Fractal matter distribution and supernovae IA

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1 Introduction and Motivations
   - Fractal models
   - LTB metric
   - Distance measure in cosmology and in LTB

2 Data Analysis with supernovae of type Ia (SNe Ia)

3 Conclusion and perspectives
General introduction on fractal

Fractals

Spatial power law scaling, self-similarity and structure recursiveness

\[ N(r) \sim r^d, \]  

where \( d \) : fractal dimension, \( r \) : the scale measure and \( N(r) \) : the distribution which manifests a fractal behaviour.
General introduction on fractal

Application to Nature

Usefull to describe irregular shapes. Fractal dimension encodes how "broken" the structure is. Nice applications: from Norway irregular coast to clouds modelling.

Result: Fractal dimension of the Norway coast $D = 1.52$ (J. Feder); Britain coast $D = 1.25$ (Richardson)
Application to Nature

Useful to describe irregular shapes. Fractal dimension encodes how "broken" the structure is. Nice applications: from Norway irregular coast to clouds modelling.

Fig. 3: Average fractal dimensions for different temperature thresholds

Apply fractal to describe **Nature** : [Mandelbrot, 1982]
Fractal in many other domains: computer science (e.g., image compression), economics, geo-statistics, fluids mechanics (turbulence, model flames, percolation), biology, surface physics (model roughness of surfaces), medicine (biosensor interaction), telecommunication (create antennas), **cosmology (?)**
Fractals in cosmology

The precursors

- D’Albe (1907) and Charlier (1908) propose a mathematical framework of a self similar universe to solve Olber’s paradox.
- Carpenter (1938) studied clusters in the context of hierachical cosmology.

The fractal fashion in the 80’s

- De Vaucouleurs, 1970: hierarchical cosmology with power law
- [Mandelbrot, 1982] fractals describing the Universe
- [Pietronero, 1987]: large scale distribution of galaxies form a single fractal model
- [Ruffini et al., 1988]: fractal model has an upper cutoff to homogeneity.

One possible development

Use a fractal model to describe the inhomogeities of our universe within the Lemaître-Tolman-Bondi (LTB) metric.
Generalities on LTB

**LTB metric**

\[ ds^2 = dt^2 - \frac{R'(r,t)}{f^2(r)} dr^2 - R^2(r,t) d\Omega^2 \]  

The equation of motion for the LTB model reads:

\[ \frac{\dot{R}^2(r,t)}{2} - \frac{M(r)}{R(r,t)} = \frac{f(r)^2 - 1}{2} \]

**Interpretation of the parameters**

- \( M(r) \): the mass function: represents the mass inside a sphere of comoving radius \( r \).
- \( f(r) \): a measure of the local curvature (\( f^2(r) = 1 \rightarrow \) flat universe, \( f^2(r) > 1 \rightarrow \) hyperbolic universe. [Durrive and Stahl, 2011]) and the energy per unit mass in a sphere of radius \( r \).
- \( R(r,t) \): Generalisation of the scale factor in the LTB metric. Can be interpreted as the angular distance (cf later).
Our choice of model

Assumptions

Only a class of cosmologically relevant LTB solutions:
- One unique big-bang: $t_B$ constant
- Flat geometry: $f(r)=1$
- Fractal distribution: $M(r) = M_g N(r) = M_g \sigma r^d$

Remark: $d=3$ gives FLRW, $\sigma$ related to lower cutoff of the fractal.

