On the foundations of black hole thermodynamics

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Kiril Maltsev
Physics of Stellar Objects (PSO) Group
Heidelberg Institute for Theoretical Studies (HITS)
kiril.maltsev@h-its.org

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**Introduction**


vacuum superradiance around semi-classical rotating horizon

Ruffini & Wheeler (1971):

classical horizon constitutes a one-way membrane to information flow


increase in classical horizon area is irreversible

Bekenstein (1972, 1973):

Generalized Second Law to resolve the Geroch-Wheeler (1971) paradox

Bardeen, Carter & Hawking (1973):

Four Laws of Black Hole Mechanics

Hawking (1975):

spectrum of radiation from semi-classical stationary horizon is thermal
1. The Four Laws

2. Temperature

3. Entropy
## 1. The Four Laws: Outline

<table>
<thead>
<tr>
<th></th>
<th>Phenomenological Thermodynamics</th>
<th>Classical Mechanics</th>
<th>Black Hole Mechanics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Zeroth Law</strong></td>
<td>Two systems in thermal equilibrium with a third are in thermal equilibrium with each other.</td>
<td>( \kappa ) is constant across the event horizon.</td>
<td></td>
</tr>
<tr>
<td><strong>First Law</strong></td>
<td>( dU = \delta Q - \delta W )</td>
<td>( dM = \frac{\kappa}{8\pi} \delta A + \Omega_{BH} , dJ_{BH} )</td>
<td></td>
</tr>
<tr>
<td><strong>Second Law</strong></td>
<td>( dS_{TD} \geq 0 )</td>
<td>( \delta A \geq 0 )</td>
<td></td>
</tr>
<tr>
<td><strong>Third Law</strong></td>
<td>( T = 0 ) is not achievable by any process within a finite number of steps</td>
<td>( \kappa = 0 ) is not achievable by any process within a finite number of steps</td>
<td></td>
</tr>
</tbody>
</table>
1. The Four Laws: 
Thermodynamic analogies

Classical Kerr BH: temperature and entropy analogies

- 2nd law: horizon area $A$ analogous to entropy $S_{TD}$
- 0th and 3rd law: surface gravity $\kappa$ analogous to temperature $T_{TD}$

Classical Kerr BH: heat and work analogies

- 1st law:
  \[
  \delta Q = TdS \equiv \frac{\kappa}{8\pi}\delta A
  \]
  \[
  \delta W = -pdV + \Omega dJ \equiv \Omega_{BH} \delta J_{BH}
  \]
  
  - $+\Omega dJ$: work done by thermodynamic system in state of rotation
  - $-pdV$: work done onto the thermodynamic system

Q: Are the laws of BHM endowed with genuine thermodynamic significance?
2. Temperature: Phenomenological definition

3 aspects of phenomenological temperature:

1. Characterization of heat flow,
2. Capacity of a system to perform work,
3. Quantification of efficiency in a (reversible) Carnot cycle.

“Thermodynamics investigates the conditions that govern the transformation of heat into work. It teaches us to recognize temperature as the work-value of heat. Heat of higher temperature is richer, is capable of doing more work.”

Sommerfeld (1964)

Fig. Source: http://www.physics.louisville.edu/cldavis/phys298/notes/therm_carnot_fig1.jpg
2. Temperature: Application to Black Hole Mechanics

Is phenomenological temperature well-grounded?

- **Classical Schwarzschild BH:**
  - if stationary: No
  - if perturbed: Possibly / Not yet (gravitational radiation, Carnot-Geroch cycle)

- **Classical Kerr Black Hole:**
  - Possibly / Not yet (Penrose process, “Carnot-Penrose” cycle)

- **Semi-classical Schwarzschild BH:**
  - Yes (Zel’dovich-Hawking radiation pressure, “Carnot-Hawking” cycle)
2. Temperature: Statistical definition

3 aspects of statistical temperature:

1. Measure of ensemble of (electromagnetic) fluctuations,
2. Black body radiation (in equilibrium),
3. Mean value encoded by peak of spectrum.

Fig. source: [https://i.stack.imgur.com/wROIT.gif](https://i.stack.imgur.com/wROIT.gif)
Is statistical temperature well-grounded?

