High energetic Gamma Ray Bursts and their spectral properties

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Introduction

Gamma Ray Bursts (GRBs) are short and intense pulses of high energy gamma rays and represent one the most fascinating phenomena in the Universe. It is difficult to trace a “typical” GRB, nevertheless it is possible to establish some observational features to characterize them. The first GRB peculiarity is their enormous observed fluence, between $10^{-7}$ and $10^{-4}$ ergs/cm$^2$. This corresponds to a total isotropic energy emitted of $10^{51} - 10^{55}$ ergs, making them the most luminous objects in the sky (Piran, 2004). Another characteristic is their temporal evolution. Usually we can identify two main components in the light curve of GRBs: a prompt emission, which is a flash of γ-rays typically lasting a few tenths of seconds, and the afterglow emission, which is the long-lasting (from days to weeks) multiwavelength counterpart. The prompt emission light curves are characterized by a great variety of temporal profiles: they range from single-peaked pulses to highly structured multi-pulses, with a time variability up to 1 ms (Piran, 2004) and a duration ranging from $10^{-3}$ to $10^3$ s. The afterglow emission is characterized by a less complex behavior than the prompt one. In particular, it is possible to recognize a “canonical” behavior which includes one or more power-law segments (Nousek et al., 2006; Evans et al., 2009); sometimes X-ray flares are observed superimposed on this canonical behaviour. Another peculiarity of GRBs is related to the prompt emission spectrum: it displays a clear non-thermal shape and is well fitted by two power-laws joined smoothly at a given break energy (Band et al., 1993).

Over the years, many efforts have been made to explain the observational
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properties of GRBs and this has led to the development of different theoretical models such as, for example (see chapter 3): the standard “fireball” model (Goodman, 1986; Paczyński, 1986), the “electromagnetic” model (Lyutikov & Blandford, 2003; Lyutikov, 2006) and the “turbulent” model (Narayan & Kumar, 2009). In this work I focus on the fireshell model, proposed by Ruffini and collaborators (see chapter 2 and Ruffini et al., 2001a,b, 2003, 2005b; Bianco & Ruffini, 2004, 2005).

Within the fireshell model, GRBs originate from the gravitational collapse of the star progenitor to a Reissner-Nordstöm black hole (Ruffini et al., 2001b); the star progenitor could be, for example, a neutron star (NS) in a binary system (Dainotti et al., 2007; Caito et al., 2009). When the black hole electric field exceeds the critical value $E_c = \frac{m_e^2 c^3}{\hbar e}$ (Heisenberg & Euler, 1936; Schwinger, 1951) a vacuum polarization process occurs and $e^\pm$ pairs form (Damour & Ruffini, 1975). This plasma self-accelerate outwards reaching an ultrarelativistic velocity; it engulfs the baryonic material left over by the gravitational collapse of the star progenitor, then it continues its expansion until the transparency condition is reached. At this stage all the pairs annihilate and a flash of photons is emitted: the Proper Gamma-Ray Burst (P-GRB). The remaining accelerated baryons interacts with the CircumBurst Medium (CBM), which is assumed to have spherical symmetry: these interactions produce the “extended” afterglow emission, which comprises both the so called “prompt” emission and the “afterglow” emission. Concerning the spectrum, the fireshell model postulates that the emission process is thermal in the comoving frame of the fireshell (Ruffini et al., 2004). The observed GRB non-thermal spectral shape is due to the convolution of various thermal spectra over the EquiTemporal Surfaces (EQTS, Bianco & Ruffini, 2004, 2005), which are the surfaces of constant arrival time of photons at the detector and over the observation time.

The fireshell model has been successfully tested on many GRBs such as, for example: GRB 970228 (Bernardini et al., 2007), GRB 031203 (Bernardini et al., 2005b), GRB 050315 (Ruffini et al., 2006), GRB 060218 (Dainotti...
et al., 2007) and GRB 060614 (Caito et al., 2009); nevertheless, there are several aspects that need some improvements. For example, there are very energetic GRBs (bursts with a total isotropic energy emitted $\gtrsim 10^{54}$ ergs) for which the assumption of thermal spectrum in the comoving frame of the fireshell does not allow to correctly reproduce the observational data (see chapter 5). Another problem is related to flares, observed in the afterglow phase. According to the fireshell model, they have the same nature than the peaks observed in the prompt emission, namely they are produced by the interaction of the fireshell with different CBM clumps. What is peculiar in the late afterglow phases is that the typical dimensions of the clumps become smaller than the visible area of the fireshell. Therefore, the assumption of spherical symmetry of the CBM is not correct, as it does not take into due account the structure of the clumps (see chapters 4 and 5). Another drawback of the fireshell model is the lack of a complete theory describing how the Reissner-Nordstrøm black hole forms from the star progenitor.

The aim of this work is to test the validity of the various assumptions of the fireshell model by comparing the theoretical results with the main GRB observational properties and to propose some improvements when discrepancies between the numerical simulations and the observational data are present. In particular, the novelties introduced with this work are:

- A new spectral energy distribution of the photons in the comoving frame of the fireshell. I have investigated the possibility of better reproducing the spectral properties of GRBs starting from a different spectral energy distribution of the photons in the comoving frame of the fireshell. In particular, I have introduced a phenomenologically “modified” thermal spectrum: a spectrum characterized by a different asymptotic low energy slope with respect to the thermal one, fixed by the free parameter $\alpha$. I have obtained all the equations involved in the determination of the source luminosity and spectrum, then I have tested the model by comparing the numerical simulations with the observed light curves and spectra of several GRBs.
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- **A bi-dimensional structure of the CBM clump along the line of sight.** I have introduced a simplified bi-dimensional model describing the CBM structure along the line of sight to reproduce the observed characteristics of X-ray flares. In particular, I have modified the numerical code simulating the GRB emission in such a way that the integration of the emitted flux over the emitting surface is performed only up to a certain angle $\theta_c$ from the line of sight, corresponding to the transverse dimension of the CBM clump producing the flare. I have tested this model by comparing the numerical simulations with the observed X-ray flares of GRB 060607A and GRB 050904.

- **A preliminary approach to the study of NSs.** I have studied the electrodynamics of NSs to investigate the possibility of having systems with an electric field of the order or greater than $E_c$, from which a Reissner-Nordstöm black hole could originate. I have introduced a general approach based on the relativistic Thomas-Fermi equation to study the inner region of NSs: the core, described as a system composed of electrons, protons and neutrons in $\beta$-equilibrium. With this model I have investigated how the electric field at the core surface changes by assuming various different profiles for the proton number density. I have also presented a general relativistic model to study the outer region of NSs: the crust. In particular, I have investigated the possibility that the baryonic material encountered by the fireshell during its expansion is the crust, left as a remnant when the NS underwent the process of gravitational collapse to a black hole. To verify this possibility, I have determined the mass of the crust, then I have compared it with the one of the baryonic remnant estimated for various GRBs.

The description of the work done is structured as follows:

- **Chapter 1:** I present an historical overview of GRBs, together with the description of the most important missions devoted to their observation in different energy bands. I also describe the general observational
features of GRBs, with particular attention to their light curves and spectra.

- **Chapter 2:** I present the fireshell model of GRBs. I describe how the $e^\pm$ pair plasma creates and evolves: from the optically thick fireshell expansion to the optically thin phase, during which the P-GRB and the extended afterglow are emitted. I also introduce all the equations needed to determine the GRB luminosity and spectrum.

- **Chapter 3:** I present a brief overview of some GRB models alternative to the fireshell model. In particular, I focus on the standard fireball model, together with the electromagnetic model and the turbulent model. Particular attention is given to the description of the various possible scenarios for GRB progenitors.

- **Chapter 4:** I present the study of GRB 060607A. This burst represents a good example of a “canonical light curve” within the fireshell model; furthermore, it is of great interest for the presence of X-ray flares in its afterglow. I analyse the BAT and XRT light curves and compare them with our numerical simulations, in order to test the validity of the model. I present the simplified bi-dimensional model for the CBM clump along the line of sight, introduced to reproduce X-ray flares, then I show the comparison between the theoretical predictions and the observational data. I also analyse the prompt emission time-integrated spectra.

- **Chapter 5:** I introduce the phenomenologically “modified” thermal spectrum, then I describe all the equations involved in the determination of the source luminosity by considering this new spectrum. Finally, I present the comparison between the numerical simulations and the observed light curves and spectra of two bursts: GRB 080319B and GRB 050904.

- **Chapter 6:** I present the generalized relativistic Thomas-Fermi equa-
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...tion used to study the core of NSs. I explain how with this model it is possible to derive theoretically a universal relation between the total number of protons $N_p$ and the mass number $A$, valid for nuclei as well as NS core, then I show how the intensity and the shape of the electric field at the core surface changes by assuming various different profiles for the proton number density. I also present the general relativistic model introduced to describe the outer crust of NSs. I determine the mass of the crust ($M_{\text{crust}}$) with this model, then I show the comparison between $M_{\text{crust}}$ and the mass of the baryonic remnant estimated for various GRBs analysed within the fireshell model.
Chapter 1

Gamma-Ray Bursts

1.1 Historical overview of GRBs

Gamma Ray Bursts (GRBs) are flashes of gamma rays occurring at an average rate of a few per day throughout the universe. They were discovered in 1967 by the U.S. Vela satellites, which were built to monitor nuclear explosions in verification of the Nuclear Test Ban Treaty. The announcement of this discovery was made in 1973 by Klebesadel, Strong and Olson, who reported the detection of sixteen short burst of photons in the energy range 0.2-1.5 MeV in the period between July 1969 and July 1972 (Klebesadel et al., 1973). Since then, several dedicated satellites have been launched to observe the bursts and give order to the great number of theories put forward to explain their origin: in a review article at the 1975 Texas Symposium on Relativistic Astrophysics, no fewer than 100 different possible theoretical models of GRB were listed (Ruderman, 1975), most of which could not be ruled out by the observations available.

The first important step toward the comprehension of GRBs was made with the NASA Burst And Transient Source Experiment (BATSE) on board the Compton Gamma Ray Observatory (CGRO, 1991-2000), whose results have been summarized in Preece et al. (2000). The all sky survey from BATSE showed that GRBs were isotropically distributed in the sky, strongly sug-
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![Histogram of the $T_{90}$ of GRBs observed by BATSE](http://www.batse.msfc.nasa.gov/batse/grb/duration/).

suggesting a cosmological origin of these phenomena. Furthermore, based on the $T_{90}$\(^1\) distribution, it was possible to identify two main classes of GRBs: long ($T_{90} > 2$ s) and short ($T_{90} < 2$ s) GRBs (see fig. 1.3).

The confirmation that GRBs occur at cosmological distances came with the Italian-Dutch satellite *BeppoSAX*\(^2\) (1997-2002). This satellite was equipped with a gamma-ray detector (Gamma-Ray Burst Monitor, GRBM), together with the X-ray Wide Field Cameras (WFCs) and the Narrow Field Instruments (NFIs). On February 28, 1997 the GRBM was triggered by a GRB event. The WFCs detected with great accuracy the position of the burst, so that the NFIs could repoint the same position about eight hours after the burst and detect a long-lasting X-ray emission from the GRB: the “afterglow” (Costa et al., 1997). This detection, followed by others, allowed the follow-up of afterglows at optical and longer wavelengths. This paved the way for the measurement of redshifts and the confirmation of the cosmological origin of GRBs.

A consolidation of the progress made with *BeppoSAX* was obtained with

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\(^1\)The $T_{90}$ is defined as the time during which the cumulative counts increase from 5% to 95% above background, encompassing 90% of the total GRB counts.

\(^2\)Beppo in honor of Giuseppe Occhialini, SAX means “Satellite per l’astronomia X” (X-ray astronomy satellite)
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the HETE-2\textsuperscript{3} mission, which was active from 2000 to 2006. One of its main results was that it localized GRB 030329, which resulted in the second clear spectroscopic association with a supernova (SN2003dh; Stanek et al., 2003; Hjort et al., 2003) after GRB 980425 (Galama et al., 1998). Another significative progress has been made by Swift\textsuperscript{4}, a satellite designed specifically for GRB science. It was launched on November 2004 and is equipped with a wide-field hard X-ray telescope (Burst Alert Telescope, BAT), a narrow field X-ray telescope (X-Ray Telescope, XRT) and an UV-optical telescope (UV Optical Telescope, UVOT). When a GRB occurs in the BAT field of view, the spacecraft repoints the observatory to bring the burst in the XRT and UVOT fields of view: this allows Swift to perform X-ray and UV/optical follow-up within 20-70 s after the burst detection (Gehrels et al., 2004). In this way it is possible to study the transition between the prompt emission and the afterglow, revealing a rich range of X-ray light curve behaviours.

1.2 Present and Future

Swift is now the primary mission devoted to the study of GRBs and it will remain operational for many years, but there are also other important missions involved in the observation of bursts in various energy bands, that will give us the opportunity to obtain important progresses in GRB research.

The Konus\textsuperscript{5} experiment on board the WIND spacecraft, launched on November 1994, provides omnidirectional and continuous coverage of the sky in the hard-X and $\gamma$-ray domains. It provides event light curves in three energy bands within the 10-770 keV range, with 64 ms time resolution. In addition, 2 ms resolution is available for at least some parts of triggered bursts.

\textsuperscript{3}http://space.mit.edu/HETE/
\textsuperscript{4}http://swift.gsfc.nasa.gov/
\textsuperscript{5}http://heasarc.gsfc.nasa.gov/docs/heasarc/
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The INTEGRAL\(^6\) (INTErnational Gamma-Ray Astrophysics Laboratory) observatory was launched on October 2002. It is equipped with two gamma-ray instruments: a spectrometer (SPI) and an Imager (IBIS). The observatory is also equipped with X-ray and optical detectors (JEM-X and OMC respectively) to provide simultaneous observations at these wavelengths.

Other GRB missions\(^7\) operating in the X-ray band are: the NASA’s Chandra X-ray Observatory, launched on July 23, 1999, which is one of the most sofisticated X-ray observatory; the XMM-Newton (X-ray Multi-Mirror Mission), launched on Dicember 10, 1999, which provides highly sensitive observations, thanks to a very large collecting area and a great ability to make long uninterrupted exposures; the Suzaku mission, launched on July 10, 2005, which covers the energy range 0.2 - 600 keV with the X-ray CCDs (X-ray Imaging Spectrometers, or XISs) and the hard X-ray detector (HXD).

Observations in the optical energy range are also very important: they allow us to measure redshifts, firmly identify host galaxies and accurately localize GRBs within them. Now there are several operating ground-based optical telescopes: ROTSE-III (Robotic Optical Transient Search Experiment), an array of 4 robotic telescopes designed to provide optical observations of GRB afterglows as close as possible to the start of the γ-ray emission (Akerlof et al., 2003; Yost et al., 2006); RAPTOR (RAPid Telescopes for Optical Response) (Vestrand et al., 2002), designed to identify and make follow-up observations of optical transients in real time; Pi-of-the-sky, which has been optimized for GRB optical counterpart (Burd et al., 2005); REM, operating also in the Near Infrared energy band (Zerbi et al., 2001); VLT\(^8\) (Very Large Telescope), an array of 4 telescopes which can work independently or together, collecting photons in the visible and in the infrared; TNG\(^9\) (Telescopio Nazionale Galileo), an optical/infrared telescope whose main feature is the presence of an Active Optics (AO) system to perform real-time, low

\(^6\)http://sci.esa.int/science-e/www/object/index
\(^7\)http://heasarc.gsfc.nasa.gov/docs/heasarc/
\(^8\)http://www.eso.org/public/astronomy/teles-instr/
\(^9\)http://www.tng.iac.es/instruments/telescope_description.html
frequency correction of the optical components in order to ensure the best optical performances in all conditions.