Explicit solution in this case

\[ R = \left( \frac{9M(r)}{2} \right)^{1/3} (t_B - r)^{2/3} \] (4)

Units

Time unit: $3.26 \times 10^9$ s ($t_B = 4.3$)
Mass unit: $2.09 \times 10^{22} M_\odot$ ($M_g = 1$)
Distances unit: Gpc
Angular distance, [Ellis, 1971] :

\[ d_A^2 \equiv \frac{d\sigma_{\Lambda}}{d\Omega_{\Lambda}} \]  

(5)

**Figure**: Angular distance defined as the ratio between the intrinsic cross-sectional area of the source and the observed solid angle (Source: Ribeiro 2005)

**In LTB case**

\[ d_A^2 = R^2 \]  

(6)
Luminosity distance

**Etherington reciprocity theorem (1933)**

Assumption of the theorem: source and observer are connected through null geodesics:

\[ d_L = (1 + z)^2 d_A = (1 + z)^2 R \]  \hspace{1cm} (7)

**Single Radial Geodesic (SNR) assumption**

Can have LTB redshift: [Mustapha et al., 1997]

\[ 1 + z(r) = \frac{t_B^{2/3}}{(t_B - r)^{2/3}} \]  \hspace{1cm} (8)

**Luminosity distance**

\[ d_L = (\sigma M_g)^{1/3} \left( \frac{9}{2} \right)^{1/3} t_B^{d+2/3} \frac{d^{d+2}}{(1 + z)^{3/2} - 1}^{d/3} \frac{(1 + z)^{d/2}}{(1 + z)^{d/2}} \]  \hspace{1cm} (9)
Generalities about Supernovae

**Characteristics**
- Duration: 30 days
- Absolute magnitude: $-19 \ (10^{10} \text{ suns})$, sometimes brighter than the host galaxy

**Characteristics**
- Luminosity peak in the blue
- **Specific** spectrum with Si absorption line for 630 nm

**Theoretical model**
- White dwarf explosion around Chandrasekhar mass
Supernovae and cosmology

- For all SNe, known luminosity peak: **standard candles**
- Riess (1998) with 10+6 SNe Ia and Perlmutter (1999) with 42 SNe Ia: **Acceleration of the expansion of the universe.**
- Concordance model: flat FLRW with "dark energy": $\Omega_\Lambda = 0.7$

**Idea:**

The acceleration of the expansion of the Universe is a "mirage" caused by the inhomogeneities.
Motivation: Ricci-Weyl problem

**Figure**: The standard interpretation of SNe Ia data assumes that light propagates in purely homogeneous and isotropic space (top). However, thin light beams are expected to probe the inhomogeneous nature of the actual Universe (bottom) down to a scale where the continuous limit is no longer valid. From [Fleury et al., 2013]
Results

Idea:
The acceleration of the expansion of the Universe is a "mirage" caused by the inhomogeneities.

Realisation
Use the SCP "Union2.1" SN Ia compilation (580 SNe) [Suzuki et al., 2012] and fit these data with:
- the standard flat FLRW model (red) with cosmological constant
- the fractal LTB model (orange)
## Results and discussion

<table>
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<tr>
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<th>parameter 1</th>
<th>parameter 2</th>
<th>$R^2$/adjusted</th>
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<tbody>
<tr>
<td>flat FLRW</td>
<td>$\Omega_M = 0.30 \pm 0.03$</td>
<td>$h = 0.704 \pm 0.006$</td>
<td>$R^2 = 0.994108/0.994088$</td>
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<tr>
<td>fractal LTB</td>
<td>$d = 2.94 \pm 0.02$</td>
<td>$\sigma = 0.148 \pm 0.001$</td>
<td>$R^2 = 0.993495/0.993472$</td>
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Discussion

- Both models fit the data good.
- Theorems for LTB metric [Mustapha et al., 1997]: under some asymptotic and regularity assumption for the LTB free functions, an LTB model can be found to fit any luminosity distance data.
- In the fractal LTB case, choosing $N(r) = \sigma r^d$ is quite a generic choice.
Conclusion and perspectives

- Interesting fractal model fits also the data (weaker assumption than the data fitting theorem of LTB [Mustapha et al., 1997])
- can be generalized to more general LTB cases
- More diverse (GRB, galaxies surveys) data are needed to reinforce the results.
- will not solve the cosmological constant problem
- No microscopic description of the fractal yet, need for more motivation to be an alternative to FLRW.
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