On the magnetic field screening

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**Inner engine model for BdHNe**

In GRB the model of **BdHNe** (Binary Driven HyperNovae) is a binary system: a NS and a CO\text{core} \rightarrow SN explosion (\nu NS)+BH, with \( P \approx 5 \) min.

In the "**Inner-engine**" model (R.Ruffini et al., 2018) strong electric field occurs around a stationary axisymmetric Kerr-BH placed in an uniform magnetic field \( B_0 \).

**Assumptions of the model** :
- \( B_0 \) originates from: toroidal field of the \nu NS or fossil field of the original NS\Rightarrow BH;
- magnetic field and spin of the BH are parallel.

We consider a series of **impulsive emissions** (R.Ruffini et al., 2019), originated from the discharge of an induced Electric field (\( J \) angular momentum; \( r_+ \) horizon radius)

\[
E_r = - \frac{B(t)J}{2M^2} \frac{r_+^2}{r^2} \Rightarrow E_r \approx \frac{J}{2M^2} \frac{c^2}{2 \gamma} B(t) = \frac{1}{2} \gamma B(t) \text{ (near the horizon)}
\]

**GRB 130427A parameters** : \( \gamma = \frac{J}{M^2} = 0.3M, \ M = 2.28M_\odot \)

In our configuration \( \vec{E} = E \hat{y} \) and \( \vec{B} = B \hat{z} \).
Accelerated $e^-/e^+$ pairs, due to the electric field, emit synchrotron photons (MeV-GeV-TeV-PeV), depending on the emission angle;

- The synchrotron photons interact with the magnetic field $B_0$ mean the Magnetic pair production process (MPP): $\gamma + B \rightarrow e^- + e^+$;
- The pairs continue to emit photons while circularize around the magnetic field creating an induced magnetic field, $B_{ind}$, opposite to the background one $B_{tot}(t) = B_0 - B_{ind}(t)$ decreases with time, reducing $B$.

The induced magnetic field is given by

$$\frac{dB_{z,ind}(t)}{dt} = e \frac{\sqrt{p_x^2(t) + p_y^2(t)}}{R_c(t)^2} \frac{dN_+}{dt},$$

(inspired by the Biot-Savart law) with the curvature radius:

$$R_c(t) = \frac{\gamma_e(t) m_e c^2}{e} \left[ \left( \vec{E}(t) + \vec{\beta}(t) \times \vec{B}(t) \right)^2 - \left( \vec{\beta}(t) \cdot \vec{E}(t) \right)^2 \right]^{-\frac{1}{2}}.$$

The aim of this study is to decrease the background magnetic field in order to have the right conditions for the transparency of synchrotron MeV and GeV photons.
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Equations of the system

\[
\begin{align*}
\frac{dr^2}{dt} &= c\beta(t), \\
\frac{d\beta}{dt} &= \frac{e}{mc\gamma(t)} \left[ \vec{E}(t) + \beta(t) \times \vec{B}(t) - \beta(t) (\vec{E}(t) \cdot \beta(t)) \right], \\
\frac{d\gamma}{dt} &= \frac{e}{mc} \left( \vec{E}(t) \cdot \beta(t) \right) - \frac{l(t)}{mc^2}, \\
\frac{dN}{dt} &= N(t) \frac{l(t)}{\epsilon(t)} \\
\frac{dN_{\pm}}{dt} &= N_{\gamma}(t) \zeta(t) \\
\frac{dB_z}{dt} &= -\frac{dB_{z,\text{ind}}}{dt}
\end{align*}
\]

(1)

The energy loss is given by (see Kelner et al., 2015):

\[I(t) \equiv \left| -\frac{dE}{dt} \right| = \frac{e^2 m^2 c^3}{\sqrt{3} \pi h^2} \overline{H}(x),\]

with:

\[
\overline{H}(x) \approx \frac{8\pi \sqrt{3}}{27} x^2 \left[ 1 + \frac{3}{4} \left( \frac{2x}{\sqrt{3}} \right)^{2/3} \right]^{-2} \left[ 1 + \frac{0.52 \sqrt{x} (1 + 3 \sqrt{x} - 3.2x)}{1 + 0.3 \sqrt{x} + 17x + 11x^2} \right].
\]

The parameter \( x = \frac{\epsilon_{\gamma}}{2 \epsilon_e} \) (\( \epsilon_{\gamma} = \text{photon energy}; \epsilon_e = e^\pm \text{ energy} \)) identifies the strength of the MPP process (MPP process becomes important for \( x \gtrsim 0.1 \)), where

\[
\epsilon_{\gamma}(t) = \frac{3e\hbar}{2mc} \gamma^2(t) \sqrt{\left( \vec{E}(t) + \beta(t) \times \vec{B}(t) \right)^2 - \left( \beta(t) \cdot \vec{E}(t) \right)^2}.
\]
Pair production rate