Observations in the High Energy (HE) band (30 MeV - 30 GeV) allow us to investigate the high energy region of GRB spectrum and to put some constraints on the various emission models. There are now two main space-based instrument working in the HE band: AGILE and Fermi. The AGILE (Astrorivelatore Gamma ad Immagini ultra LEggero) satellite, launched on 23 April 2007, is an Italian Space Agency mission dedicated to high energy astrophysics, with the study of GRBs among its main scientific targets. It is equipped with the GRID (Gamma-ray Imaging Detector) instrument, together with the hard-X ray Imager (SuperAGILE) and a Mini-Calorimeter (MCAL), operating in the 30 MeV - 50 GeV, 18 - 60 keV and 350 keV - 100 MeV energy ranges respectively (Tavani et al., 2009).

The Fermi Gamma-ray space telescope was launched on 11 June 2008. It is equipped with the Large Area Telescope (LAT), sensitive in the 20 MeV - 300 GeV energy band (Atwood et al., 2009) and the Gamma Bursts Monitor (GBM), especially designed for the detection of GRBs, sensitive in the 8 keV - 40 MeV energy band (Megan et al., 2009).

Extremely important is also the study of the emission in the Very High Energy (VHE) band (30 GeV - 30 TeV), that will allow us to understand the behavior, the composition and the dynamics of the most accelerated particles in GRBs. This study can be performed thanks to the IACT (Imaging Atmospheric Cherenkov Telescope) arrays: multi-telescope arrays designed to detect the Cherenkov light emitted in the atmosphere by secondary electrons, which are produced in the cascades initiated by primary cosmic rays. There are currently four operating IACT arrays: MAGIC (Major Atmospheric Gamma ray Imaging Cherenkov), located on the Canary island of La Palma (Cortina et al., 2005); HESS (High Energy Stereoscopic System), located in the Khomas Highland of Namibia (Hinton et al., 2004); VERITAS (Very Energetic Radiation Imaging Telescope Arrays System), located near Amado in Arizona (Krenrich et al., 2007) and CANGAROOIII (Col-
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laboration of Australia and Nippon for a GAmma Ray Observatory in the Outback), located near Woomera in the South of Australia (Enomoto et al., 2006).

1.3 Observational properties

GRBs are short and intense pulses of high energy gamma rays isotropically distributed in the sky. It is difficult to trace a “typical” GRB, nevertheless it is possible to establish some observational features to characterize them. The first GRB peculiarity is their enormous observed fluence, between $10^{-7}$ and $10^{-4}$ ergs/cm$^2$. This corresponds to a total isotropic energy emitted of $10^{51} - 10^{55}$ ergs, making them the most luminous objects in the sky (Piran, 2004).

Usually we can identify two main components in the light curve of GRBs: a prompt emission, which is a flash of $\gamma$-rays typically lasting a few tenths of seconds, and the afterglow emission, which is the long-lasting (from days to weeks) multiwavelength counterpart, softening from X-rays to optical to radio; we will see in the next chapters that, within the fireshell model, these two components contribute to the so-called “extended afterglow” and are produced by the same physical mechanism.

1.3.1 Prompt emission

The prompt emission light curves are characterized by a great variety of temporal profiles (see fig 1.2): they range from single-peaked pulses to highly structured multi-pulses, with a time variability up to 1 ms (Piran, 2004) and a duration ranging from $10^{-3}$ to $10^3$ s. The prompt emission spectrum display a clear non-thermal shape and is well fitted by two power-laws joined smoothly at a break energy $(\alpha - \beta) E_0$ (Band et al., 1993):

$$N_E(E) = \begin{cases} 
(h\nu)^\alpha \exp\left(-\frac{h\nu}{E_0}\right) & \text{if } h\nu < (\alpha - \beta) E_0 \\
[(\alpha - \beta) E_0]^{\alpha-\beta} (h\nu)^\beta \exp(\beta - \alpha) & \text{if } h\nu > (\alpha - \beta) E_0.
\end{cases}$$
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Figure 1.2: Prompt emission light curves observed by BATSE (Credit: J.T. Bonnell, NASA/GSFC).

In fig 1.3 is shown, as an example, the fit of the spectrum of GRB 990123. Several GRBs also display bright optical flashes during their prompt emission; this prompt optical emission was observed: (i) to be decoupled from that at γ-rays, as in the case of GRB 990123 (Akerlof et al., 1999); (ii) to be a superposition of a variable component tracking the X-ray/γ-ray light curve and a smooth one peaking after the γ-ray prompt emission, as in the case of GRB 050820A (Vestrand et al., 2006); (iii) to fluctuate in phase with that at γ-rays, as in the case of GRB 041219 (Vestrand et al., 2005). There are several correlations between various prompt emission properties. Some of the most important ones are:
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Figure 1.3: Observed spectrum by BATSE for GRB 990123 and best-fitting Band function (Briggs et al., 1999).

**Amati correlation:** it was obtained by Amati et al. (2002). It is the correlation between the photon energy at which the $\nu F_\nu$ GRB spectrum peaks (the so called “peak energy”, indicated as $E_p$ when referring to the observed spectrum or $E_{p,i} = E_p \times (1 + z)$ for the cosmological rest-frame spectrum) and the total isotropic-equivalent radiated energy of the prompt emission $E_{iso}$: $E_{p,i} \propto E_{iso}^m$, with $m \sim 0.5$.

**Yonetoku correlation:** it is the correlation between $E_{p,i}$ and the isotropic luminosity emitted at the peak of the light curve (the peak luminosity, indicated as $L_{p,iso}$): $E_{p,i} \propto L_{p,iso}^{0.5}$ (Yonetoku et al., 2004).

**Firmani correlation:** it is the correlation between $L_{p,iso}$, $E_{peak}$ and the time interval spanned by the brightest 45% of the total counts above the background (the ‘high signal’ time-scale, indicated as $T_{0.45}$): $L_{iso} = E_{pk}^{1.62} \times T_{0.45}^{-0.49}$ (Firmani et al., 2006).
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1.3.2 Afterglow emission

The afterglow shows a less complex behavior than the prompt one. If we look at the X-ray light curve, it is possible to recognize a “canonical” behavior (Nousek et al., 2006; Evans et al., 2009), which includes one or more of the following contributions: an initial very steep decay ($F \propto t^{-\alpha_1}$), with $3 \leq \alpha_1 \leq 5$, followed by a very shallow decay ($F \propto t^{-\alpha_2}$), with $0.5 \leq \alpha_2 \leq 1$ and finally a steeper decay ($F \propto t^{-\alpha_3}$), with $1 \leq \alpha_3 \leq 1.5$. Sometimes, a further steepening is detected after the normal decay phase, which is consistent with a jet break\(^\text{10}\) (Zhang et al., 2006). The transition between the three main contributions occur at two break-times: $t_{\text{break,1}} \lesssim 500\,\text{s}$ and $10^3\,\text{s} \lesssim t_{\text{break,2}} \lesssim 10^4\,\text{s}$. The spectrum, except in few cases, remains constant throughout all these stages of the afterglow (Nousek et al., 2006).

On top of this canonical behavior, many events have superimposed X-ray flares. Most flares have a very steep rise and decay, with very large temporal rise/decay indices when fitted with a power-law (Nousek et al., 2006). Flares are spectrally harder than the underlying afterglow and evolves from hard to soft (Burrows et al., 2005). Furthermore, the fluxes before and after the flare lie approximately on the same power-law decay. Representative examples of Swift GRB X-ray afterglows are shown in fig.1.4.

Many GRBs also show an optical/UV afterglow. This signal decays as a power law ($F \propto t^{-\alpha}$) and sometimes a further steepening is observed at late times ($t \gtrsim 1\,\text{day}$), which is consistent with a jet break. The observed optical spectrum is also a power law, generally with some absorption lines superimposed. While X-ray afterglows are detected in nearly all GRBs, only about 50% of bursts show optical/UV afterglows; the bursts without optical counterparts are called dark bursts. The nature of dark bursts is still unclear and several explanations have been proposed, such as absorption by molec-

\(^{10}\)If the energy is emitted into a narrow jet, when the Lorentz factor of the relativistic ejecta responsible for GRB emission becomes larger than the jet opening angle we observe the so called “jet break”: an achromatic steepening of the afterglow light curve (see also section 3.1.2).
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Figure 1.4: Representative examples of X-ray afterglows of long and short Swift GRBs with steep-to-shallow transitions (GRB 050315, GRB 050724), large X-ray flares (GRB 050502B, GRB 050724), fastly declining (GRB 051210) and gradually declining (GRB 051221a, GRB 050826; the flux scale is divided for 100) afterglows. Picture from Gehrels et al. (2009).

1.4 Association with supernovae

Intensive optical, infrared and radio follow-up of GRBs has established that at least a significant fraction of long-duration GRBs are directly connected with supernova (SN) explosions. The main evidence for this association arises from observations of supernova features in the spectra of a few GRB afterglows. Examples of clear cases of the GRB/SN connection are GRB 980425/SN 1998bw (Galama et al., 1998), GRB 030329/SN 2003dh (Stanek et al., 2003; Hjort et al., 2003), GRB 031203/SN 2003lw (Malesani et al., 2004), GRB 050525A/SN 2005nc (Della Valle et al., 2006c) and GRB 060218/SN 2006aj (Masetti et al., 2006; Modjaz et al., 2006; Campana et al., 2006; Sollerman
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et al., 2006; Pian et al., 2006; Mirabel et al., 2006). In addition, there are several afterglows which show, days to weeks after the gamma-ray events, a rebrightening (or flattening) in their light curves (Zeh, Klose & Hartmann, 2004). These bumps are interpreted as SNe emerging out of the GRBs’ afterglows (Bloom et al., 1999; Castro-Tirado & Gorosabel, 1999).

There are also bursts for which the association with a SN is ruled out by the observations: this is the case of two nearby GRBs, GRB 060505 (z = 0.09) and GRB 060614 (z = 0.13). Due to their proximity, deep follow-up campaigns to search for SNe were done, but none were found in either case: the eventually associated SNe would have been more than 2 orders of magnitude fainter than the prototypical GRB-associated SN, 1998bw (Fynbo et al., 2006; Della Valle et al., 2006b).

If we focus on GRBs with a spectroscopically confirmed associated SN (GRB 980415, GRB 030329, GRB 031203 and GRB 060218), we found that all but GRB 030329 have γ-ray energy budgets between 2-4 orders of magnitude fainter than those exhibited by “standard” GRBs. The increasing number of discovery of these underenergetic events (with an associated SN component) can no longer be considered a simple collection of peculiar, atypical cases. These bursts were so faint, that they would have been easily missed at cosmological distances, therefore it is very possible that they are the most frequent GRBs in the universe (Della Valle et al., 2006a).

Concerning the SNe associated with these events, they are type Ib/c SNe more luminous with respect to the typical ones; furthermore, they have longer risetime and eject material at very high velocities (Pian et al., 2006). The peculiarities common to all GRB supernovae allow us to consider them a “subclass” of type Ib/c SNe, called hypernovae (Iwamoto et al., 1998) in order to emphasize the extremely high energy involved in these explosions.

11Type I SNe are characterized by the absence of hydrogen lines in their optical spectra. Among them, three subclasses are known: SN Ia, Ib and Ic. SNe Ia have Si absorption lines in their spectrum; SNe Ib have not Si lines but show He lines; SNe Ic display neither Si nor He lines. SNe Ib/c probably result from core collapse of massive stars largely stripped of their hydrogen (Ib) and helium (Ic) envelopes (Filippenko, 1997).
1. Gamma-Ray Bursts

The GRB/SN connection provides a direct evidence, at least in some cases, of a common scenario between GRBs and the death of massive stars, that is useful to constrain different progenitor models.
Chapter 2

The fireshell model of GRBs

In this chapter we describe in detail the fireshell model we use to interpret the GRB observational properties. As it will be clarified here and in chapter 3, there are many differences between our approach and the one currently addressed as “standard model for GRBs”. Surely the basic novelty of our approach is that it represents the first attempt to analyze the GRB phenomenon as a whole. In fact, starting from the initial conditions which characterize the progenitor, we fix uniquely the dynamics of the system since its creation (Ruffini et al., 2001a,b, 2003, 2005b). Furthermore, we calculate the exact solutions for the equations of motion of the system (Bianco & Ruffini, 2004, 2005), without the approximations usually adopted in the “standard model” (Mészáros, Laguna & Rees, 1993; Sari, 1997, 1998; Waxman, 1997a; Rees & Mészáros, 1998; Granot, Piran & Sari, 1999; Panaitescu & Mészáros, 1998; Piran, 1999; Gruzinov & Waxman, 1999; Van Paradijs, Kouveliotou & Wijers, 2000; Mészáros, 2002).

To interpret GRB spectra and light curves within the fireshell model, we assume that the radiation emitted in the comoving frame of the fireshell is thermal (see section 2.4 and Ruffini et al., 2004), although the observed spectrum is clearly non-thermal. In fact, each single instantaneous spectrum is the result of an integration of hundreds of thermal spectra over the corresponding “EQuiTemporal Surface” (see section 2.3.2 and Bianco & Ruffini, 2004,
2. The fireshell model of GRBs

2005); this calculation produces a non-thermal behavior of the instantaneous spectrum. Time integrated spectra are obtained by the convolution of such instantaneous spectra over the observational time Bernardini et al. (2005b) and they also have a clear non-thermal shape. Another assumption is that the emitted radiation is produced in inelastic collisions between the fireshell and the Circumburst Medium (CBM). The behavior of the light curve depends on the CBM structure; in particular, the CBM is arranged in spherical shells of width $\approx 10^{15}$ cm located in such a way that the modulation of the emitted flux coincides with the observed peaks of the light curve (see section 2.5).

2.1 The $e^{\pm}$ creation.

According to the fireshell model, GRBs are produced by a process of energy extraction from a black hole. All the properties of a generic black hole are described by three parameters: its mass $M$, its angular momentum $L$ and its electric charge $Q$. We assume, for simplicity, that in our case $L = 0$. Christodoulou & Ruffini (1971) showed that for a Reissner-Nordstrom black hole ($Q \neq 0, L = 0$) a large fraction of energy (up to 50%) can be stored as extractable electromagnetic energy. The mass-energy formula for a charged black hole is (Christodoulou & Ruffini, 1971):

$$E^2 = M^2 c^4 = \left( M_{irr} c^2 + \frac{Q^2}{2r_+} \right)^2$$

$$S = 4\pi r_+^2 = 16\pi \left( \frac{G^2}{c^4} \right) M_{irr}^2,$$

where $M_{irr}$ is the black hole irreducible mass (Christodoulou & Ruffini, 1971), namely the minimum value of the mass left after the energy extraction process, $r_+$ is the horizon radius and $S$ is the horizon surface.

Damour & Ruffini (1975) demonstrated that in the neighborhood of a charged black hole the vacuum polarization process (Heisenberg & Euler, 1936; Schwinger, 1951) can occur if the electric field exceeds the critical
2.1. The $e^\pm$ creation.

The value
\[ E_c = \frac{m_e^2 c^3}{\hbar e} , \]  
(2.1)

where $m_e$ and $e$ are the mass and the charge of the electron. They found that (see also Preparata, Ruffini & Xue, 1998):

- the polarization process can occur around a black hole of mass between $3.2M_\odot$ and $7.2 \times 10^6 M_\odot$;

- the pair creation process occurs on very short time scales ($t \approx \frac{\hbar}{m_e c^2}$) and it is a completely reversible process according to the definition by Christodoulou & Ruffini (1971), leading to an extremely efficient mechanism for the energy extraction;

- the energy extracted is $\approx 10^{54}$ erg, compatible with the observed GRB isotropic energy.

