Applying no-z approximation for modeling dynamo action in accretion discs

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Introduction

• It is now firmly established that a number of astrophysical objects - such as the sun, stars, galaxies, etc. - possess large-scale magnetic fields.

• There are reasons to believe that they are also present in accretion disks formed by massive objects.

• For their observational study, various manifestations of magnetism are used, such as the Zeeman effect, the Faraday rotation of the plane of polarization, the "blockage" of synchrotron radiation, etc.

• From a theoretical point of view, the excitation of a magnetic field is described using dynamo theory.
• The dynamo mechanism describes the transition of the energy of turbulent motions into the energy of a magnetic field.

• It is based on the combined action of the alpha effect and differential rotation.

• They are counteracted by turbulent diffusion, which tends to destroy large-scale field structures.

• In view of this, the dynamo mechanism is a threshold: the generation of the field is possible only with sufficiently intense motions, when the first two effects are able to resist the "blurring" due to diffusion.
The basic dynamo equation

- The generation of a magnetic field is described using the Steenbeck-Krause-Radler equation (mean-field dynamo equation):

\[ \frac{\partial \hat{B}}{\partial t} = \text{rot}(\alpha \hat{B}) + \text{rot}[\hat{V}, \hat{B}] + \eta \Delta \hat{B} \]

- This equation is three-dimensional, and its explicit solution is rather difficult.

- For this reason, specific representations are used for each type of object.
Magnetic fields of accretion disks

• Accretion disks that form around black holes, white dwarfs, etc. play an important role in astrophysics.

• It is important to be able to calculate magnetic fields. [Rüdiger, Shalybkov, 2002].

• The magnetic field of accretion disks can be formed by the dynamo mechanism in them [von Rekowsky, Brandenburg, 2004].

• It can be expected that the magnetic fields in them will be described by similar patterns [Moss, Sokoloff, Suleimanov 2016].
No-z approximation

• The so-called no-z approximation, which is based on the fact that the disk is thin enough, is quite effective.

• In this case, the magnetic field will lie with high accuracy in the plane of the galactic disk; therefore, the z-component can be neglected.

• Moreover, it is convenient to assume that the field is axisymmetric.
Vertical magnetic field structure

• In this case, we can assume that the magnetic field depends on the coordinates according to the cosine law:

\[ \vec{B}(r, \varphi, z, t) = \vec{B}(r, 0, 0, t) \cos \left( \frac{\pi z}{2h} \right) \]

• Then the second derivatives of the field can be written using the expression:

\[ \frac{\partial^2 \vec{B}}{\partial z^2} = -\frac{\pi^2}{4h^2} \vec{B} \]
Applying the no-z approximation

• The no-z approximation can be used to study the magnetic field in accretion disks.

• At the same time, it is necessary to take into account that the kinematic parameters of the medium in accretion disks will be fundamentally different.

• In addition, we will not be interested in “solid disks”, but objects with a sufficiently large inner radius:

\[ r_{in} < r < R \]
The dynamo governing parameters

- We will assume that the speed is described by Kepler's law [Shakura, Sunyaev, 1973]:

\[
\Omega = \frac{(GM)^2}{r^2},
\]

where \( M \) is the mass of the star, \( r \) is the distance to the center.

- For the speed in the radial direction, we have [Lipunov, 1982]:

\[
V_r = -\frac{3\nu}{2r} \frac{1}{\sqrt{1 - \frac{r_{in}}{r}}},
\]

where \( r_{in} \) is the inner radius, \( \nu \) is the viscosity.
Dimensionless parameters

- \( R_\alpha = \frac{\Omega(R)l^2}{\nu} \), where \( l \) is the typical lengthscale of the turbulence — characterizes the alpha effect

- \( R_\omega = \frac{\Omega(R)h(R)^2}{\nu} \) — characterizes differential rotation,

- \( \lambda = \frac{h}{R} = 0.037r^{\frac{1}{8}} \left(1 - 0.9 \sqrt{\frac{r_{in}}{r}}\right)^{\frac{3}{20}} \) — number describing the dissipation of the magnetic field in the plane of the disk [Suleimanov, Lipunova, Shakura, 2007]

- The dynamo number in this case is equal to:
  \[ D = R_\alpha R_\omega. \]
If we take into account these assumptions, the equations for the magnetic field in dimensionless form look like this:

\[
\frac{\partial B_r}{\partial t} = -R_\alpha B_\phi - \frac{\pi^2}{4} B_r + \lambda^2 \left( \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (rB_r) \right) \right) - V_r \left( \frac{\partial B_r}{\partial r} \right); \\
\frac{\partial B_\phi}{\partial t} = - \frac{3R_\omega}{2r^2} B_r - \frac{\pi^2}{4} B_\phi + \lambda^2 \left( \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (rB_\phi) \right) \right) - V_r \left( \frac{\partial B_\phi}{\partial r} \right). 
\]
Parameter values

