Holographic Bound on Area of Compact-binary-merger-remnant

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Partha Sarathi Majumdar

Indian Association for the Cultivation of Science
Kolkata 700032, India

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GW150914 Basics (LIGO)

GW150914: FACTSHEET

First direct detection of gravitational waves (GW) and first direct observation of a black hole binary

- **observed by**: LIGO L1, H1
- **source type**: black hole (BH) binary
- **date**: 14 Sept 2015
- **time**: 09:50:45 UTC
- **likely distance**: 0.75 to 1.9 Gly
- **redshift**: 0.054 to 0.136
- **signal-to-noise ratio**: 24
- **false alarm prob.**: < 1 in 5 million
- **false alarm rate**: < 1 in 200,000 yr
- **duration from 30 Hz**: ~ 200 ms
- **# cycles from 30 Hz**: ~10
- **peak GW strain**: $1 \times 10^{-21}$
- **peak displacement of interferometers arms frequency/wavelength**: 150 Hz, 2000 km
- **peak speed of BHs**: ~ 0.6 c
- **peak GW luminosity**: $3.6 \times 10^{56}$ erg s$^{-1}$
- **radiated GW energy**: 2.5-3.5 M$_{\odot}$
- **remnant ringdown freq.**: ~ 250 Hz
- **remnant damping time**: ~ 4 ms
- **remnant size, area**: 180 km, $3.5 \times 10^5$ km$^2$
- **consistent with general relativity?**: passes all tests performed
- **graviton mass bound**: $< 1.2 \times 10^{-22}$ eV
- **coalescence rate of binary black holes**: 2 to 400 Gpc$^{-3}$ yr$^{-1}$
- **online trigger latency**: ~ 3 min
- **# offline analysis pipelines**: 5
- **CPU hours consumed**: ~ 50 million (=20,000 PCs run for 100 days)

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**Acronyms**

L1=LIGO Livingston, H1=LIGO Hanford; Gly=giga lightyear=9.46 x 10$^{12}$ km; Mpc=mega parsec=3.2 million lightyear, Gpc=10$^{3}$ Mpc, fm=femtometer=10$^{-15}$ m, M$_{\odot}$=1 solar mass=2 x 10$^{30}$ kg
What constitutes the Binary Inspiral or Remnant in GW150914 obs. ?

LIGO: GW150914 results consistent with both inspiral and remnant config.s being black holes

- No evidence of horizon !
- No evidence of accretion from either inspiral components or post-merger remnant
- Smoking gun signal ?

Exotic Compact Objects : Wormholes, Boson Stars, Gravastars, Fuzzballs, ...  
Cardoso, Pani, Liv. Rev. 2019

- How to distinguish ECOs from black holes ?
- Are there theoretical predictions for both delineations of inspiral or remnant config.s ?
Gravitational Instabilities of ECOs (Addazi et. al., arXiv 1905.08734)

How compact can ECOs be?

Hoop Conjecture: \( \exists \) Critical closed 2-surface of revolution of circular hoop of radius \( r_S \) (Thorne 1972)

‘Adiabatically’ accreting ECO without compactness \( \searrow \) may collapse to a black hole

But, no information on accretion of binary inspiral in GW150914
If inspiral and remnant are black holes, then black hole area theorem (Hawking 1972) \[ A_{\text{rem-bh}} > A_{\text{bh1}} + A_{\text{bh2}} \]

Mass estimates appear to agree with area law (Dasgupta, arXiv 1604.00951; Pooh-Kolb et. al. arXiv 2006.03939)

What if remnant is not a black hole?

Alternative approach based on:

- **Generalized Second Law** (Bekenstein 1973)
- **Entropy bound** (Bekenstein 1974) tightened by Loop Quantum Gravity corrections (Kaul, PM, arXiv gr-qc/0002040)

Together, these \[ \exists \] a lower bound on cross-sectional area of remnant of binary black hole coalescence
Generalized Second Law

- Generalization of classical area theorem for black holes
- Valid where entropy outside event horizon decreases whenever matter falls into a black hole
- Defines black hole entropy as function of horizon area in any quantum gravity framework where such entropy can be computed ab initio
- Where two black holes coalesce into a final config.

\[ S_{\text{fin}} > S_{bh1}(A_1) + S_{bh2}(A_2) \]