The pair creation occurs in a region called “dyadosphere” (Preparata, Ruffini & Xue, 1998), between the horizon radius
\[ r_+ = \frac{GM}{c^2} \left( 1 + \sqrt{1 - \frac{Q^2}{GM^2}} \right) \]  
(2.2)

and the “dyadosphere radius” $r_{ds}$ (Preparata, Ruffini & Xue, 1998), defined as
\[ \mathcal{E}(r_{ds}) = \frac{Q}{r_{ds}^2} = E_c . \]  
(2.3)

The total $e^\pm$ energy stored in this region is (Preparata, Ruffini & Xue, 1998)
\[ E_{c^\pm}^{tot} = \int_{r_+ < r < r_{ds}} \epsilon(r) \alpha d\Sigma , \]  
(2.4)

\[ \epsilon(r) = \frac{(\mathcal{E}^2(r) - \mathcal{E}_c^2)}{8\pi} = \frac{Q^2}{8\pi r^4} \left[ 1 - \left( \frac{r}{r_{ds}} \right)^4 \right] , \]  
(2.5)

where $\epsilon(r)$ is the electromagnetic energy density measured by a static observer, $\alpha = \sqrt{1 - \frac{2GM}{c^2 r} + \frac{GQ^2}{c^4 r^2}}$ is the gravitational redshift and $d\Sigma = \alpha^{-1} r^2 dr d\Omega$ is the 3-dimensional volume element.
2. The fireshell model of GRBs

2.2 The optically thick fireshell expansion

2.2.1 The equations of the theory - I

We study now the evolution of a plasma composed by $e^\pm$, photons and baryons (Ruffini et al., 1999, 2000). Such a plasma is described by the stress-energy tensor:

$$T^{\mu\nu} = p g^{\mu\nu} + (p + \rho) U^\mu U^\nu,$$

(2.6)

where $\rho$ is the total energy density of the plasma, $p$ the pressure and $U^\mu$ its 4-velocity.

The energy-momentum conservation law is

$$(T^\mu_{\;\nu})_{;\nu} = 0.$$  (2.7)

The conservation law for the baryonic number is

$$(n_B U^\mu)_{;\mu} = 0.$$  (2.8)

In our analysis we include also the rate equation for $e^\pm$:

$$(n_{e^\pm} U^\mu)_{;\mu} = \bar{\sigma} v (n_{e^-}(T)n_{e^+}(T) - n_{e^0} - n_{e^0}),$$

(2.9)

where $\bar{\sigma}$ is the mean pair annihilation cross section, $v$ is the thermal velocity of $e^\pm$, $n_{e^\pm}(T)$ are the proper number densities of $e^\pm$, given by appropriate Fermi integrals with zero chemical potential, and $n_{e^0}$ are the proper number densities at the initial time of the expansion. The equilibrium temperature $T$ is determined by the thermalization processes occurring in the expanding plasma with a total energy density $\rho = \rho_\gamma + \rho_{e^+} + \rho_{e^-} + \rho_B$, given by the sum of Fermi or Bose integrals, one for each fluid component. We can also, analogously, evaluate the total pressure $p$. We have, then, the equation of state ($\Gamma$ is the thermal index)

$$\Gamma = 1 + \frac{p}{\rho}.$$  (2.10)
2.2. The optically thick fireshell expansion

2.2.2 The numerical integration

In order to integrate the conservation laws for the plasma it is necessary to simplify the problem. First of all, we have assumed that the system evolves in a Minkowski space-time. This is reasonable if we consider the value of $g_{00}$ for a Reissner-Nordstøm space-time evaluated at a distance from the black hole $\bar{r} = 2 \, r_{ds}$ (for $M = 10 M_\odot$ and $Q/(\sqrt{GM}) = 7.0 \times 10^{-3}$). We see that

$$g_{00}(\bar{r}) = 1 - \frac{GM}{c^2 \bar{r}} + \frac{GQ^2}{c^4 \bar{r}^2} \approx 0.98,$$

so the space-time in which the plasma expands is approximately flat even at short distances from the dyadosphere.

We assume also that the width of the plasma remains constant in the laboratory frame, namely the one in which the progenitor is at rest, throughout the expansion. This proposal, together with other possible descriptions of the expanding plasma, was compared with the exact solutions of the hydrodynamic equations obtained with the Livermore numerical code resulting, among all, the best approximation (see fig. 2.1 and Ruffini et al., 1999).

2.2.3 The equations of the theory - II

We derive explicitly now, starting from the assumptions presented in section 2.2.2, the equations we use to describe the evolution of the plasma.

From the differential conservation law (2.7), the following integral conservation law can be derived (Synge, 1960):

$$\int_{\Sigma_t} \xi_\mu T^{\mu \nu} d\Sigma_\nu = E, \quad (2.11)$$

where $\xi_\mu$ is the static vector field normalized at unity at spatial infinity, $\Sigma_t$ is the space-like hypersurface orthogonal to $\xi_\mu$ and $d\Sigma_\nu$ is the vector surface element of $\Sigma_t$. Assuming the constancy of the pulse width and recalling the equation of state (2.10), we get

$$E = (\Gamma \rho \gamma^2 + p) \mathcal{V}, \quad (2.12)$$
Figure 2.1: The exact solution of the 1-dimensional hydrodynamic equations obtained by the Livermore numerical code (black squares) is compared with several approximated description of the expanding plasma. Among all, the best assumption has been found the so-called “Slab 1” (solid line), which corresponds to a constant width in the laboratory frame.
2.2. The optically thick fireshell expansion

where $V$ is the volume of the pulse in the laboratory frame and $V = \gamma V$ is the same volume in the comoving frame. Since the Lorentz factor $\gamma$ at this stage is high ($\approx 50$), in what follows we can neglect the pressure term, namely

$$E = \Gamma \rho \gamma^2 V.$$  \hspace{1cm} (2.13)

This means that

$$\Gamma \rho_0 \gamma_0^2 V_0 = \Gamma \rho \gamma^2 V,$$  \hspace{1cm} (2.14)

where the subscript “0” refers to quantities evaluated at the initial time. This equality is used to define the evolution of the pulse Lorentz factor

$$\gamma = \gamma_0 \sqrt{\frac{\rho_0 V_0}{\rho V}}.$$  \hspace{1cm} (2.15)

Analogously, we can write the integral conservation law for the baryonic number (eq. (2.8)):

$$\int_{\Sigma_t} n_B U^\nu d\Sigma_\nu = N_B = n_B \gamma V,$$  \hspace{1cm} (2.16)

and, again,

$$n_B \gamma_0 V_0 = n_B \gamma V,$$  \hspace{1cm} (2.17)

$$n_B = n_B \gamma_0 \gamma V.$$  \hspace{1cm} (2.18)

In order to obtain the evolution of the internal energy density $\epsilon = \rho - \rho_B$ ($\rho_B = n_B m_p c^2$) during the expansion of the pulse we must impose the adiabaticity condition:

$$d(\epsilon V) + pdV = 0.$$  \hspace{1cm} (2.19)

Recalling from eq. (2.10) that $p = (\Gamma - 1)\epsilon$, we get

$$d\ln \epsilon + \Gamma d\ln V = 0.$$  \hspace{1cm} (2.20)

If we integrate this equation we obtain for the internal energy density

$$\epsilon = \epsilon_0 \left(\frac{V_0}{V}\right)^\Gamma.$$  \hspace{1cm} (2.21)
2. The fireshell model of GRBs

Figure 2.2: Lorentz gamma factor versus radial coordinate for two different positions of the baryonic remnants: $r_B = 50 r_{ds}$ and $r_B = 5 r_{ds}$. It is evident the convergence to a common behavior when the system approaches the transparency.

2.2.4 The expansion of the “fireshell”

After its creation around the progenitor, the $e^\pm$ plasma starts to expand as a “fireshell” in a region with very low baryonic contamination because of the gravitational collapse of the central black hole (Ruffini et al., 1999). We label the expanding plasma “fireshell” in order to emphasize the difference with the “fireball” (see chapter 3). In our treatment the plasma behaves as a “shell”, namely it moves as a single shell with a Lorentz factor $\gamma$ and with constant thickness in the laboratory frame, and all the particles stored inside the plasma move with the same Lorentz factor $\gamma$.

After this first stage, the pulse reaches the baryonic remnants left over by the collapse of the progenitor, necessary to guarantee the neutrality of the system (Ruffini et al., 2003). Such baryonic remnants are assumed to be arranged in a spherical shell of width $\Delta \approx 10 r_{ds}$ which is outside the dyadosphere, but near enough that it can be reached by the plasma before it has become transparent. For simplicity, we choose $r_B = 50 r_{ds}$. From our numerical simulations we have found that the following expansion of the pulse is quite insensitive to the position of the baryonic remnants (Ruffini et al., 2003, see fig. 2.2).
2.2. The optically thick fireshell expansion

The total mass of the baryonic remnants \( M_B = N_B m_p \), where \( N_B \) is the number of baryons and \( m_p \) is the proton mass, written as a dimensionless parameter in function of \( E_{\text{tot}}^{\pm} \),

\[
B = \frac{M_B c^2}{E_{\text{tot}}^{\pm}},
\]

is the second free parameter of our theory defining completely, together with \( E_{\text{tot}}^{\pm} \), the dynamics of the system (Ruffini et al., 2000).

In order to study the interaction between the expanding \( e^\pm \) plasma and the shell of baryonic remnants we have to consider some assumptions (Ruffini et al., 2000, 2003):

- the \( e^\pm \) pulse does not change its geometry during the interaction;
- the interaction between them is a totally inelastic collision;
- the baryonic remnants reach the thermal equilibrium with the incoming pulse.

These assumptions are valid if (Ruffini et al., 2000):

- the total energy of the plasma is much larger than the one of the baryonic matter, namely \( B < 10^{-2} \);
- the ratio between the \( e^\pm \) number density \( n_{e^\pm} \) and the baryon number density \( n_B \) is large (e.g. \( n_{e^\pm}/n_B > 10^6 \));
- the Lorentz gamma factor of the incoming pulse is high (\( \gamma \approx 100 \)).

Now we write the energy and momentum conservation for the system. If we consider the collision of the pulse with a sub-shell of baryonic matter between the radii \( r_1 \) and \( r_2 \), with \( r_B < r_1 < r_2 \) and \( r_2 - r_1 < \Delta \), the amount of baryonic matter loaded by the incoming pulse is:

\[
\Delta M = \frac{M_B 4\pi}{V_B} \frac{4\pi}{3} (r_2^3 - r_1^3).
\]
The conservation of energy and momentum can be written as (Ruffini et al., 2000, 2003):

\[
(\Gamma_0 + \rho_B^0)\gamma_0^2 \nu_0 + \Delta M = \left( \Gamma(\epsilon + \Delta \epsilon) + \rho_b + \frac{\Delta M}{V} \right) \gamma^2 \nu
\]

\[
(\Gamma_0 + \rho_B^0)\gamma_0 U_r^0 \nu_0 = \left( \Gamma(\epsilon + \Delta \epsilon) + \rho_b + \frac{\Delta M}{V} \right) \gamma U_r \nu,
\]

(2.24)

where \(\Delta \epsilon\) is the increase in the plasma internal energy due to the collision in the comoving frame, \(\nu\) and \(V = \gamma \nu\) are, respectively, the volume of the incoming pulse in the laboratory and in the comoving frames and \(U_r\) is the radial component of the 4-velocity of the pulse. The quantities with (without) the index “0” are evaluated before (after) the collision. So we get (Ruffini et al., 2000, 2003):

\[
\Delta \epsilon = \frac{1}{\Gamma} \left[ \Gamma \epsilon_0 + \rho_B^0 \frac{\gamma_0 U_r^0 \nu_0}{\gamma U_r \nu} - \left( \Gamma \epsilon + \rho_b + \frac{\Delta M}{V} \right) \right],
\]

\[
\gamma = \frac{a}{\sqrt{a^2 - 1}}, \quad a \equiv \frac{\gamma_0}{U_r^0} + \frac{\Delta M}{(\Gamma \epsilon_0 + \rho_B^0) \gamma_0 U_r^0 \nu_0}.
\]

(2.25)

The global results of the interaction between the \(e^\pm\) pulse and the baryonic remnants are (Ruffini et al., 2000, 2003, see fig. 2.3):

\(\bullet\) an abrupt decrease in the Lorentz factor of the system;

\(\bullet\) an increase of its comoving internal energy, with a consequent heating of the plasma in the comoving frame, an increase in the \(e^\pm\) number and in the opacity of the system.

2.2.5 The approach to the transparency

After the baryonic matter loading, the pulse composed by \(e^\pm - \gamma\)–baryons continues its expansion until the plasma cools and becomes optically thin (Ruffini et al., 2000):

\[
\int_R dr (n_{e^\pm} + Z n_B) \sigma_T \approx O(1),
\]

(2.26)
2.2. The optically thick fireshell expansion

Figure 2.3: The Lorentz gamma factor of the system versus the radial coordinate. After a first stage of free expansion, the plasma impacts with the baryonic remnants with the consequent abrupt decrease on the Lorentz factor. Then, the system accelerates again. We notice that there is still a good agreement between our simulation (dashed line), performed assuming a constant width of the shell in the laboratory frame, and the Livermore one (squared dots), at least for small values of the baryon loading ($B = 10^{-4}$ in this simulation).
2. The fireshell model of GRBs

where $\sigma_T = 0.665 \cdot 10^{-24}$ cm$^2$ is the Thomson cross section, $Z$ is the average number of electrons per baryon and the integration is performed over the width of the pulse in the comoving frame. We have to notice that the presence of the baryons increases the opacity and delays the instant of transparency (Ruffini et al., 1999, 2000, 2003).

Now all the photons stored inside the pulse are emitted in a flash of radiation: the Proper Gamma-Ray Burst (P-GRB; Ruffini et al., 1999, 2000, 2001a, 2003). The remaining accelerated baryons continue their expansion and, interacting with the CircumBurst Medium (CBM), produce a prolonged emission: the Extended Afterglow (Ruffini et al., 2001a, 2003).

It is important to check if the constant width approximation is still valid until the transparency. As we can see in fig. 2.3, the assumption of a constant width in the laboratory frame for the shell is in perfect agreement with the Livermore simulation even when we include the baryons in the plasma but only for a small value of baryonic matter ($B = 10^{-4}$ in this simulation). When $B$ goes to its limiting value $10^{-2}$ this approximation is not valid anymore. In fact turbulent motions inside the shell can occur and they can even stop the pulse, preventing the GRB occurrence. de Barros et al. (submitted to A&A) are currently relaxing this approximation. In analogy with the works of Piran et al. (1993) and Mé{sz}áros, Laguna & Rees (1993), they solved numerically the conservation equations for the optically thick fireshell and they found that the constant width approximation is not the unique solution: structures can form and survive in the expanding fireshell up to large radii. The formation of a trailing edge of matter distribution could be responsible for the emission of the decaying afterglow phase of GRBs (see appendix A).