- Let us estimate the values for $R_\alpha$ and $R_\omega$ for accretion disks.
- Let's take the characteristic values for the disk parameters:
  \[ R = 5 \cdot 10^7 \text{ m}, \quad M = 5 \cdot 10^{30} \text{ kg}, \quad \nu = 15 \frac{\text{km}}{s} = 1.5 \cdot 10^4 \frac{m}{s} \]
- Then:
  \[ \Omega = \sqrt{\frac{GM}{R^3}} = 0.05 \text{ s}^{-1}, \quad h = 0.01R = 5 \cdot 10^5 \text{ m}, \quad l = \frac{h}{5} = 1 \cdot 10^5 \text{ m}, \]
  \[ \eta = \frac{1}{3}l\nu = 5 \cdot 10^8 \frac{m^2}{s} \]
  \[ R_\alpha = \frac{\Omega l^2}{\eta} \approx 1, \quad R_\omega = \frac{\Omega h^2}{\eta} \approx 10 \]
• The dynamo effect is characterized by a dynamo number:

\[ D = R_\alpha R_\omega \]

• The dynamo effect is a threshold effect. For \( D > D_{cr} \), the magnetic field increases exponentially, otherwise it decreases.

• We can take constant dynamo number in the hole disk or changing one (the field will be characterized by “medium” effective value \( D_{eff} \)).
Time dependence of the magnetic field (linear case) for different cases. Solid line shows constant value of dynamo number ($D = 1$), dashed line shows dynamo number, which depends on the distance to the center of the accretion disk ($D_{eff} = 1$).
Time dependence of the magnetic field (linear case) for different cases. Solid line shows constant value of dynamo number ($D = 3$), dashed line shows dynamo number, which depends on the distance to the center of the accretion disk ($D_{\text{eff}} = 3$).
Dependence of the magnetic field on the distance to the center of the accretion disk (linear case) for different cases. Solid line shows constant value of dynamo number ($D = 1$), dashed line shows dynamo number, which depends on the distance to the center of the accretion disk ($D_{\text{eff}} = 1$)
Dependence of the magnetic field on the distance to the center of the accretion disk (linear case) for different cases. Solid line shows constant value of dynamo number ($D = 3$), dashed line shows dynamo number, which depends on the distance to the center of the accretion disk ($D_{\text{eff}} = 3$).
Equations of the no-z approximation in the nonlinear case

\[
\frac{\partial B_r}{\partial t} = -R_\alpha B_\phi \left(1 - \frac{B^2}{B_{max}^2}\right) - \frac{\pi^2 B_r}{4} + \lambda^2 \left(\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r B_r)\right)\right) - V_r \left(\frac{\partial B_r}{\partial r}\right);
\]

\[
\frac{\partial B_\phi}{\partial t} = -\frac{3R_\omega}{2r^2} B_r - \frac{\pi^2 B_\phi}{4} + \lambda^2 \left(\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r B_\phi)\right)\right) - V_r \left(\frac{\partial B_\phi}{\partial r}\right),
\]

где \( B_{max} = 420r^{-21/16} \left(1 - 0.9 \sqrt{\frac{r_{in}}{r}}\right)^{17/40} \)

[Moss, Sokoloff, Suleimanov, 2016]
Boundary conditions

• For numerical simulation, we used different boundary conditions.
• At the outer border, the condition was used:
  \[
  \frac{\partial B_r}{\partial r} (R) = 0, \frac{\partial B_\varphi}{\partial r} (R) = 0
  \]
• The first version of the boundary condition (I) was obtained from the basic relations:
  \[
  B_r(r_{in}) = 0, B_\varphi(r_{in}) = 0
  \]
• For this case, we can take the initial condition as
  \[
  B_\varphi(t = 0) = B_0 \sin \left(\frac{\pi (r - r_{in})}{R - r_{in}}\right)
  \]
Boundary conditions

• We also used the conditions which can describe the interaction between the central body and the accretion disc:

\[ B_r(r_{in}) = 0, \quad B_\varphi(r_{in}) = 0.5B^*. \]
\[ B_\varphi(t = 0) = \frac{0.5B^* \exp^{-\left(\frac{r-r_{in}}{\Delta}\right)}}{\Delta} + B_0 \sin\left(\frac{(r-r_{in})}{\pi(R-r_{in})}\right), \]

• where \( \Delta \sim 10^{-2} \) is a small parameter which describes the penetration depth of the central body field to the accretion disc.
The dependence of the magnetic field on the distance to the center of the accretion disk for different cases ($D_{\text{eff}} = 9$). Solid line shows the results from the work of Moss et al. 2016, dotted line shows the zero condition and dashed line shows the condition with interaction with central body.
The dependence of the magnetic field on the distance to the center of the accretion disk for different cases. Solid line shows constant value of dynamo number \((D = 9)\), dashed line shows dynamo number, which depends on the distance to the center of the accretion disk \((D_{\text{eff}} = 9)\)
The dependence of the magnetic field on the distance to the center of the accretion disk for different $r_{in}$ with zero boundary condition. Dashed line shows the case $r_{in} = 0.01$, solid line – the case $r_{in} = 0.1$ and the dotted line – the case $r_{in} = 0.2$. 