• **Classical Schwarzschild BH:**
  No

• **Classical Kerr Black Hole:**
  No

• **Semi-classical Schwarzschild BH:**
  Yes (thermal Planck distribution of Zel’dovich-Hawking quanta)
3. Entropy: Phenomenological definition

3 aspects of phenomenological entropy:

1. Measure of how much energy is required to transform heat into work,
2. Function of state of thermodynamic system in a Carnot cycle,

“Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.”

Clausius (1854)

\[ ds = \frac{\delta Q_{rev}}{T} \]

“It is impossible to devise a cyclically operating heat engine, the effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work.”

Kelvin-Planck statement
Is phenomenological entropy well-grounded?

- **Classical Schwarzschild BH:**
  - if stationary: No
  - if perturbed: Possibly / Not yet (work from gravitational radiation)

- **Classical Kerr Black Hole:**
  - Possibly / Not yet (work from rotational energy)

- **Semi-classical Schwarzschild BH:**
  - Yes (Generalized Second Law, Carnot-Hawking cycle)
3. Entropy: Application to Black Hole Mechanics

Generalized Second Law:

\[ \delta \left( S_{\text{exterior}} + \frac{c^3}{4Gh} A \right) \geq 0 \]

"Carnot-Hawking" cycle (Schwarzschild BH - photon gas):

Given restricted parameter value ranges, thermodynamic meaning of \( S_{\text{BH}} \) is rigorously well-grounded upon efficiency \( \mu \) of reversible, quasi-static Carnot Cycle

Fig. source: Prunkl & Timpson (2019)
3. **Entropy: Statistical definition**

3 versions of statistical entropy:

1. **Boltzmann entropy:**
   \[ S = k_B \log \Omega \]
   measure over the sum of combinatorial ways a given macroscopic equilibrium state can be represented by non-interacting single-particle microstates in phase space.

2. **Gibbs - von Neumann entropy:**
   \[ S = -k_B \sum_i p_i \log p_i \]
   \[ S = -k_B \Tr(\hat{\rho} \log(\hat{\rho})) \]
   measure over the probability distribution, defined in state space of all the possible (interacting) many-particle microstates, for a given statistical ensemble.

3. **Shannon entropy:**
   \[ H(X) = -\sum_{i=1}^{n} p(x_i) \log p(x_i) \]
   measure of the amounts of (missing) information to fully specify a given macrostate.
3. Entropy: Application to Black Hole Mechanics

Is statistical entropy well-grounded?

- **Classical Schwarzschild BH:**
  No

- **Classical Kerr Black Hole:**
  No

- **Semi-classical Schwarzschild BH:**
  - if **Boltzmann:** Possibly / Not yet
  - if **Gibbs:** Possibly / Not yet (quantum gravity research paradigms)
  - if **Shannon:** Debated Yes:
    - # of possible internal configurations of the “No hair” BH,
    - # of possible initial states riving rise to the “No hair” BH,
    - (...)
### SUMMARY:

<table>
<thead>
<tr>
<th></th>
<th>Phenomenological Temperature</th>
<th>Statistical Temperature</th>
<th>Phenomenological Entropy</th>
<th>Statistical Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical Schwarzschild BH</td>
<td>Yellow</td>
<td>Red</td>
<td>Yellow</td>
<td>Red</td>
</tr>
<tr>
<td>Classical Kerr BH</td>
<td>Yellow</td>
<td>Red</td>
<td>Yellow</td>
<td>Red</td>
</tr>
<tr>
<td>Semi-classical Schwarzschild BH</td>
<td>Green</td>
<td>Yellow</td>
<td>Green</td>
<td>Yellow</td>
</tr>
</tbody>
</table>
CONCLUSION:

- The establishment of bona fide thermodynamic significance in the laws of black hole mechanics is an ongoing research program, which has partial success.

- Even for purely classical black holes there are interesting prospects in grounding their phenomenological temperature and entropy in a Carnot cycle.

- Zel’dovich’s merit consisted in being the first to propose a mechanism for semi-classical black hole horizons to emit radiation, although he did not ascribe any thermodynamic properties to it.

- The link between Zel’dovich-Hawking radiation and (statistical) temperature was achieved by Hawking.
References