We turn now to evaluate the Lorentz factor of the pulse $\gamma_0$ at the transparency. There is a maximum value for $\gamma_0$ (Ruffini et al., 2000):

$$\gamma_{\text{asym}} = \frac{E_{\text{tot}}^{\gamma+\gamma}}{M_B c^2} = \frac{1}{B}$$

(2.27)

that corresponds to a situation in which all the energy of the pulse is converted into baryon kinetic energy. As we can see in fig. 2.4, when $B$ tends to
2.2. The optically thick fireshell expansion

Figure 2.4: Lorentz gamma factors for selected values of the baryon loading $B$ are represented versus the radial coordinate until the transparency is reached. The Lorentz factor at transparency tends to $\gamma_{\text{asym}}$ when $B$ increases.
2. The fireshell model of GRBs

its maximum value $\gamma_0$ tends to that “asymptotic” value,

$$\gamma_0 \xrightarrow{B-10^{-2}} \gamma_{asym}.$$  

It means that the fraction of energy emitted in the P-GRB and the one converted into baryonic kinetic energy depends on the amount of baryons stored inside the pulse (Ruffini et al., 2000, 2003). This could provide a possible explanation for the different observational properties of short and long GRBs (Ruffini et al., 2001b, 2003).

2.3 The optically thin fireshell evolution

After the emission of the P-GRB, the optically thin fireshell composed by baryons continues its expansion and interacts with the surrounding CBM. In order to study this interaction, we assume that:

- the interaction between the fireshell and the CBM is represented as a sequence of inelastic collisions of the baryons with a series of cold, thin shells of CBM, at rest in the laboratory frame;
- the fireshell width remains constant in the laboratory frame;
- the energy released in the collision is emitted instantaneously (“fully radiative” condition, see Ruffini et al., 2003).

Each CBM shell is assumed to have a mass $\Delta M_{CBM}$ and a width $\Delta r$ in the laboratory frame. Its collision with the fireshell releases an amount of internal energy $\Delta E_{int}$. Let $\Delta \epsilon$ be the increase in the internal energy density of the baryonic shell during the collision with a single CBM shell,

$$\rho_B = \frac{(M_B + M_{CBM})c^2}{V} \quad (2.28)$$

the baryonic matter energy density stored inside the fireshell, both measured in the comoving frame and

$$M_{CBM} = \frac{4}{3} \pi (r^3 - r_0^3)m_p n_{CBM} \quad (2.29)$$
2.3. The optically thin fireshell evolution

the amount of CBM mass swept up within the radius \( r \).

The conservation of energy and momentum during each collision can be written as

\[ \rho_B \gamma_1^2 V_1 + \Delta M_{CBM} c^2 = \left( \rho_B \frac{V_1}{V_2} + \frac{\Delta M_{CBM} c^2}{V_2} + \Delta \epsilon \right) \gamma_2^2 V_2, \quad (2.30) \]

\[ \rho_B \gamma_1 U_{r_1} V_1 = \left( \rho_B \frac{V_1}{V_2} + \frac{\Delta M_{CBM} c^2}{V_2} + \Delta \epsilon \right) \gamma_2 U_{r_2} V_2, \quad (2.31) \]

where the quantities with the index “1” (“2”) are evaluated before (after) the collision, \( V \) is the volume of the fireshell in the comoving frame and \( V = \gamma V \) is the same volume in the laboratory frame, \( U_r \) is the radial component of the 4-velocity (Ruffini et al., 1999, 2000, 2003).

From eqs. (2.30) and (2.31) we get (Ruffini et al., 2003)

\[ \Delta \epsilon = \rho_B \gamma_1 \gamma_1 U_{r_1} V_1 \left( \frac{\rho_B}{V_2} + \frac{\Delta M_{CBM} c^2}{V_2} \right), \quad (2.32) \]

\[ \gamma_2 = \frac{a}{\sqrt{a^2 - 1}}, \quad a = \frac{\gamma_1}{U_{r_1}} + \frac{\Delta M_{CBM} c^2}{\rho_B \gamma_1 U_{r_1} V_1}. \quad (2.33) \]

We can write

\[ \Delta \epsilon = \frac{E_{int_2}}{V_2} - \frac{E_{int_1}}{V_1} = \frac{E_{int_1} + \Delta E_{int_1}}{V_2} - \frac{E_{int_1}}{V_1} = \frac{\Delta E_{int_1}}{V_2}, \quad (2.34) \]

since we have assumed a fully radiative regime (\( E_{int_1} = 0 \)). Substituting this expression into eqs. (2.32) and (2.33) we obtain (Ruffini et al., 2003)

\[ \Delta E_{int} = \rho_B V_1 \sqrt{1 + 2\gamma_1 \frac{\Delta M_{CBM} c^2}{\rho_B V_1}} + \left( \frac{\Delta M_{CBM} c^2}{\rho_B V_1} \right)^2 - \rho_B V_1 \left( 1 + \frac{\Delta M_{CBM} c^2}{\rho_B V_1} \right), \quad (2.35) \]

\[ \gamma_2 = \frac{\gamma_1 + \frac{\Delta M_{CBM} c^2}{\rho_B V_1}}{\sqrt{1 + 2\gamma_1 \frac{\Delta M_{CBM} c^2}{\rho_B V_1}} + \left( \frac{\Delta M_{CBM} c^2}{\rho_B V_1} \right)^2}. \quad (2.36) \]

These are the equations which are numerically integrated in order to follow the fireshell expansion during the afterglow. In fig. 2.5 is represented the Lorentz factor of the system versus the radial coordinate from the initial
2. The fireshell model of GRBs

Figure 2.5: Lorentz gamma factor of the system versus the radial coordinate for all the stages. I: free expansion of the $e^\pm$ plasma; II: collision with the baryonic remnants; III: expansion of the pulse composed by $e^\pm-\gamma$-baryons; IV: afterglow phase; V: non-relativistic regime. In the point 4 the system reaches the transparency.
2.3. The optically thin fireshell evolution

stages of the expansion to the approach to the non-relativistic regime (Ruffini et al., 2001a, 2003).

If we consider the limit in which the CBM shells are infinitesimally thin, so that the relation

\[ \eta = \frac{\Delta M_{\text{CBM}} c^2}{\rho_{B_1} V_i} \ll 1 \]  

(2.37)

is satisfied, we can expand eqs. (2.35) and (2.36) to the first order in \( \eta \) obtaining in the differential form

\[ dE_{\text{int}} = (\gamma - 1) dM_{\text{CBM}} c^2, \]

\[ d\gamma = -\frac{2\gamma-1}{M} dM_{\text{CBM}}, \]

\[ dM = \frac{1}{\varepsilon^2} dE_{\text{int}} + dM_{\text{CBM}}, \]  

(2.38)

\[ dM_{\text{CBM}} = 4\pi m_p n_{\text{CBM}} r^2 dr, \]

A first integral of these equations has been found (Piran, 1999; Chiang & Dermer, 1999; Bianco & Ruffini, 2004, 2005), leading to expressions for the Lorentz gamma factor as a function of the radial coordinate. In the “fully adiabatic condition” (i.e. \( \varepsilon = 0 \)) we have:

\[ \gamma^2 = \frac{\gamma_0^2 + 2\gamma_0 (M_{\text{CBM}}/M_B) + (M_{\text{CBM}}/M_B)^2}{1 + 2\gamma_0 (M_{\text{CBM}}/M_B) + (M_{\text{CBM}}/M_B)^2}, \]  

(2.39)

while in the “fully radiative condition” (i.e. \( \varepsilon = 1 \)) we have:

\[ \gamma = \frac{1 + (M_{\text{CBM}}/M_B) (1 + \gamma_0^{-1}) [1 + (1/2) (M_{\text{CBM}}/M_B)]}{\gamma_0^{-1} + (M_{\text{CBM}}/M_B) (1 + \gamma_0^{-1}) [1 + (1/2) (M_{\text{CBM}}/M_B)]}, \]  

(2.40)

where \( \gamma_0, M_B \) and \( r_0 \) are respectively the values of the Lorentz gamma factor, of the mass of the accelerated baryons and of the radius \( r \) at the beginning of the extended afterglow phase.

A major difference between our treatment and the other ones in the current literature is that we have integrated the above equations analytically, obtaining the explicit form of the equations of motion for the expanding shell in the afterglow for a constant CBM density. For the adiabatic case we have explicitly integrated the differential equation for \( r(t) \) in eq. (2.39),
2. The fireshell model of GRBs

recalling that $\gamma^{-2} = 1 - [dr / (cdt)]^2$, where $t$ is the time in the laboratory reference frame. We have then obtained a new explicit analytic solution of the equations of motion for the relativistic shell in the entire range from the ultra-relativistic to the non-relativistic regimes (Bianco & Ruffini, 2005):

$$t = \left( \gamma_0 - \frac{m_i^0}{M_B} \right) \frac{r - r_0}{c\sqrt{\gamma_0^2 - 1}} + \frac{m_i^0}{4M_B r_0^3 c\sqrt{\gamma_0^2 - 1}} - t_0. \quad (2.41)$$

Correspondingly, in the fully radiative case we have (Bianco & Ruffini, 2005):

$$t = \frac{M_B - m_i^0}{2c\sqrt{C}} (r - r_0) + \frac{r_0\sqrt{C}}{12cm_i^0 A^2} \ln \left\{ \frac{[A + (r/r_0)]^3 (A^3 + 1)}{[A^3 + (r/r_0)^3] (A + 1)^3} \right\} - \frac{m_i^0 r_0}{8c\sqrt{C}}$$

$$+ t_0 + \frac{m_i^0 r_0}{8c\sqrt{C}} \left( \frac{r}{r_0} \right)^4 + \frac{r_0 \sqrt{3C}}{6cm_i^0 A^2} \left[ \arctan \left( \frac{2(r/r_0) - A}{A\sqrt{3}} \right) - \arctan \left( \frac{2 - A}{A\sqrt{3}} \right) \right]$$

where $A = \sqrt[3]{(M_B - m_i^0) / m_i^0}$, $C = M_B^2 (\gamma_0 - 1) / (\gamma_0 + 1)$ and $m_i^0 = (4/3) \pi m_p n_{CBM} r_0^3$.

In the current literature, following Blandford & McKee (1976), a so-called “ultrarelativistic” approximation $\gamma_0 \gg \gamma \gg 1$ has been widely adopted by many authors to solve eqs. (2.38) (see e.g. Sari, 1997, 1998; Waxman, 1997a; Rees & Mészáros, 1998; Granot, Piran & Sari, 1999; Panaitescu & Mészáros, 1998; Piran, 1999; Gruzinov & Waxman, 1999; Van Paradijs, Kouveliotou & Wijers, 2000; Mészáros, 2002, and references therein). This leads to simple constant-index power-law relations:

$$\gamma \propto r^{-a}, \quad (2.43)$$

with $a = 3/2$ in the fully adiabatic case and $a = 3$ in the fully radiative case. In the same spirit, instead of eq. (2.41) and eq. (2.42), some authors have assumed the following much simpler approximation for the relation between the time and the radial coordinate of the expanding shell, both in the adiabatic radiative and in the fully radiative cases:

$$ct = r, \quad (2.44)$$
2.3. The optically thin fireshell evolution

while others, like e.g. Panaitescu & Mészáros (1998), have integrated the approximate eq. (2.43), obtaining:

\[ ct = r + \left[ 2 (2a + 1) \gamma_0^2 \right]^{-1} r_0 \left( r / r_0 \right)^{2a+1}. \]  

(2.45)

2.3.1 The arrival time

We now have to study the relation between the time at which the photons are emitted from the external surface of the expanding pulse and the time at which they are detected. First of all, we have to take into account the relativistic beaming effect. In fact only a small area centered on the line of sight emits photons that can reach the observer at infinity. The condition for that is (Bianco, Ruffini & Xue, 2001; Ruffini et al., 2002, 2003)

\[ \cos \vartheta \geq \frac{v(t)}{c}. \]  

(2.46)

The maximum allowed \( \vartheta \) value corresponds to \( \cos \vartheta_{\text{max}} = (v/c) \). In the earliest GRB phases \( v \sim c \) and so \( \vartheta_{\text{max}} \sim 0 \). On the other hand, in the latest phases of the extended afterglow the baryonic pulse velocity decreases and \( \vartheta_{\text{max}} \) tends to 90° (see fig. 2.6 and Ruffini et al., 2002, 2003).

We can obtain the relation between the emission time in the laboratory frame \( t \) and the arrival time \( t_a \) of the photons. The general expression for that is (Ruffini et al., 2001a, 2003):

\[ t_a = t - \int_0^t \frac{v(t') dt'}{c} \cos \vartheta + \frac{r_{ds}}{c}. \]  

(2.47)

We want to include also the effect on the arrival time due to the expansion of the Universe. In fact the frequency of a photon detected on Earth is redshifted with respect to its value when the photon is emitted. So a time interval between two photons \( \Delta t_a^d \) received on Earth (at a certain redshift \( z \) from the source) and the corresponding time interval \( \Delta t_a \) measured in the neighborhood of the source (at \( z = 0 \)) is (Ruffini et al., 2003):

\[ \Delta t_a^d = \Delta t_a (1 + z). \]  

(2.48)
2. The fireshell model of GRBs

Figure 2.6: In this figure we show $\theta_{\text{max}}$ (i.e. the angular amplitude of the visible area of the baryonic shell) in degrees as a function of the arrival time at the detector for the photons emitted along the line of sight.
2.3. The optically thin fireshell evolution

Usually in the study of GRBs many authors (see i.e. Fenimore et al., 1999; Waxman, 1997b,a; Rees & Mészáros, 1998; Piran, 1999; Van Paradijs, Kouveliotou & Wijers, 2000; Mészáros, 2002) use an approximated version of eq. (2.47),

\[ t_a = \frac{t}{2\gamma^2}. \]  (2.49)

This expression for the arrival time is completely unrealistic because it requires that \( \gamma \) is constant during the motion. This assumption is, of course, not valid for GRBs. In order to prevent this problem, this formula has been modified introducing “by hand” a dependence on time in the Lorentz factor:

\[ t_a = \frac{t}{2\gamma(t)^2}. \]  (2.50)

This formula is still contradictory because it is obtained firstly assuming \( \gamma \) constant and varying with time.

It has been proposed by Sari (1997) to consider eq. (2.50) only in a differential way,

\[ dt_a = \frac{dt}{2\gamma(t)^2}, \]  (2.51)

and to obtain the exact relation integrating it:

\[ t_a = \int_0^t \frac{dt'}{2\gamma(t')^2}. \]  (2.52)

Assuming the scaling law (2.43) with \( a = 3/2 \) and assuming also \( dt = dr/c \), from eq. (2.52) we have, then

\[ t_a = \frac{r}{16\gamma^2c}. \]  (2.53)

It is worth to notice that this formula corresponds to a first order expansion in \( 1/\gamma^2 \) of the exact expression (2.47) in the particular case in which \( \cos \vartheta = 1 \). This means that such approximate expression is valid only for \( \gamma^2 \gg 1 \), so it cannot be valid during all the GRB’s stages.
2. The fireshell model of GRBs

2.3.2 The EQuiTemporal Surfaces (EQTSs)

As pointed out long ago by Couderc (1939), in all relativistic expansions the crucial geometrical quantities with respect to a physical observer are the locus of source points of the signals arriving at the observer at the same time.

For a relativistically expanding spherically symmetric source such “EQuiTemporal Surfaces” (EQTSs) are surfaces of revolution about the line of sight. The general expression for their profile, in the form $\vartheta = \vartheta(r)$, corresponding to an arrival time $t_a$ of the photons at the detector, can be obtained from (see Ruffini et al., 2003; Bianco & Ruffini, 2004, 2005, and figs. 2.6–2.7):

$$ct_a = ct(r) - r \cos \vartheta + r^*, \quad (2.54)$$

where $r^*$ is the initial size of the expanding source and $t = t(r)$ is its equation of motion, expressed in the laboratory frame, obtained by eqs. (2.41) and (2.42). From the definition of the Lorentz gamma factor $\gamma^{-2} = 1 - (dr/cdt)^2$, we have in fact:

$$ct(r) = \int_0^r \left[1 - \gamma^{-2}(r') \right]^{-1/2} dr', \quad (2.55)$$

where $\gamma(r)$ comes from the integration of eqs. (2.38).