In this case, if remnant is an ECO,

\[ S_{\text{ECO}} + S_{\text{GW}} > S_{bh1}(A_1) + S_{bh2}(A_2) \]

\( S_{bh}(A) \) can be computed ab initio in Loop Quantum Gravity
Isolated Hor : Null inner boundary of sptm - marginally closed trapped
Induced metric on IH - degenerate ⇒ only topological field theories on IH
LQG description of QIH : $SU(2)$ Chern-Simons Gauge theory with $k \simeq A/A_{Pl}$ coupled to punctures of spin network states of bulk gravity
Counting Chern-Simons states (Kaul, PM, arXiv gr-qc/9802111)

For large $k \gg 1 \Rightarrow A \gg A_{Pl}$ (macroscopic IH), count the number of (conformal blocks) of $SU(2)$ WZW model on punctured $S^2$, using fusion rules

$$\mathcal{N}_T = \sum_{\{j_i\}} \sum_{P} \prod_{i=1}^{P} \sum_{m_i = -j_i} j_i \left[ \delta_{\sum_{n=1}^{P} m_n, 0} - \frac{1}{2} \delta_{\sum_{n=1}^{P} m_n, -1} - \frac{1}{2} \delta_{\sum_{n=1}^{P} m_n, 1} \right]$$

Holographic: horizon states of 4 dim black hole counted by conformal blocks of 2 dim WZW model

QIH Entropy (Kaul, PM arXiv gr-qc/0002040)

$$S_{bh}(A) = \log \mathcal{N}_T = S_{BH}(A) - \frac{3}{2} \log S_{BH} + \mathcal{O}(S_{BH})^{-1}, \quad S_{BH}(A) = \frac{A}{4A_{Pl}}$$

BHAL with LQG corrections from quantum sptm fluctuations
Holographic Entropy Bound

Every compact config. has a maximum entropy

\[ S_{ECO} < S_{bh}(A_{ECO}) \]

\( S_{bh}(A) \) is computed holographically yielding

\[ S_{BH}(A) - (3/2) \log S_{BH}(A) + \cdots \]

Original Bekenstein bound \( S_{ECO} < S_{BH}(A_{ECO}) \) is tightened by LQG corrections

Any ECO not satisfying bound can always acrete matter adiabatically, without changing its cross-sectional area, to increase its entropy upto the limiting value \( S_{bh}(A_{ECO}) \)

Together, Gen. 2nd law & Entropy bound \( \Rightarrow (A_{E} \equiv A_{ECO}) \)

\[ S_{bh}(A_{E}) + S_{GW} > S_{E} + S_{GW} > S_{bh1}(A_{1}) + S_{bh2}(A_{2}) \]

Holds irrespective of remnant being black hole or ECO
Holographic Entropy Bound (contd)

Alternative expression

\[
\frac{\exp \bar{A}_E}{\bar{A}_E^{3/2}} > \frac{\exp(\bar{A}_1 + \bar{A}_2 - S_{GW}^{EQ})}{(\bar{A}_1 \bar{A}_2)^{3/2}}
\]

\[\bar{A} \equiv S_{BH}(A) = \frac{A}{A_{Pl}}\]

Replaced \( S_{GW} \) in rhs by \( S_{GW}^{EQ} \)

With estimates for \( S_{GW} \) and \( \bar{A}_{1,2} \) from GW150914 data, lower bound on \( A_E \) can be determined

But what if binary inspiral is not just a black hole pair? Does bound generalize?
Generalizing the bound

Recall that, if binary inspiral consists of ECOs

\[ S_{E1} < S_{bh}(A_{E1}) \quad , \quad S_{E2} < S_{bh}(A_{E2}) \]

\[ \Rightarrow S_{bh}(A_{E1}) + S_{bh}(A_{E2}) > S_{E1} + S_{E2} \]

Does this \[ \Rightarrow \]

\[ S_{bh}(A_{E}) + S_{GW} > S_{bh}(A_{E1}) + S_{bh}(A_{E2}) \? \]

Not immediately, mathematically!

But from a physical perspective, if bound has to work also for inspiralling black holes, it must hold!