![Graph showing the dependence of magnetic field $B/B_{max}$ on radius $r$. The graph has three lines: dashed line for $r_{in} = 0.01$, solid line for $r_{in} = 0.1$, and dotted line for $r_{in} = 0.2$. The x-axis represents the radius $r$, ranging from 0.0 to 1.0, and the y-axis represents $B/B_{max}$, ranging from 0.0 to 1.0.]}
Time dependence of the magnetic field for different $r_{in}$ with zero boundary condition. Dashed line shows the case $r_{in} = 0.01$, solid line – the case $r_{in} = 0.1$ and the dotted line – the case $r_{in} = 0.2$. 
• There is magnetic helicity:

\[ \alpha_m = (\vec{A}, \vec{H}), \]

where \( \vec{A} \) – vector potential, \( \vec{H} \) – magnetic field strength

• In the alpha effect we use:

\[ \alpha = \alpha_k + \alpha_m \]
Model with a helicity flows

• There are also vertical flows of helicity. Convection must be taken into account.

• To do this, you need to solve the equations:

\[
\frac{\partial B_r}{\partial t} = -R_\alpha B_\phi (1 + \alpha) - \left( R_U + \frac{\pi^2}{4} \right) B_r - V_r \frac{\partial B_r}{\partial r} + \lambda^2 \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r B_r) \right);
\]

\[
\frac{\partial B_\phi}{\partial t} = -\frac{3}{2} \frac{R_\omega}{r^{3/2}} B_r - \left( R_U + \frac{\pi^2}{4} \right) B_\phi - V_r \frac{\partial B_\phi}{\partial r} + \lambda^2 \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r B_\phi) \right);
\]

\[
\frac{\partial \alpha}{\partial t} = -R_U \alpha - C \left\{ (1 + \alpha) \left( B_r^2 + B_\phi^2 \right) + \frac{3}{8} \frac{B_r B_\phi}{R_\alpha} \sqrt{\frac{3}{2} \frac{R_\alpha R_\omega}{r^{3/2}} (1 + \alpha) + \frac{\alpha}{R_m}} \right\} - V_r \frac{\partial \alpha}{\partial r} + \lambda^2 \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial \alpha}{\partial r} \right) \right\}.
\]
• Boundary and initial conditions:

\[ B_r \mid_{r=r_{in}} = B_r \mid_{r=R} = B_\varphi \mid_{r=r_{in}} = B_\varphi \mid_{r=R} = \alpha \mid_{r=r_{in}} = \alpha \mid_{r=R} = 0; \]
\[ B_r \mid_{t=0} = 0; \]
\[ B_\varphi \mid_{t=0} = B_0 \sin \pi r \]
\[ \alpha \mid_{t=0} = -\alpha_0 \sin \pi r \]

• For parameters, you can write the following:

\[ R_m = 10000; \quad C = 50; \quad R_U = 0.3; \quad \alpha_0 = 0.001. \]
Parameter $R_U$

• In the model taking into account helicity, a coefficient arises that characterizes the typical intensity of vertical flows:

$$R_U = \frac{Vh}{\nu}$$

where $\nu$ - turbulent viscosity, $V$ – typical vertical speed.

• In addition, the situation will be of interest when it changes, decreasing with distance from the center of the disc. For example, we can assume that the speed of convective currents is proportional to the speed of sound, and use the following formula:

$$R_U = \frac{R_{U0}}{r^{3/8}}.$$
The dependence of the magnetic field on the distance to the center of the accretion disk for different cases. Dashed line shows the case $Ru = 0$, $E = 1.14 \cdot 10^{-6}$ solid line – the case, when $Ru = 1.5$, $E = 5.98 \cdot 10^{-6}$.
The dependence of the magnetic field on the distance to the center of the accretion disk for different cases. Dashed line shows the case \( R_u = 0 \), solid line – the case, when \( R_u = 1.5 \).
• The no-z approximation, which is widely used to study the magnetic fields of galaxies, has been used to simulate the magnetic fields of accretion disks.

• The values of the control parameters of the dynamo corresponding to the properties of accretion disks are calculated.

• The dependence of the magnetic field on the coordinates and time for the typical characteristics of these objects is constructed.