We have obtained the expressions in the adiabatic and in the fully radiative cases respectively (Bianco & Ruffini, 2005):

$$\cos \vartheta = \frac{m_i^0}{4M_B\sqrt{\gamma_0^2 - 1}} \left[ \left( \frac{r}{r_0} \right)^3 - \frac{r_0}{r} \right] + \frac{ct_0}{r} \quad (2.56)$$

$$\cos \vartheta = \frac{M_B - m_i^0}{2r\sqrt{\gamma_0^2 - 1}} (r - r_0) + \frac{m_i^0r_0}{8r\sqrt{\gamma_0^2 - 1}} \left[ \left( \frac{r}{r_0} \right)^4 - 1 \right] + \frac{ct_0}{r} - \frac{ct_a}{r} \quad (2.57)$$

$$+ \frac{r_0\sqrt{\gamma_0}}{12rm_i^0A^2} \ln \left\{ \frac{[A + (r/r_0)]^3 (A^3 + 1)}{[A^3 + (r/r_0)^3] (A + 1)^3} \right\} + \frac{ct_0}{r} - \frac{ct_a}{r}$$

$$+ \frac{r^*}{r} + \frac{r_0\sqrt{3}\gamma_0}{6rm_i^0A^2} \left[ \arctan \frac{2(r/r_0) - A}{A\sqrt{3}} - \arctan \frac{2 - A}{A\sqrt{3}} \right].$$
2.3. The optically thin fireshell evolution

Figure 2.7: Temporal evolution of the visible area of the baryonic pulse. The green half-circles are the expanding baryonic pulse at radii corresponding to different laboratory times. The red curve marks the boundary of the visible region. In the earliest GRB phases the visible region is squeezed along the line of sight, while in the final part of the extended afterglow phase almost all the emitted photons reach the observer.
2. The fireshell model of GRBs

Figure 2.8: Surfaces of photon emission corresponding to selected values of the photon arrival time at the detector: the EQTSs. The EQTSs represented here (red lines) correspond respectively to values of the arrival time ranging from 5 s (the smallest surface on the left of the plot) to 60 s (the largest one on the right). Each surface differs from the previous one by 5 s. To each EQTS contribute emission processes occurring at different values of the Lorentz gamma factor. The green lines are the boundaries of the visible area of the baryonic pulse.
2.4. The GRB luminosity and the spectrum

This two solutions for the adiabatic and the radiative cases are represented for selected values of the arrival time $t_a$ in fig. 2.9. The initial conditions at the beginning of the extended afterglow era are in this case given by $\gamma_0 = 310.131$, $r_0 = 1.943 \times 10^{14}$ cm, $t_0 = 6.481 \times 10^3$ s, $r^* = 2.354 \times 10^8$ cm (Ruffini et al., 2001a,b, 2002, 2003).

2.4 The GRB luminosity and the spectrum

In this section we introduce the equations needed to determine theoretically the spectrum of GRBs. As explained in section 1.3, the GRB observed spectrum shows a clear non-thermal behaviour and is well fitted by the Band function: two power laws joined smoothly at a given break energy (Band et al., 1993). This function provides an excellent fit to most of the observed spectra but there is no particular theoretical model that predicts this spectral shape.

To interpret GRB spectra within the fireshell model, we adopt two basic assumptions (Ruffini et al., 2004):

- the resulting radiation as viewed in the comoving frame has a thermal spectrum, with the number density of photons per unit of energy given by

$$\frac{dN_\gamma}{dVde} = \left(\frac{8\pi}{\hbar^3c^3}\right) \frac{e^2}{\exp\left(\frac{e}{k_B T}\right) - 1},$$

where $h$ is the Planck constant, $c$ is the light velocity, $k_B$ is the Boltzmann constant and $T$ is the temperature in the comoving frame.

- the CBM swept up by the front of the baryonic shell is responsible for this thermal emission.

The assumption of a pure thermal spectrum in the comoving frame has been chosen for simplicity as an ansatz on general thermal emission processes expected in high energy collisions. In chapter 5 we will investigate a different assumption about the spectral energy distribution in the comoving frame.
2. The fireshell model of GRBs

Figure 2.9: Comparison between EQTSs in the adiabatic regime (red lines) and in the fully radiative regime (green lines). The upper panel shows the EQTSs for $t_a = 5$ s, $t_a = 15$ s, $t_a = 30$ s and $t_a = 45$ s, respectively from the inner to the outer one. The lower panel shows the EQTS at an arrival time of 2 days.
2.4. The GRB luminosity and the spectrum

Within the fireshell model the radiation is produced in the inelastic collision between the accelerated baryons and the CBM. The structure of the collision is determined by mass, momentum and energy conservation, i.e. by the constancy of the specific enthalpy, which are standard conditions in the expanding matter rest frames (Zel’dovich & Rayzer, 1966). The only additional free parameter to model this emission process is the size of the “effective emitting area” of the emitting shell: \( A_{\text{eff}} \).

The power emitted in the interaction of the baryonic shell with the CBM inhomogeneities measured in the comoving frame is:

\[
\frac{\Delta E_{\text{int}}}{\Delta \tau} = \pi r^2 c R \int_0^\infty \left( \frac{dN_\gamma}{dV d\epsilon} \right) \epsilon d\epsilon, \tag{2.59}
\]

where \( \Delta \tau \) is the time interval in which the energy \( \Delta E_{\text{int}} \) is developed and

\[
R = \frac{A_{\text{eff}}}{A_{\text{vis}}} \tag{2.60}
\]

is the “surface filling factor”, which accounts for the fraction of the shell’s surface becoming active, being the ratio between the “effective emitting area” and the total visible area \( A_{\text{vis}} \). With the assumption of thermal spectrum in the comoving frame of the fireshell eq. (2.59) becomes

\[
\frac{\Delta E_{\text{int}}}{\Delta \tau} = 4\pi r^2 \sigma R T^4 \tag{2.61}
\]

where \( \sigma \) is the Stefan-Boltzmann constant and \( T \) is the comoving temperature.

We are now ready to evaluate the GRB luminosity in a given energy band (Ruffini et al., 2004). The GRB luminosity at the detector arrival time \( t_d \) per unit solid angle \( d\Omega \) and in the energy band \([\nu_1, \nu_2]\) is given by (Ruffini et al., 2003)

\[
\frac{dE[\nu_1, \nu_2]}{dt_d d\Omega} = \int_{E_{\text{QTS}}} \frac{\Delta \epsilon}{4\pi} v \cos\theta \Lambda^4 \frac{dt}{dt_d} W(\nu_1, \nu_2, T_{\text{arr}}) d\Sigma, \tag{2.62}
\]

where \( \Delta \epsilon = \frac{\Delta E_{\text{int}}}{V} \) is the emitted energy density released in the interaction of the accelerated baryons with the CBM measured in the comoving frame, \( \Lambda = \{\gamma[1 - (v/c)\cos\theta]\}^{-1} \) is the Doppler factor, \( W(\nu_1, \nu_2, T_{\text{arr}}) \) is an “effective
2. The fireshell model of GRBs

weight” required to evaluate only the contributions in the energy band \([\nu_1, \nu_2]\),
d\Sigma is the surface element of the EQTS at detector arrival time \(t_0^d\) on which
the integration is performed and \(T_{\text{arr}}\) is the observed temperature of the
radiation emitted from \(d\Sigma\):

\[
T_{\text{arr}} = \frac{\Lambda^{-1} T}{(1 + z)} .
\]  \tag{2.63}

The “effective weight” \(W(\nu_1, \nu_2, T_{\text{arr}})\) is defined as ratio between the energy
density emitted in a given energy band \([\nu_1, \nu_2]\) and the bolometric energy
density:

\[
W(\nu_1, \nu_2, T_{\text{arr}}) = \frac{\int_{\epsilon_1}^{\epsilon_2} \left( \frac{dN_e}{d\epsilon d\Omega} \right) \epsilon d\epsilon}{\int_{0}^{\infty} \left( \frac{dN_e}{d\epsilon d\Omega} \right) \epsilon d\epsilon} .
\]  \tag{2.64}

With the assumption of thermal spectrum in the comoving frame of the
fireshell we have

\[
W(\nu_1, \nu_2, T_{\text{arr}}) = \frac{\int_{\epsilon_1}^{\epsilon_2} \left( \frac{dN_e}{d\epsilon d\Omega} \right) \epsilon d\epsilon}{a T^4} ,
\]  \tag{2.65}

where \(a\) is the radiation constant.

Once we have the luminosity in a given energy band in the same way
we can evaluate the instantaneous and the time-integrated photon number
spectra.

In fig. 2.10 are shown samples of time-resolved spectra for five different
values of the arrival time, together with their own temporal convolution. It
is manifest from this picture that, although the spectrum in the comoving
frame of the expanding fireshell is thermal, the shape of the final spectrum
in the laboratory frame is clearly non thermal. In fact, the temperature of
the fireshell is evolving with the comoving time, therefore each single instan-
taneous spectrum is the result of an integration of many thermal spectra
over the corresponding EQTS (Ruffini et al., 2004), i.e. it is a convolution,
weighted by appropriate Lorentz and Doppler factors, of many thermal spec-
tra with variable temperature. This calculation produces a non thermal in-
stantaneous spectrum in the observer frame.

Concerning the time-integrated spectra, Pozdniakov et al. (1983) argued
that it is possible to obtain power-law spectra from a convolution of many non
2.4. The GRB luminosity and the spectrum

Figure 2.10: Five different theoretically predicted instantaneous photon number spectrum $N(E)$ for $t_d = 2, 6, 10, 14, 18$ s are here represented (colored curves) together with their own temporal convolution (black bold curve). The shape of the instantaneous spectra is not thermal due to the spatial convolution over the EQTS (see text).

power-law instantaneous spectra evolving in time. This result was recalled and applied to GRBs by Blinnikov et al. (1999) assuming for the instantaneous spectra a thermal shape with a temperature changing with time. They showed that the integration of such energy distributions over the observation time gives a typical power-law shape possibly consistent with GRB spectra. The fireshell model is more sophisticated than the one considered by Blinnikov et al. (1999): in fact, as previously said, the instantaneous spectrum is not thermal. The time-integrated spectra is given by convolutions of convolutions of thermal spectra and also in this case it has a clear non-thermal behavior possibly consistent with GRB spectra (see fig. 2.10).

The assumption of thermal spectrum in the comoving frame of the fireshell has been successfully tested on many GRBs such as, for example, GRB 031203 (see fig. 2.11 and Bernardini et al., 2005b) and GRB 060607A (see chapter 4). Nevertheless, a departure from this picture has been found for
2. The fireshell model of GRBs

Figure 2.11: Three theoretically predicted time-integrated photon number spectra $N(E)$ are here represented for $0 \leq t_d \leq 5$ s, $5 \leq t_d \leq 10$ s and $10 \leq t_d \leq 20$ s (colored curves). The theoretically predicted time-integrated photon number spectrum $N(E)$ corresponding to the first $20$ s of the “prompt emission” of GRB 031203 (black bold curve) is compared with the data observed by INTEGRAL (Sazonov, Lutovinov & Sunyaev, 2004).

several very energetic bursts ($E_{iso} \gtrsim 10^{54}$ ergs) such as, for example, GRB 080319B (see chapter 5), in particular when considering spectra integrated just over few seconds. We will see in chapter 5 that in these cases numerical simulations that correctly reproduce the observational data can be obtained with a different assumption about comoving the spectrum.

2.5 The light curve

In this section we summarize the main characteristics of GRB light curve within the fireshell scenario.

In the fireshell model we define a “canonical GRB” light curve with two sharply different components: the P-GRB, sometimes observed as a “precur- sor” and the extended afterglow, which comprises both the so called “prompt”
2.5. The light curve

emission and the initial steep decay of the so called “afterglow” emission. Concerning the late afterglow phase observed in the X-ray light curves (see section 1.3), we will introduce in chapters 4 and 5 a possible scenario in which it arises from the injection of slower material into the fireshell.

The P-GRB is emitted when the optically thick fireshell reaches the condition of transparency (see section 2.2.5). The structure of the P-GRB is directly linked to the physics of the gravitational collapse generating the GRB (Caito et al., 2009). Recently Aksenov et al. (2009) showed that the characteristic time constant for the thermalization of an electron-positron plasma with a baryon loading of the order of the one observed in GRBs is \( \lesssim 10^{-11} \text{s} \). The shortness of such a time scale, as well as the knowledge of the dynamical equations of the optically thick phase preceding the P-GRB emission, implies that the structure of the P-GRB is a faithful representation of the gravitational collapse process leading to the formation of the black hole (Caito et al., 2009).

The “extended” afterglow emission is produced by the interaction of the accelerated baryons with the CBM, which is modeled as inelastic collisions (see section 2.3). The number of such collisions depends on the CBM density: in this sense if, as it is, the CBM is inhomogeneous the observed light curve is not smooth but presents a temporal behavior which roughly follows the density profile. In fig. 2.12 is shown an example of CBM behavior compared with the prompt emission in order to better understand how its trend influences the light curve.

In order to understand how this mechanism works it is useful to analyze the time scales characteristic of the bursts. Suppose to have a burst lasting \( T_a \approx 20 \text{s} \) with substructures \( \delta t_a \approx 1 \text{s} \). If we evaluate these times in the laboratory frame, assuming \( \gamma \approx 400 \), we have roughly:

\[
\delta t \approx \gamma^2 \delta t_a \approx 1.6 \times 10^5 \text{s}
\]

(for the exact relation between \( t \) and \( t_a \) see section 2.3.1 and Ruffini et al., 2005b; Bianco & Ruffini, 2004, 2005). This determines the characteristic dimension for the inhomogeneity \( \delta L \approx c\delta t = 4.8 \times 10^{15} \text{ cm} \), which is consistent
2. The fireshell model of GRBs

Figure 2.12: Upper panel: particle number density of the CBM ($n_{cbm}$) versus distance from the progenitor. Lower panel: theoretically simulated light curve of GRB 050904 prompt emission in the 15-150 keV energy band (black solid curve) is compared with observed data (red points) for GRB 050904 (see section 5.4). The labels $a$, $b$, $c$, $d$, $e$ indicate the peaks present in the light curve and the corresponding values of $n_{cbm}$. 
2.5. The light curve

with small clouds of interstellar matter.

The simplest way to model the CBM structure is to assume that \( n_{CBM} \) is a function only of the radial coordinate, \( n_{CBM} = n_{CBM}(r) \) (radial approximation). The CBM is arranged in spherical shells of width \( \approx 10^{15} \) cm positioned in such a way that the modulation of the emitted flux coincides with the observed peaks.

Of course our “radial approximation” is valid until the visible area of the incoming baryonic pulse is comparable with the characteristic dimensions of the clouds. The transverse dimension of such area is \( R_T = r \sin \vartheta \), where \( \vartheta \sim 1/\gamma \) is the relativistic beaming angle, so we have \( R_T \approx r/\gamma \). When the Lorentz factor is too small (see, for example, Ruffini et al., 2002, 2006) and/or the shell is too distant (see, for example, Guida et al., 2006) from the progenitor, the visible area of the shell is comparable with the density cloud and this approximation fails. We will present in chapters 4 and 5 a 2-dimensional model introduced to solve this problem and we will show how we successfully tested it on the X-ray flares observed in GRB 060607A and GRB 050904.
2. The fireshell model of GRBs
Chapter 3

Other Models for GRBs

Over the years, many efforts have been made to explain the observational properties of GRBs and this has led to the development of different theoretical models. One of the most accepted is the standard “fireball” model, that was proposed by Goodman (1986) and Paczyński (1986). Alternative models are, for example, the “electromagnetic” model (Lyutikov & Blandford, 2003; Lyutikov, 2006) and the “turbulent” model (Narayan & Kumar, 2009).