The pair production rate in our configuration of the fields is given by (Daugherty & Lerche, 1975):

\[
\zeta = 0.23 \frac{\alpha_f c}{\lambda_c} \frac{B_z}{B_{cr}} \left(1 - \frac{E_y^2}{B_z^2}\right) \frac{\sqrt{\eta_y^2 \left(1 - \frac{E_y^2}{B_z^2}\right) + \left(\eta_x - \frac{E_y}{B_z}\right)^2}}{1 - \frac{E_y}{B_z} \eta_x} \times 
\exp \left\{-\frac{8}{3} \frac{m c^2}{\varepsilon_\gamma} \frac{B_{cr}}{B_z} \left[\eta_y^2 \left(1 - \frac{E_y^2}{B_z^2}\right) + \left(\eta_x - \frac{E_y}{B_z}\right)^2\right]^{-1/2}\right\}.
\]

(3)

This formula is valid until the following condition is satisfied

\[
\Psi = \frac{3}{4} \frac{e \hbar}{m c} \frac{B_z^2}{B_{cr}} \gamma^2 \sqrt{\beta_y^2 \left(1 - \frac{E_y^2}{B_z^2}\right) + \left(\frac{E_y}{B_z} - \beta_x\right)^2} \sqrt{\eta_y^2 \left(1 - \frac{E_y^2}{B_z^2}\right) + \left(\eta_x - \frac{E_y}{B_z}\right)^2} \ll 1.
\]

This condition for \(\Psi\) imposes the values for the set of initial conditions \((\beta_0, \gamma_0, B_0)\) to use.
Normalization

We adopt the following normalization for the variables:

\[ \tilde{t} = \frac{t}{\tau_c} \text{ with } \tau_c = \frac{\hbar}{m_e c^2} = 1.3 \times 10^{-21} \text{ sec} \]

\[ \tilde{B} = \frac{B}{B_{cr}} \text{ with } B_{cr} = \frac{m_e^2 c^3}{e\hbar} = 4.414 \times 10^{13} \text{ Gauss} \]

\[ \tilde{E} = \frac{E}{E_{cr}} \text{ with } E_{cr} = B_{cr} \]

\[ \tilde{R}_c = \frac{R_c}{\lambda_c} \text{ with } \lambda_c = \frac{\hbar}{m_e c} = 3.862 \times 10^{-11} \text{ cm} \]

\[ \tilde{\varepsilon}_\gamma = \frac{\varepsilon_\gamma}{m_e c^2} \text{ with } m_e c^2 = 0.511 \text{ MeV} \]

\[ \tilde{\zeta} = \zeta \times \tau_c. \]

In the *Generic* direction: \( \theta = 75^\circ \) (polar angle), \( \phi = 30^\circ \) (azimuthal angle)
Results- $B$ screening ($N_{±,0} = 10^{10}$)

$B_0 = 0.3B_{cr}$
$\gamma = 1$
$\gamma_0 = 2.27, 2.14$

$B_0 = 0.1B_{cr}$
$\gamma = \frac{1}{5}$
$\gamma_0 = 4.18, 3.71, 22.66$
Results- $B$ screening

\[ B_0 = 0.1 B_{cr} \]
\[ \gamma = \frac{1}{50} \]
\[ \gamma_0 = 3.81, 3.71 \]
\[ N_{\pm,0} = 10^{10} \]

\[ B_0 = 0.1 B_{cr} \]
\[ \gamma_0 = 6.48, 4.18, 3.81 \]
\[ N_{\pm,0} = 10^{15} \]
Results

- Graph showing $\varepsilon_{\gamma}(t)$, (MeV) vs. Log(t), (s)
  - Gen, Y, Z

- Graph showing Log($N_{\gamma}(t)$) vs. Log(t), (s)
  - $N_{\gamma,0}=10^3$, $N_{\gamma,0}=10^6$, $N_{\gamma,0}=10^{10}$

- Graph showing Log($\zeta(t)$), (s$^{-1}$) vs. Log(t), (s)
  - $N_{\pm,0}=10^3$, $N_{\pm,0}=10^6$, $N_{\pm,0}=10^{10}$, $N_{\pm,0}=10^{15}$, $N_{\pm,0}=10^{18}$
Summary and Conclusions

We have built a set of one-particle equations to describe the screening of strong and crossed magnetic ($\vec{B} = B\hat{z}$) and electric ($\vec{E} = E\hat{y}$) field;

The fields screening occurs when a huge number of particles ($N_{\pm,0} \geq 10^{10}$) is inserted in the system;

The screening depends by
- the BH spin \( \Upsilon \) (its efficiency decreases if \( \Upsilon \) decreases);
- initial direction of the emission (less efficiency for emission in the z-direction);
- initial number of particles (inefficient for $N_{\pm,0} < 10^{10}$);
- photons energy.

The photon energy presents an oscillatory behaviour due to the motion of the particles and the transformation between energy gain and energy lost;

The low value of the rate $\zeta(t)$ (and than of the efficiency of the screening) is due to the low values of photons energy ($\sim\text{MeV}$). Higher photons energy leads to a stronger MPP rate ($\varepsilon_\gamma \sim\text{GeV}$).
Study of the case with higher photons energy \((\varepsilon_{\gamma} \sim \text{GeV, TeV})\), consistently with the condition \(\Psi \ll 1\);

Study of the MPP rate \(\zeta(t)\) for any value of \(\Psi\);

Study the pair production and the screening effect for the configuration with \(\vec{E} \times \vec{B} = 0\) \((\vec{E} \parallel \vec{B})\) and \(\vec{E} \cdot \vec{B} \neq 0\).
Thanks for the attention
Other results
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