Bound is independent of actual config of inspiral constituents or remnant
More about the bound

Alternative expression:

\[
\frac{\exp \bar{A}_E}{\bar{A}_E^{3/2}} > \frac{\exp(\bar{A}_{E1} + \bar{A}_{E2} - S_{GW}^E)}{(\bar{A}_{E1} \bar{A}_{E2})^{3/2}}
\]

Define compactness ratio

\[C_E \equiv \frac{R_E}{r_{SE}}, \quad R_E = \text{size}, \quad r_{SE} = \text{Schwarzschild radius}\]

\[\Rightarrow A_E(R_E) = A(C_E r_{SE}, C_E r_{2E}) = C_E^2 A_E(r_{SE}, \ldots) = C_E^2 M_E^2 (\text{for spin} = 0)\]

Assuming (cf. GW150914) that the remnant is spinning very slowly, with scaling as above,

\[
\frac{\exp[C_E^2 \bar{M}_E^2]}{(C_E \bar{M}_E)^3} > \frac{\exp[C_{E1}^2 \bar{A}_{E1} + C_{E2}^2 \bar{A}_{E2} - S_{GW}^E]}{(C_{E1}^2 \bar{A}_{E1} C_{E2}^2 \bar{A}_{E2})^{3/2}}
\]
Estimating the entropy of gravitational waves

Follow the method of ‘Coincidence Probability of recurrence of trajectory ’ (Ma, 1984)

Config. space $\Omega = \bigoplus_\lambda \Omega_\lambda$, $\Gamma_\lambda \equiv \text{vol}(\Omega_\lambda)$, $p_\lambda \equiv T_\lambda / T$

Coincidence probability of recurrence $= p_\lambda / \Gamma_\lambda$

Taking average over all groups $\lambda$

$$S = \sum_\lambda p_\lambda \log \frac{\Gamma_\lambda}{p_\lambda}$$

Adapt to GW: characterise config. space groups by Fourier modes of frequency $\omega$, $p(\omega) \equiv I(\omega) / I_0$, $\Gamma(\omega)$. Then, the entropy is

$$S_{GW}^{EQ} = \int_{\omega_1}^{\omega_2} \frac{d\omega}{2\pi} p(\omega) \log \left[ \frac{\Gamma(\omega)}{p(\omega)} \right]$$
How do we estimate $\Gamma(\omega)$?

Adapt ‘coincident trajectories’ by considering ‘coherent modes’ of GW: $N_t(\omega) = \#(GW \ \omega \ modes)$, $N_c(\omega) = \#(coherent \ \omega \ modes)$

$\Rightarrow (\Gamma(\omega))^{-1} = N_c/N_t \Rightarrow p(\omega)(\Gamma(\omega))^{-1} = N_c/N_0 \ (N_0 \propto l_0)$

The GW entropy can now be written

$$S_{GW}^{EQ} = \int_{\omega_1}^{\omega_2} \frac{d\omega}{2\pi} \frac{l(\omega)}{l_0} \log \left[ \frac{N_0}{N_c(\omega)} \right]$$

Equilibrium $\Rightarrow N_c(\omega) \ll N_t(\omega) \ll N_0$, so the maximum entropy ensues when $N_c(\omega) = 1$
Results

From GW150914 FactSheet, estimated

\[ N_0 \approx 2.7 \times 10^{77} \text{ at frequency } \]

\[ \omega_{\text{max}} \approx 150\text{hz} \Rightarrow S^E_{\text{GW}} = O(50) \]

Minimal cross-sectional area of post-coalescence remnant can be plotted as a function of inspiralling black hole masses \( M_{1,2} \) with \( C_{E1} = 1 = C_{E2} \)

Fix all masses to values measured in GW150914; the minimal value of \( C_E \) can be plotted as a function of the inspiral compactness ratios \( C_{E1,E2} \)

For various fixed values of \( C_E \) within a reasonable range, assuming very small remnant spin, minimal remnant mass can be plotted as functions of \( M_{1,2} \) for black holes (\( C_{E1} = 1 = C_{E2} \)) or as functions of \( C_{E1,E2} \) for masses given by GW150914 data.
Fig. 1 $\log \tilde{A}_E$ for fixed $C_{E1} = C_{E2} = 1$
Fig. 2 Compactness ratio \( C_E \) vs \( C_{E1} \), \( C_{E2} \)
Conclusions

- Insensitivity of inequalities on precise nature of inspiral or remnant configurations may have wide applications for GW detection in ongoing and future GW interference experiments on compact binary coalescence

- Results cover range of estimated masses for most recent black hole merger GW190521: may make predictions

- Need better theoretical underpinnings on Generalized Second Law and Entropy Bound from Quantum Gravity