There are some generally accepted ingredients in all current GRB models, for example the relation of GRBs with the death of massive stars and the birth of compact objects (see section 3.1.4). Another common characteristic is the relativistic motion of matter with a Lorentz factor $\Gamma > 100$. This assumption is essential to overcome the so called “compactness problem” (Ruderman, 1975): the GRB non-thermal observed spectrum indicates that the sources must be optically thin, but simple calculations shows that the optical depth for $e^\pm$ pair creation is very large (Piran, 2004). This problem can be solved if the emitting matter is moving relativistically toward the observer. In this case, in fact, the optical depth is lowered by a factor $\gamma^{4+2\alpha}$, where $\alpha$ is the high energy spectral index, so when $\gamma \sim 10^2$ the problem is solved (Piran, 2004). Afterglow observations have confirmed the relativistic motion (see Molinari et al., 2007).

The simplest way to obtain a relativistic energy flow is in the form of
kinetic energy of relativistic particles: this is what happens within the framework of the standard “fireball” model (see section 3.1). Alternatively, the kinetic energy could be mainly carried by Poynting flux: this is the case of the “electromagnetic model” (see section 3.2).

The dissipation of the kinetic energy of the relativistic outflow is responsible for both the prompt and the afterglow emission (Piran, 2004). In the “fireball” model (see section 3.1.2) the relativistic motion is dissipated by shocks, either internal (when faster moving matter takes over a slower moving shell) and external (when the moving matter is slowed down by the external medium surrounding the burst). An alternative to shocks is given by the “turbulent” model (see section 3.3), in which the kinetic energy is dissipated downstream to a combination of macroscopic relativistic random motions of the emitting clumps (Piran, 2004).

3.1 The standard “Fireball” Model

Goodman (1986) and Paczyński (1986) have shown that the release of a large quantity of gamma-ray photons into a compact region can lead to an optically thick photon-lepton “fireball” through the production of $e^\pm$ pairs. The term “fireball” refers here to an opaque plasma whose initial energy is significantly greater than its rest mass (Piran, 1999). Both Goodman (1986) and Paczyński (1986) considered a pure radiation fireball in which there are no baryons. Later Shemi & Piran (1990) and Paczyński (1990) considered the effect of a baryonic load, showing that the ultimate outcome will be the transfer of all the energy of the fireball to the kinetic energy of the baryons.

We now explain how an homogeneous fireball evolves. Consider, first, a pure radiation fireball. The photons with energy $E_1$ could interact with lower energy photons (with energy $E_2$) and produce $e^\pm$ pairs via $\gamma \gamma \rightarrow e^+e^-$ if $\sqrt{E_1E_2} > m_e c^2$. Because of the opacity due to pairs, the radiation cannot escape. The pairs-radiation plasma behaves like a perfect fluid described by the stress-energy tensor $T^{\mu\nu}$ with pressure $p$, energy density $\epsilon$ and equation
of state $p = \epsilon/3$. In addition to radiation and $e^\pm$ pairs, astrophysical fireballs may also include some baryonic matter which may be injected with the original radiation or may be present in an atmosphere surrounding the initial explosion (Piran, 1999). These baryons can influence the fireball evolution in two ways. The electrons associated with this matter increase the opacity, delaying the escape of radiation. Moreover, the baryons are accelerated with the rest of the fireball and convert part of the radiation energy into bulk kinetic energy.

The expansion of the plasma is ruled by the relativistic conservation equations of baryon number, energy and momentum

\[
\begin{align*}
(\rho_B U^\mu)_{,\mu} &= 0, \\
(T^{\mu\nu})_{,\nu} &= 0,
\end{align*}
\]  

(3.1) (3.2)

where $\rho_B$ is the baryon mass density.

The expanding fireball has two basic phases: a radiation dominated phase and a matter dominated phase; the transition from the radiation dominated phase and the matter dominated phase takes place when the fireball has a size (Piran, 1999)

\[
R_\eta = \frac{R_i E}{M c^2},
\]  

(3.3)

where $R_i$, $E$ and $M$ are the initial radius, the energy and the mass of the fireball respectively.

During the radiation dominated phase the conservation equations reduce to these simple scaling laws (Piran, 1999):

\[
\gamma \propto r, \quad \rho_B \propto r^{-3}, \quad \epsilon \propto r^{-4}.
\]  

(3.4)

The matter dominated phase is characterized by (Piran, 1999):

\[
\gamma \rightarrow \text{const}, \quad \rho_B \propto r^{-2}, \quad \epsilon \propto r^{-8/3}.
\]  

(3.5)

These scaling of $\rho_B$ and $\epsilon$ arise because the fireball moves with a constant radial width in the comoving frame (it behaves like a “frozen” profile). Moreover, since $\epsilon \ll \rho_B$, the radiation has no important dynamical effect on the
motion and produces no significant radial acceleration. Therefore, $\gamma$ remains constant on streamlines and the fluid coasts with a constant asymptotic radial velocity. Of course, since each shell moves with a velocity that is slightly less than $c$ and that is different from one shell to the next, the frozen pulse approximation must ultimately break down at some large radius.

At some point during the expansion, the fireball will become optically thin. From this stage on the radiation and the baryons no longer move with the same velocity and the radiation pressure vanishes. Any remaining radiation will escape freely now and the baryon shells will coast with their own individual velocities.

### 3.1.1 Energy conversion

Within the standard fireball model the energy transport is in the form of the kinetic energy of a shell of relativistic particles with a width $\Delta$. The kinetic energy is converted to energy of relativistic particles via shocks (Piran, 1999). These particles then release this energy and produce the observed radiation. There are two modes of energy conversion (Piran, 1999): external shocks (Rees & Mészáros, 1992), which are due to interaction with an external medium like the InterStellar Medium (ISM), or internal shocks (Rees & Mészáros, 1994), that arise due to shocks within the flow when fast moving particles catch up with slower ones.

In the standard fireball model, internal shocks are believed to provide the best way to explain the observed temporal structure in GRB prompt emission (Piran, 1999). These shocks, that take place at distances of $\sim 10^{15}$ cm from the center, convert two to twenty percent of the kinetic energy of the flow to thermal energy (Piran, 1999), and with this mechanism it is possible to extract at most half of the shell’s energy (Kobayashi, Piran & Sari, 1997; Katz, 1997). Highly relativistic flow with a kinetic energy and a Lorentz factor comparable to the original one remains after the internal shocks.

Sari & Piran (1997) pointed out that if the shell is surrounded by ISM and collisionless shock occurs, the relativistic shell will dissipate by external
3.1. The standard “Fireball” Model

Figure 3.1: Schematic representation of the different phases of GRB fireball model, from the fireball generation to the external shock dissipation. Figure from http://www.swift.ac.uk/grb.shtml

shocks as well. This predicts an additional smooth burst, with a comparable or possibly greater energy. This is most probably the source of the observed afterglow (Piran, 1999). Therefore, the standard fireball model is characterized by an “internal-external” scenario (Sari & Piran, 1997) in which the prompt is produced by internal shocks, while the afterglow is produced by external shocks (see fig. 3.1).

3.1.2 The GRB light curve within the fireball model

We now explain in more detail how GRB light curves are explained within the “internal-external” scenario.

Prompt emission light curve with internal shocks

According to the fireball model, the prompt emission pulse duration and the separation between the prompt emission pulses are determined by the same parameter, namely the time interval between the shells emitted by the inner
Consider two shells separated by a distance $L$ (see i.e. Piran, 2004). The Lorentz factor of the slower outer shell is $\Gamma_S = \Gamma$ and the one of the inner faster shell is $\Gamma_F = a\Gamma$ ($a > 2$ for an efficient collision). Both are measured in the observer frame. The shells are ejected at $t_{a1}$ and $t_{a2} \approx t_{a1} + L/c$, where $t_a$ is the time measured in the observer frame. The collision takes place at a radius $R_s \approx 2\Gamma^2 L$. The emitted photons from the collision will reach the observer at time (omitting the photons flight time, and assuming transparent shells):

$$t_{a0} \approx t_{a1} + R_s/(2c\Gamma^2) \approx t_{a1} + L/c \approx t_{a2}.$$

(3.6)

The photons from this pulse are observed almost simultaneously with a photon that was emitted from the inner engine together with the second shell. This argument has been investigated by various numerical simulations (Kobayashi, Piran & Sari, 1997; Daigne & Mochkovitch, 1998; Panaitescu et al., 1999), which argue that for internal shocks the observed light curve replicates the temporal activity of the source.

In order to determine the time interval $\Delta t_a$ between the pulses we should consider multiple collisions. All combinations of multiple collisions can be divided into three types (Piran, 2004).

Consider four shells emitted at times $t_{ai}$ ($i = 1, 2, 3, 4$) separated by a distance $L$. In the collisions of the first type there are two collisions - between the first and the second shells and between the third and the fourth shells. The first collision will be observed at $t_{a2}$ while the second one will be observed at $t_{a4}$. Therefore the separation between each pulse results to be $\Delta t_a \approx t_{a4} - t_{a2} \approx 2L/c$.

A different collision scenario occurs if the second and the first shells collide, and afterward the third shell takes over and collide with them (the fourth shell does not play any role in this case). The first collision will be observed at $t_{a2}$ while the second one will be observed at $t_{a3}$. Therefore, $\Delta t_a \approx t_{a3} - t_{a2} \approx L/c$. Numerical simulations (Nakar & Piran, 2002) show that more then 80% of the efficient collisions follows one of these two scenar-
3.1. The standard “Fireball” Model

Therefore one can estimate:

\[ \Delta t_a \approx L/c . \]  \hspace{1cm} (3.7)

A third type of multiple collisions arises if the third shell collides with the second shell first, and then the merged shell collides with the first one (again the fourth shell does not participate in this scenario). In this case the two pulses merge and they will arrive almost simultaneously, at the same time with a photon that would have been emitted from the inner engine simultaneously with the third (fastest) shell. Only a 20% fraction exhibits this type of collision (Piran, 2004).

The pulse width is given by

\[ \delta t_a = \frac{R_s}{2c\Gamma_s^2} \]

where \( \Gamma_s \) is the Lorentz factor of the shocked emitting region (Piran, 2004). If the shells have an equal mass \( (m_1 = m_2) \) then \( \Gamma_s = \sqrt{a}\Gamma \) while if they have equal energy \( (m_1 = am_2) \) then \( \Gamma_s \approx \Gamma \). Therefore the pulse’s width is

\[ \delta t_a \approx \begin{cases} R_s/2a\Gamma^2c & \text{equal mass,} \\ R_s/2\Gamma^2c & \text{equal energy.} \end{cases} \]  \hspace{1cm} (3.8)

The ratio of the Lorentz factors \( a \) determines the collision’s efficiency. For efficient collisions the variations in the shells Lorentz factor (and therefore \( a \)) must be large. It follows from eqs. (3.7) and (3.8) that for equal energy shells a correlation between the separation and the width of each pulse \( (\Delta t_a - \delta t_a) \) arises naturally from the reflection of the shells initial separation in both variables\(^1\). However, for equal mass shells, \( \delta t_a \) is shorter by a factor \( a \) than \( \Delta t_a \). This shortens the pulses relative to the intervals. Additionally, the large variance of \( a \) would wipe off the \( \Delta t_a - \delta t_a \) correlation. This suggests that equal energy shells are more likely to produce the observed light curves (Piran, 2004).

The internal shock model has been for a long time the favourite model to interpret GRB prompt emission, as it can explain various properties of GRB

\(^1\)Nakar & Piran (2002) found that the pulse duration is correlated with the duration of the time interval preceding (and following) it.
light curves, for example the temporal variability (Nakar & Piran, 2002; Ramirez-Ruiz & Fenimore, 2000). There are, however, several observations and/or theoretical considerations that pose difficulties for it. First of all, internal shocks have a low efficiency for converting bulk kinetic energy to photons (see Kumar, 1999, and references therein). Another problem is the large value of $R_{\gamma}^2$ constrained by the observational data, which is significantly larger than what one expects in the internal shock model (see Kumar et al., 2007, and section 5.1).

Afterglow emission light curve with external shocks

Consider a cold shell (whose internal energy is negligible compared to the rest mass) that overtakes another cold shell or moves into the cold CBM. Generally, two shocks form: an outgoing shock that propagates into the CBM or into the external shell, and a reverse shock that propagates into the inner shell, with a contact discontinuity between the shocked material. Two quantities determine the shocks’ structure: the Lorentz factor $\Gamma$ of the motion of the inner shell relative to the outer one or the CBM, and the ratio $f$ between the particle number densities in these regions.

According to the standard fireball model the afterglow is produced when the relativistic ejecta is slowed down by the surrounding matter (Mészáros & Rees, 1997a). At this stage external shocks become predominant (Piran, 1999).

Initially the process might be radiative, namely a significant fraction of the kinetic energy is dissipated and the radiation process affects the hydrodynamics of the shock (Piran, 2004). Later the radiation processes become less efficient and an adiabatic phase begins during which the radiation losses are minor and do not influence the hydrodynamics (Piran, 2004). Finally, a transition into the Newtonian regime takes place when $\Gamma \approx 1.5$.

The theory of a relativistic shell propagating into the CBM has been worked out in a classical paper by Blandford & McKee (1976). This model is

\footnote{$R_{\gamma}$ is the distance of the $\gamma$-ray source from the center of the explosion.}
3.1. The standard “Fireball” Model

a self-similar spherical solution describing an adiabatic ultra relativistic blast wave in the limit \( \Gamma \gg 1 \). For this blast wave, the total energy is proportional to \( r^3 \Gamma^2 \), leading to the scaling law:

\[ \Gamma \propto r^{-3/2}. \tag{3.9} \]

Analogously, it is possible to find for a fully radiative regime another scaling law (Blandford & McKee, 1976):

\[ \Gamma \propto r^{-3}. \tag{3.10} \]

The simple adiabatic model assumes that the energy of the GRB is constant. However, the energy could change if additional slower material is ejected behind the initial matter (Piran, 2004). As the initially faster moving matter is slowed down by the circumburst matter, this slower matter eventually catch up and produces “refreshed shocks” (Rees & Mészáros, 1998; Kumar & Piran, 2000; Sari & Mészáros, 2000). There are two implications for the refreshed shocks: first, the additional energy injection will influence the dynamics of the blast wave (Rees & Mészáros, 1998; Sari & Mészáros, 2000). This would change the decay slope from the canonical one and produce a slower decay in the light curve (Piran, 2004). A second effect is the production of a reverse shock propagating into the slower material when it catches up with the faster one (Kumar & Piran, 2000). This is of course in addition to the forward shock that propagates into the outer shell. This reverse shock could be episodal or long lasting, depending on the profile of the additional matter (Piran, 2004). The calculation of the shock is slightly different than the calculation of a shell propagating into a cold material (Piran, 2004): here the outer shell has already collided with the CBM, hence it is hot with internal energy exceeding the rest mass energy.

