICRANet

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Bose-Einstein condensation

The phenomenon of quantum condensation of bosons was predicted by Bose and Einstein in 1924. In physics textbooks it is associated with cooling to low temperatures, and it was indeed observed for ultracold atoms (Nobel prize in Physics, 2001) and for quasi-particles. It was also observed for photons in a microcavity, where special boundary conditions ensured necessary pre-requisites for Bose-Einstein condensation: (a) photon number conservation or (b) generation of effective mass via spatial confinement.
Photon condensation

In a pure photon gas such phenomenon does not occur, because photons are massless particles and cooling leads to disappearance of photons. Nevertheless, it was predicted in hot opaque plasma in a pioneering work by Zeldovich and Levich (1969). In absence of photon absorption, the dominant interaction process in plasma is Compton scattering. Considering the properties of Kompaneets equation, the mechanism of condensation was illustrated. It was shown that, unlike ideal Bose gases, BEC manifests itself as an excess of photons over the Planck distribution, which is only possible at intermediate energies: between the spectral peak and the critical energy, below which absorption dominates. It was proposed that such phenomenon may occur in astrophysical conditions, when hot radiation passes through cold plasma.
Kinetic vs thermodynamic equilibria

Considering relaxation of non-equilibrium electron-positron-photon plasma with arbitrary initial conditions we found (PRL, 2007; PLA 2019) that this process may occur in two steps:

- detailed balance is established in two-particle (binary) interactions: Compton and Coulomb scattering, pair creation and annihilation. This kinetic equilibrium state is characterized by the same temperature $T_k$ of all particles, and nonzero chemical potentials $\mu_i$. The distribution function of particles with energy $E$ in this state has the form

$$f = \frac{2}{(2\pi\hbar)^3} \exp\left(\frac{E-\mu_i}{kT_k}\right) \pm 1.$$

- detailed balance is established in three-particle (triple) interactions: relativistic bremsstrahlung, double Compton scattering, radiative pair production and three-photon annihilation. Then $\mu_i \to 0$ and plasma comes to thermodynamic equilibrium.
Conditions for photon condensation

The possibility of condensation of photons in kinetic equilibrium state occurs if the number density of photons $n_\gamma$ exceeds the one with zero chemical potential of photons $\mu_\gamma = 0$, namely

$$n_\gamma > \frac{2\zeta(3)}{\pi^2} \left( \frac{\hbar}{mc} \right)^{-3} \left( \frac{kT}{mc^2} \right)^3.$$  \hspace{1cm} (1)

If photon annihilation process is not fast enough, binary interactions (essentially Compton scattering) will bring plasma to kinetic equilibrium with an excess of photons over the Bose-Einstein distribution. We have found that BEC of photons indeed occurs for a broad class of initial distribution functions, including Gaussian and Wien distributions. In what follows we give several examples.
Kinetic equations I

We solve relativistic Boltzmann equations for one-particle distribution functions $f_i(\epsilon, t)$ with Uehling-Uhlenbeck collision integrals

$$\frac{1}{c} \frac{\partial f_i}{\partial t} = \sum_q (\eta^q_i - \chi^q_i f_i),$$

The emission and absorption coefficients are multidimensional integrals. For binary interactions

$$\eta^2_{p} = \int d^3p_2 d^3p_3 d^3p_4 \ W_{(3,4|1,2)} f_{III} f_{IV} (1 + \zeta f_I) (1 + \zeta f_{II}),$$

$$\chi^2_p f_I = \int d^3p_2 d^3p_3 d^3p_4 \ W_{(1,2|3,4)} f_I f_{II} (1 + \zeta f_{III}) (1 + \zeta f_{IV}).$$
Kinetic equations II

For triple interactions

\[ \eta_3^p = \int d^3p_2 d^3p_3 d^3p_4 d^3p_5 \, W_{(3,4,5|1,2)} f_{III} f_{IV} f_{V} (1 + \xi f_I) (1 + \xi f_{II}), \]

\[ \chi_3^p f_I = \int d^3p_2 d^3p_3 d^3p_4 d^3p_5 \, W_{(1,2|3,4,5)} f_{I} f_{II} (1 + \xi f_{III}) (1 + \xi f_{IV}) (1 + \xi f_{V}). \]

The transition coefficients are related to QED matrix elements

\[ W_{(3,4,5|1,2)} d^3p_3 d^3p_4 d^3p_5 = V d\omega_{(3,4,5|1,2)} \] and

\[ W_{(1,2|3,4,5)} d^3p_1 d^3p_2 = V^2 d\omega_{(1,2|3,4,5)}, \] where

\[ d\omega = c (2\pi\hbar)^4 \delta(p_{\mu, in} - p_{\mu, fin})|M_{fi}|^2 V \left( \prod_{in} \frac{\hbar c}{2e_{in} V} \right) \left( \prod_{fin} \frac{d^3p_{fin}}{(2\pi\hbar)^3} \frac{\hbar c}{2e_{fin}} \right). \]
Results

Figure: Time evolution of energy density (top) and particle number density (bottom) of photons (blue), electrons/positrons (orange), all together (green) in nonrelativistic case. Black line represents the final equilibrium quantity. Final equilibrium temperature is $\theta = k_B T / m_e c^2 \approx 0.1$. 

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Results

Figure: Top: The spectral energy density (dots) with the associated Planck fit (solid) for selected time moments from left to right: $10^{-15}, 10^{-11}, 10^{-8}$ s, in nonrelativistic case. Bottom: emission and absorption coefficients for photons (binary reactions: emission (blue) and absorption (cyan); triple reactions: emission (purple) and absorption (red)).
Figure: The time evolution of the spectral index of the power law distribution of photons $d\rho/d\epsilon \sim \epsilon^a$, below the peak starting at the moment when it is first established.
Figure: The spectral energy density and reaction rates for relativistic case with an initial degeneracy $n_{tot}^{in} = 8.5n_{tot}^{fin}$ and final equilibrium temperature $\theta = 3$ at selected time moments (from left to right): $3.8 \times 10^{-20}, 5 \times 10^{-20}, 10^{-15}$ s.
Conclusions I

- First principles calculations demonstrating BEC of photons in relativistic plasma. This phenomenon was predicted in 1969 and still awaits confirmation in the laboratory.

- Condensation of photons may occur as a transient phenomenon both in nonrelativistic and in relativistic cases and it manifests in photon spectra described by the Planck law with an excess formed in the energy range above the critical energy for the dominance of triple interactions and below the peak of the spectrum.

- At nonrelativistic temperatures the excess is well described by a power law, while at relativistic temperatures it represents a bump. In our nonrelativistic example with $k_B T \simeq 0.1 m_e c^2$ the condensation persist until about $\sim 10^{-8}$ sec, when complete thermodynamic equilibrium is established.
Conclusions II

- It is found that necessary condition for the development of BEC is an excess of photon number over the equilibrium number, as well as initial distribution of photons not broader than Wien spectrum with the peak of the distribution located above the critical energy below which triple interactions dominate over the binary ones.

- Broader initial distributions, even the Planck spectrum, contain too many photons at low energies, and triple interactions such as bremsstrahlung quickly eliminate excess photons, preventing the condensation. This is the reason why the cooling of photons by electrons proposed by Zeldovich and Levich does not lead to photon condensation.