The afterglow theory becomes much more complicated if the relativistic ejecta is not spherical. The commonly called “jets” correspond to relativistic matter ejected into a cone of opening angle \( \theta \). The simplest implication of a jet geometry, that exists regardless of the hydrodynamic evolution, is that
Figure 3.2: Schematic representation of the jet geometry: when the relativistic beaming angle becomes equal to the jet opening angle (left panel, time $t_4$) a break in the light curve is observed (right panel).

once $\Gamma \sim \theta^{-1}$ relativistic beaming of light will become less effective (see fig. 3.2). The radiation was initially beamed locally into a cone with an opening angle $\Gamma^{-1}$ remained inside the cone of the original jet. Now with $\Gamma^{-1} > \theta$ the emission is radiated outside of the initial jet. This has two effects: (i) an “on axis” observer, one that sees the original jet, will detect a “jet break” due to the faster spreading of the emitted radiation; (ii) an “off axis” observer, that could not detect the original emission, will be able to see now an “orphan afterglow”, namely an afterglow without a preceding GRB. Jet breaks are intrinsically achromatic as they only reflect the ejecta geometry, therefore they are expected to manifest in the optical as well as in the X-ray light curves.

In the pre-Swift era it was common to observe a steepening in the optical and X-ray light curves several days after the GRB triggers and to associate this epoch with the jet break (see section 1.3.2). In particular, the achromaticity of breaks has been confirmed in various pre-Swift GRBs (see, for example, Harrison et al., 2001; Klose et al., 2004). However, things have changed during the Swift-era. In particular, several studies (Burrows & Racusin, 2006; Liang et al., 2008; Kocevski & Butler, 2008; Willingale et al., 2007; Evans et al., 2009) have searched for jet breaks in the XRT data and agree that
there is a substantial deficit relative to the pre-Swift expectations. This raises concerns about interpreting the breaks as jet breaks.

### 3.1.3 The GRB spectrum within the fireball model

Within the standard fireball model the observed GRB spectrum (both of the prompt and the afterglow emission) is explained as Synchrotron emission of the fireball relativistic electrons (Rybicki & Lightman, 1979). The typical energy of Synchrotron photons depends on the Lorentz factor of the relativistic electrons under consideration $\gamma_e$:

$$h\nu_{\text{syn}} = \frac{h e B}{m_e c^3} \gamma_e \Gamma,$$

where $\Gamma$ is the Lorentz factor of the emitting material and $B$ is the magnetic field.

The instantaneous Synchrotron spectrum of a single relativistic electron with an initial energy $\gamma_e m_e c^2$ is approximately a power-law with $F_\nu \propto \nu^{1/3}$ ($F_\nu$ is the flux in the observer frame) up to $\nu_{\text{syn}}(\gamma_e)$ and an exponential decay above it. This description is suitable when the electron does not loose a significant fraction of its energy into radiation. This requires $\gamma_e$ to be less than a critical value $\gamma_c$ given by $\Gamma \gamma_c m_e c^2 = P(\gamma_c) t$, where $P(\gamma)$ is the radiation power and $t$ refers to the time in the observer frame. When $\gamma_e > \gamma_c$, $F_\nu \propto \nu^{-1/2}$.

To calculate the overall spectrum one need to integrate over the electrons’ Lorentz factor distribution. The simplest distribution is $N(\gamma_e) \sim \gamma_e^{-p}$, with $p > 2$. The minimum Lorentz factor of the distribution $\gamma_m$ is

$$\gamma_m = \frac{p-2}{p-1} \langle \gamma_e \rangle,$$

which plays an important role in this treatment as it characterizes the “typical” Synchrotron frequency $\nu_m = \nu_{\text{syn}}(\gamma_m)$.

The lowest part of the overall spectrum is always the sum of the contributions of the tails of all the electrons’ emissions: $F_\nu \propto \nu^{1/3}$. The most energetic
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electrons, instead, will always be cooling rapidly and emit practically all their
energy at their Synchrotron frequency, thus \( F_\nu \propto \nu^{-p/2} \).

In the intermediate frequency region the spectrum varies depending if the
electrons are in a “fast cooling” \((\nu_m > \nu_c)\) or in a “slow cooling” \((\nu_c > \nu_m)\)
regimes. The net spectrum in the first case results to be:

\[
F_\nu \propto \begin{cases} 
\left(\frac{\nu}{\nu_m}\right)^{1/3} F_{\nu,\text{max}}, & \nu < \nu_c \\
\left(\frac{\nu}{\nu_c}\right)^{-1/2} F_{\nu,\text{max}}, & \nu_c < \nu < \nu_m \\
\left(\frac{\nu_m}{\nu_c}\right)^{-1/2} \left(\frac{\nu}{\nu_m}\right)^{-p/2} F_{\nu,\text{max}}, & \nu_m < \nu ,
\end{cases}
\]

(3.13)

with \( F_{\nu,\text{max}} \) the observed peak flux. In the “slow cooling” regime we have, instead:

\[
F_\nu \propto \begin{cases} 
\left(\frac{\nu}{\nu_m}\right)^{1/3} F_{\nu,\text{max}}, & \nu < \nu_m \\
\left(\frac{\nu}{\nu_m}\right)^{-(p-1)/2} F_{\nu,\text{max}}, & \nu_m < \nu < \nu_c \\
\left(\frac{\nu_c}{\nu_m}\right)^{-(p-1)/2} \left(\frac{\nu}{\nu_c}\right)^{-p/2} F_{\nu,\text{max}}, & \nu_c < \nu ,
\end{cases}
\]

(3.14)

Fast cooling must take place during the GRB itself: the internal shocks
must emit their energy effectively or there will be a serious inefficiency prob-
lem (Piran, 2004). Additionally, the pulse won’t be variable if the cooling
time is too long. The electrons must cool rapidly and release all their energy.
It is most likely that during the early stages of an external shock there will
be a transition from fast to slow cooling (Mészáros & Rees, 1997a; Waxman,
1997a; Mészáros et al., 1998; Waxman, 1997b).

At low frequencies Synchrotron self-absorption may take place. It leads
to a steep cutoff of the low-energy spectrum, as \( \nu^{5/2} \) or as \( \nu^2 \) (Piran, 2004).

While a reasonable agreement between the predictions of the Synchrotron
model and the afterglow observations has been found (Piran, 2004), for the
prompt emission there are several inconsistencies between the observational
data and the theoretical predictions (see section 5.1 and Kumar & McMahon,
2008). The main alternative to Synchrotron emission is Synchrotron Self-
Compton (SSC) or Inverse Compton (IC) of external light.

IC scattering may have different effects. It can influence the spectrum
even if the system is optically thin to Compton scattering (Rybicki & Light-
man, 1979). In view of the high energies involved, a photon can be Inverse Compton scattered just once; multiple scatterings are very unlikely. The effect of IC depends on the Comptonization parameter \( Y = \gamma^2 \tau_e \) (\( \tau_e \) is the electrons’ opacity): it becomes important when \( Y > 1 \). Its effect is to add an ultra high-energy component to the GRB spectrum: this component will be typically at the GeV-TeV range (see Bottcher & Dermer, 1998; Vietri, 1997). Furthermore, IC speeds up the cooling of the emitting regions and shortens the cooling time by a factor \( Y \); at the same time this also reduces the efficiency (for producing the observed \( \gamma \)-rays) by the same factor (Piran, 2004).

Also with IC and SSC there are several inconsistencies between the observed data and the theoretical predictions; more detail will be given in section 5.1.

### 3.1.4 Models for GRB progenitors

The fireball model is independent on the details of the “inner engine” that releases the initial energy (Mészáros, 2002). Nevertheless, simple considerations on the energetic and the time scales leads to the idea that the central engine producing the required energy flow could be an accretion disk. Several scenarios could lead to a black hole - massive accretion disk system. This could include mergers, as Neutron Star-Neutron Star (NS-NS, Eichler et al., 1989; Narayan et al., 1992), Neutron Star-Black Hole (NS-BH, Paczyński, 1991) or Neutron Star-White Dwarfs (NS-WD, Fryer et al., 1999) binaries, and models based on “failed supernovae” or “Collapsars” (Woosley, 1993; Paczyński, 1998; MacFadyen & Woosley, 1999).

Within the standard model it is now believed (Narayan et al., 2002) that among all the above scenarios Collapsars could produce long bursts and NS-NS (or NS-BH) mergers could produce short bursts. The basic idea is that the duration of the accretion depends on the size of the disk. This means that short bursts must originate from small disks which are naturally produced in mergers. On the other hand long bursts require large disks. An alternative
possibility is to have a small disk that is fed continuously. In this case the
duration of the process can be longer (Piran, 2004).

The Collapsar model. Woosley (1993) proposed that GRBs arise from
the collapse of a single Wolf-Rayet star endowed with fast rotation. Paczyński
(1998) pointed out that there is evidence that many GRBs are close to star-
forming regions and that this suggests that GRBs are linked to cataclysmic
deaths of massive stars. Then MacFadyen & Woosley (1999) begun a series
of calculations of a relativistic jet propagation through the stellar envelope
of the collapsing star. The collimation of a jet by the stellar mantle was
shown to occur analytically by Mészáros & Rees (2001). Zhang et al. (2003)
numerically confirmed and extended the basic features of this collimation
process. All these ingredients led to the Collapsar model.

According to the Collapsar model the massive iron core of a rapidly ro-
tating massive star ($M > 30M_\odot$) collapses to a black hole (either directly
or during the accretion phase that follows the core collapse). An accretion
disk forms around this black hole and a funnel forms along the rotation axis,
where the stellar material has relatively little rotational support. The mass
of the accretion disk is around $0.1M_\odot$. Energy can be extracted via neu-
trino annihilation (MacFadyen & Woosley, 1999) or via the Bladford-Znajek
mechanism. The energy deposited in the surrounding matter will preferably
leak out along the rotation axis producing jets with opening angles $< 10^\circ$. If
the jets are powerful enough they would penetrate the stellar envelope and
produce the GRB.

The processes of core collapse, accretion along the polar column (which
is essential in order to create the funnel) and the jet propagation through
the stellar envelope take together $\sim 10$ sec (MacFadyen & Woosley, 1999).
The duration of the accretion onto the black hole is expected to take several
dozen seconds. The jet, as it passes through the star, is modulated by its
interaction with the surrounding medium. In this way the Collapsar model
attempts to explain the time structure of GRB prompt emission and to pro-
produce the variable Lorentz factor necessary for the internal shocks occurrence (Woosley & Bloom, 2006). Moreover it is a prediction of this model that the central engine remains active for a long time after the principal burst is over, potentially contributing to the GRB afterglow (Burrows et al., 2005). This is because the jet and the disk are inefficient in ejecting all the matter in the equatorial plane of the pre-collapse star and some continues to fall back and accrete (MacFadyen et al., 2001). All these arguments imply that Collapsars are expected to produce long GRBs (Piran, 2004).

Neutron stars merging. NS-NS (Eichler et al., 1989; Narayan et al., 1992) or NS-BH binary mergers (Paczyński, 1991) also produce a black hole-accretion disk system and are candidates for the inner engines of GRBs, specifically of short GRBs. These mergers take place because of the decay of the binary orbits due to gravitational radiation emission.

A merger releases $\sim 5 \times 10^{53}$ erg, but most of this energy is in the form of low energy neutrinos and gravitational waves. Still there is enough energy available to power a GRB but is not clear how the GRB is produced (Piran, 2004). A central question is how does a merger generate the relativistic wind required to power a GRB. Eichler et al. (1989) suggested that about one thousandth of these neutrinos annihilates and produces pairs that in turn produces gamma-rays via $\nu \bar{\nu} \rightarrow e^+e^- \rightarrow \gamma\gamma$. This idea was criticized on several grounds by different authors: the main problem is that it does not produce enough energy. For example Jaroszynksi (1996) pointed out that a large fraction of the neutrinos will be swallowed by the black hole that forms.

3.2 The “electromagnetic” Model

An alternative model for GRBs is the “electromagnetic” model (Lyutikov & Blandford, 2003; Lyutikov, 2006), that builds upon earlier models of electromagnetic and magnetohydrodynamic explosions (Blandford & Rees, 1972; Bendford, 1978; Usov, 1992, 1994; Mészáros & Rees, 1992; Ferrari, 1998;
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Kluźniak & Ruderman, 1998; Vietri, 1998; Spruit, 1999; Wheeler et al., 2000; Lyutikov & Blackman, 2000; Vlahakis & Königl, 2001; Blandford, 2002; Lyutikov & Blandford, 2002). While in the fireball model the energy flow is in the form of kinetic energy of relativistic particles, in the electromagnetic model the kinetic energy is mainly carried by a Poynting flux. Let’s introduce the magnetization parameter $\sigma$, defined as the ratio between the Poynting flux $F_{Py}$ and the particle flux $F_p$:

$$\sigma = \frac{F_{Py}}{F_p} = \frac{B^2}{4\pi \Gamma \rho c^2},$$

(3.15)

where $B$ and $\rho$ are the magnetic field and the plasma density in the laboratory frame and $\Gamma$ is the bulk Lorentz factor. The dynamical behavior of the outflow depends on the value of $\sigma$. When $\sigma \ll 1$ magnetic fields are dynamically unimportant: this is the case of the standard fireball model. For $\sigma > \sigma_{crit} = \Gamma^2/2$, there is an important qualitative change in the dynamical behavior of the flow: this is the case of the electromagnetic model (Lyutikov, 2006).

In the electromagnetic model the energy that powers a GRB comes from kinetic rotational energy of the central source (fast rotating neutron star or black hole-accretion disk system). This rotational energy is first converted into magnetic energy through the unipolar dynamo mechanism (like in pulsars), propagated in the form of relativistic, subsonic, Poynting flux-dominated wind (the outflow) and then dissipated in the emitting region (Lyutikov & Blandford, 2003; Lyutikov, 2006). We now explain the various phases of the model.

Initially, the source will inflate an electromagnetic-dominated bubble inside the star. This magnetized cavity is separated from the outside material by a contact discontinuity (CD). The velocity of expansion of the bubble is determined by the pressure balance on the CD between the magnetic pressure in the bubble and the ram pressure of the stellar material. As the magnetic field strength is strongest close to the symmetry axis, the bubble will expand most rapidly along the rotational axis of the progenitor (Lyutikov & Blandford, 2003).
3.2. The “electromagnetic” Model

The expansion of the bubble inside the star is non-relativistic; after the bubble breaks out from the stellar surface, its expansion becomes ultra-relativistic and bi-conical. After it has expanded beyond a radius \( r_{sh} \sim ct_s \) (\( t_s \) is the time interval during which the central source operates), the electromagnetic energy will be concentrated within an expanding, electromagnetic shell. The leading surface of the shell is separated by a contact discontinuity. Outside the CD an ultra-relativistic shock front may form and propagate into the surrounding circumstellar medium. The expansion will still be non-spherical. As long as the outflow is ultra-relativistic, its motion is determined by the balance between the magnetic stress at the CD and the ram pressure of the circumstellar medium (Lyutikov & Blandford, 2003).

By the time the shell radius expands to \( r_{GRB} \sim ct_s\Gamma^2 \), most of the electromagnetic Poynting flux from the source will have caught up with the CD and been reflected by it, transferring its momentum to the blast wave. Simultaneously a strong region of magnetic shear is likely to develop at the outer part of the CD (Lyutikov, 2002). Both of these effects are likely to lead to the rapid development of current instabilities in the shell that will ultimately result in the acceleration of pairs and the emission of Doppler-boosted Synchrotron emission in the \( \gamma \)-ray band (Lyutikov & Blandford, 2003).

As the magnetic shell expands, its energy is gradually transferred to the preceding forward shock wave. When \( r >> r_{GRB} \), most of the energy released by the central, spinning, magnetic rotator will reside in the blast wave. Particles accelerated at the forward shock may combine with electromagnetic field from the shell to produce afterglow emission. The initial non-spherical expansion will give the appearance of a jet with the “achromatic break” occurring when the Lorentz factor becomes comparable with the reciprocal of the observer’s inclination angle with respect to the symmetry axis (Lyutikov & Blandford, 2003).

The electromagnetic model explains the fast variability of GRB prompt emission light curves in a different way with respect to the fireball model: it is obtained by assuming that the emission is beamed in the outflow frame,
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for example due to relativistic motions of “fundamental” emitters (Lyutikov & Blandford, 2003) like in the “turbulent” model (see section 3.3). In this way the prompt emission is produced at distances larger that the values assumed in the fireball model, more consistently with observations (Lyutikov, 2006). This approach presents, however, several problems. For example it is unclear how macroscopic relativistic motions can be generated and sustained (see section 3.3 and Lazar et al., 2009). Another drawback of the model is the lack of the detailed model of energy dissipation and particle acceleration in relativistic magnetically-dominated medium (Lyutikov & Blandford, 2003).

3.3 The “turbulent” Model

We have seen in section 3.1.2 that the internal shock model for the prompt emission of GRBs presents some problems such as, for example, the low efficiency in converting bulk kinetic energy in photons. Narayan & Kumar (2009) proposed a model alternative to internal shocks to explain the prompt emission of GRBs: the relativistic “turbulent” model. Within this model the observed variability in GRB light curves is obtained by assuming that the radiating fluid in the GRB outflow is relativistically turbulent, e.g. by assuming an inhomogeneous relativistic velocity field in the GRB outflow. The beaming effect of the turbulent energy-bearing eddies causes large amplitude fluctuations in the observed flux. Despite being inhomogeneous, the model is radiatively efficient in the sense that the whole medium radiates and the observer receives a fair share of the radiated luminosity (Kumar & Narayan, 2009). This allows the model to overcome the arguments of Sari & Piran (1997) against inhomogeneous GRB models.

Let’s now explain how the outflow evolves. Consider an idealized model of a GRB in which a spherical shell, located at radius $R$ with respect to the center of the explosion, expands ultra-relativistically outward with a bulk Lorentz factor $\Gamma$. In the shell frame, let the typical Lorentz factor of an
3.3. The “turbulent” Model

energy-bearing eddy be $\gamma_t$. The size of an eddy in its own frame is

$$r_e \sim R/\Gamma \gamma_t.$$  

(3.16)

Each eddy has a volume $\sim r_e^3$, so the total number of eddies in a causal volume of the shell is expected to be

$$n_e \sim \gamma_t^3.$$  

(3.17)

Eddies are not likely to travel along perfectly straight lines. Rather, their velocities are expected to change on the light crossing time scale $\sim R/(c\gamma_t \Gamma)$ (Narayan & Kumar, 2009).

Consider now the radiation from an eddy as viewed in the shell frame. At any instant, the radiation is beamed into a cone of half-angle $1/\gamma_t$. During the life of the eddy, the orientation of the beam wanders by a few radians as a result of turbulent acceleration. Thus each eddy illuminates a total solid angle $\sim 1/\gamma_t$ in the shell frame in the course of its motion. Boosting to the observer frame, the illuminated solid angle from each eddy is $\sim 1/(\Gamma^2 \gamma_t)$. Summing over all $n_e$ eddies in a causal volume, the total solid angle illuminated by all the eddies is $\sim \gamma_t^2/\Gamma^2$. All of this radiation is beamed within a solid angle $\sim 1/\Gamma^2$. Therefore, each observer receives radiation from $\sim \gamma_t^2$ eddies.

An observer receives radiation from the entire collection of eddies (inside one causal volume) over a time $\sim R/(\Gamma^2 c)$. Narayan & Kumar (2009) associate this time with the burst duration $t_{\text{burst}}$. The radiation received from a single eddy then corresponds to an individual pulse in the GRB light curve. To estimate the duration of a pulse, Narayan & Kumar (2009) note that the thickness of an eddy in a direction parallel to its beamed radiation is $\sim R/(\Gamma^2 \gamma_t^2)$ in the observer frame. Thus, an observer receives radiation from a single eddy within a time $\sim t_{\text{burst}}/\gamma_t^2$ and, on average, $\sim \gamma_t^2$ pulses.

To summarize, in the turbulent model we have: $t_{\text{burst}} \sim R/(\Gamma^2 c)$, $t_{\text{var}} \sim R/(\Gamma^2 \gamma_t^2 c)$ and $n_{\text{pulse}} \sim \gamma_t^2$, where $n_{\text{pulse}}$ is the mean number of pulses in the burst (Kumar & Narayan, 2009).

The turbulent model can reproduce the observed temporal variability of GRB light curves as the fireball model does, with the advantage that it is
3. Other Models for GRBs

radiatively efficient and can accommodate much larger values of $R$ and smaller values of $\Gamma$ (Narayan & Kumar, 2009). There are, however, several problems. First of all, it is unclear how macroscopic relativistic motions can be generated and sustained (Lazar et al., 2009). Furthermore, there are some additional questions involving the shape and other properties of individual pulses versus those seen in observed pulses. For example, GRBs show a clear difference between the fast rise and the slow decline of individual pulses (Norris et al., 1996); furthermore, the duration of an observed pulse is correlated with the preceding interval (see section 3.1.2 and Nakar & Piran, 2002). Within the turbulent model, there is no reason for pulses to have these properties (Lazar et al., 2009).
Chapter 4

Analysis of GRB060607A: prompt emission, X-ray flares and late afterglow phase*

GRB060607A is a very distant ($z = 3.082$, Ledoux et al., 2006) and energetic event ($E_{iso} \sim 10^{53}$ erg, Molinari et al., 2007). Its BAT light curve shows a double-peaked structure with a duration of $T_{90} = (100 \pm 5)$ s (Tueller et al., 2006). The time-integrated spectrum over the $T_{90}$ is best fit with a simple power-law model with an index $\Gamma = 1.45 \pm 0.08$ (Guidorzi, private communication). The XRT light curve shows a prominent flaring activity (at least two flares) superimposed on the normal afterglow decay (Page et al., 2006).

GRB060607A main peculiarity is that the peak of the near-infrared (NIR) afterglow has been observed with the REM robotic telescope (Molinari et al., 2007). Interpreting the NIR light curve as corresponding to the afterglow onset as predicted by the fireball forward shock model (Sari & Piran, 1999; Bernardini, M.G., Dainotti, M.G., Bianco, C.L., Caiò, L., Guida, R. Ruffini, R., “Prompt emission and X-ray flares: the case of GRB 060607A”, in the Proceedings of “Probing the Stellar populations out of the Distant Universe” in Cefalù, Italy, September 7 - 19, 2008, eds. G. Giobbi, A. Tornambè, G. Raimondo, M. Limongi and L.A. Antonelli, AIP Conference Proceedings, 1111 (2009) 319-322 (Attachment 1).
4. Analysis of GRB060607A: prompt emission, X-ray flares and late afterglow phase

Mészáros, 2006, see section 3.1 and), it is possible to infer the initial Lorentz gamma factor of the emitting system that results to be $\Gamma_0 \sim 400$ (Molinari et al., 2007; Covino et al., 2008; Jin & Fan, 2007). Moreover, these measurements seem to be consistent with an interstellar medium environment within the standard fireball scenario, ruling out the wind-like medium predicted by the Collapsar model (Molinari et al., 2007; Jin & Fan, 2007).

We have chosen this burst to illustrate the features of the GRB light curves within the fireshell model because it represents a good example of what has been addressed as “canonical light curve” (see section 2.5). In fact, thanks to the data provided by the Swift satellite, we can test our theoretical predictions on the GRB structure presented in section 2.5 with a detailed comparison between the numerical simulations and the observed prompt and afterglow emission light curves. We deal only with the BAT and XRT observations, that are the basic contribution to the extended afterglow emission up to a distance from the progenitor $r \sim 4 \times 10^{17}$ cm. Such observations are usually neglected in the treatments adopted in current literature to characterize the fireball dynamics (Molinari et al., 2007; Sari & Piran, 1999). The complete numerical modeling of the fireshell dynamics (Ruffini et al., 2007), from the creation of the initial plasma up to the interaction of the optically thin fireshell with the surrounding medium, allows to calculate all its characteristic quantities. In particular, the exact value of the Lorentz gamma factor of the fireshell at the transparency can be therefore calculated ($\gamma_0 = 328$) once we fix the two free parameter from the gamma and X-ray light curves analysis.

According to the “canonical GRB” scenario we interpret the whole prompt emission as the joint contribution of both the Proper-GRB (P-GRB), emitted at the fireshell transparency, and the peak of the extended afterglow, which follows the P-GRB emission. While the variability of the P-GRB is linked to the collapse mechanism of the progenitor (Caito et al., 2009), the observed temporal variability of the peak of the extended afterglow is shown to be produced by the interaction of the fireshell with CircumBurst Medium (CBM)
clumps with density contrast $\delta n/n \sim 10$. The corresponding theoretical light curves obtained are in good agreement with the observations in all the Swift BAT energy bands.

For GRB 060607A it is also possible to analyse prompt emission time-integrated spectra: this is useful to test our theoretical predictions about GRB spectrum (see section 2.4): in particular, we can test the validity of the assumption of thermal spectrum in the comoving frame of the fireshell. We analyze the GRB060607A BAT time-integrated spectrum, finding that the theoretical time-integrated spectrum is compatible with the observed one in different time intervals covering the whole prompt emission, despite a small discrepancy at low energy.

GRB 060607A is very interesting also for the presence of X-ray flares in its light curve: the analysis of this features is essential to test the validity of the radial approximation for the CBM distribution at this stage (see section 2.5). We analysed the X-ray flares observed by Swift XRT in the early part of the decaying phase of the X-ray afterglow (Page et al., 2006). According to the fireshell model these flares have the same nature than the peaks observed in the prompt emission, namely they are produced by the interaction of the fireshell with different CBM clumps. This idea is consistent with the correlation between the late peaks in the gamma-ray light curve and the X-ray flares when they are observed simultaneously (Molinari et al., 2007; Ziaeepour et al., 2008). What is peculiar in the late afterglow phases is that the typical dimensions of the clumps become smaller than the visible area of the fireshell. Under this conditions, a three dimensional description would be necessary to substitute the assumption of spherical symmetry and to take into due account the structure of the clumps (see also section 2.5). We propose here a simplified bi-dimensional model for the CBM clump along the line of sight, the emission being limited to a small fraction of the entire EQTS. We show that even with this simplified model it is possible to fully explain flares with $\Delta t/t_{\text{tot}}$ compatible with the observations.

The NIR emission shows no significant evidence for correlation with the
4. Analysis of GRB060607A: prompt emission, X-ray flares and late afterglow phase

prompt emission (Ziaeepour et al., 2008): while the common gamma and X-ray flares light curves appear to be very close and correlate (Ziaeepour et al., 2008), in the NIR band the flaring activity, if any, is much weaker (Molinari et al., 2007). On the contrary, when both the NIR and X-ray light curves are decaying regularly, these curves appear to be correlated (Molinari et al., 2007). We propose a possible scenario in which this second decaying phase arises from the injection of slower material into the fireshell (see appendix A). This leads to a collision which is assumed to be responsible for the X-ray plateau, producing a modification in the comoving thermal spectrum, as well as in the equations of the dynamics of the afterglow. This phase, characterized by high level of instabilities, may account for both the observed X-ray and NIR emission.

4.1 GRB060607A prompt emission light curve

In fig. 4.1 we present the comparison between the numerical simulations and the observed Swift BAT light curve in the 15–150 keV energy band of GRB060607A. We identify the whole prompt emission with the peak of the extended afterglow emission, and the remaining part of the light curve with the decaying tail of the afterglow, according to our “canonical GRB” scenario (see section 2.5 and Ruffini et al., 2001b, 2007). The temporal variability of the light curve has been reproduced assuming spherical CBM regions with different densities (Ruffini et al., 2002). The detailed structure of the CBM adopted is presented in fig. 4.2: $\delta n/n \sim 31$ and $\delta n/n \sim 10$ for the initial two spikes (A and B respectively, see fig. 4.1), and $\delta n/n \sim 1$ for the third one (C, see fig. 4.1).

We therefore obtain for the two parameters characterizing the source $E_{\text{tot}}^{e\pm} = 2.5 \times 10^{53}$ erg and $B = 3.0 \times 10^{-3}$. This implies an initial $e^\pm$ plasma with a total number of $e^\pm$ pairs $N_{e^\pm} = 2.6 \times 10^{58}$ and an initial temperature $T = 1.7$ MeV. The theoretically estimated total isotropic energy emitted in the P-GRB is $E_{P-GRB}^{iso} = 1.9\% E_{\text{tot}}^{e\pm} = 4.7 \times 10^{51}$ erg, hence the P-GRB results
4.1. GRB060607A prompt emission light curve

Figure 4.1: *Swift* BAT 15–150 keV light curve (green points) compared with the theoretical one (red line). The labels “A”, “B”, “C” identifies the three major peaks.

to be undetectable, being under the noise threshold, if we assume an observed duration $\Delta t_{P-\text{GRB}} \gtrsim 10$ s.

After the transparency point at $r_0 = 1.4 \times 10^{14}$ cm from the progenitor, the initial Lorentz gamma factor of the fireshell is $\gamma_o = 328$. This value has been calculated by the numerical solution of the fireshell dynamical equation, as discussed in chapter 2, and using as initial conditions the two free parameters $(E_{tot}^{\infty}$ and $B)$ estimated from the simultaneous analysis of the BAT and XRT light curves, which are the basic contribution to the extended afterglow emission from the progenitor up to a distance $r \sim 4 \times 10^{17}$ cm. This approach is different from what has been proposed in the current literature (Molinari et al., 2007; Rykoff et al., 2009) where the maximum Lorentz factor of the fireball is estimated from the temporal occurrence of the peak of the opti-
4. Analysis of GRB060607A: prompt emission, X-ray flares and late afterglow phase

cal emission, which is identified with the peak of the forward external shock emission in the thin shell approximation (Sari & Piran, 1999). We would like to emphasize a further basic difference between the fireshell approach and the other ones in the current literature. There, the Lorentz factor is usually inferred from the late time observation by means of analytical power-law estimates. On the contrary, within the fireshell model, it is explicitly computed from the description of all the phases starting from the moment of gravitational collapse by explicit evaluation of the radial coordinate as a function of the laboratory time, the comoving time and the arrival time at the detector (see chapter 2 and Ruffini et al., 2001a). It is also important to stress that our treatment fixes uniquely the zero of the time at the moment of the creation of the \(e^+e^-\) plasma in the process of gravitational collapse, and it takes into due account the time needed to reach transparency in the optically thick phase in determining the beginning of the extended afterglow phase. This computation is carried out both in the cases in which the P-GRB is detected and in the cases in which it is under the observational noise threshold.

4.2 GRB spectrum

We turn now to the analysis of the GRB060607A prompt emission time-integrated spectrum. As discussed in chapter 2, the fireshell model postulates that:

- the resulting radiation as viewed in the comoving frame has a thermal spectrum
- the CBM swept up by the front of the optically thin fireshell is responsible for this thermal emission.

In our case the radiation is produced in the inelastic collision between the accelerated baryons and the CBM.

We recall (see section 2.4) that each single instantaneous spectrum is the result of a convolution of thermal spectra since photons observed at the same
4.2. GRB spectrum

Figure 4.2: Detailed structure of the CBM adopted: particle number density $n_{cbm}$ (upper panel) and fraction of effective emitting area $R$ (lower panel) versus the distance from the progenitor. The labels “A”, “B” and “C” indicate the values corresponding to the peaks in the BAT light curve (see fig. 4.1). The two X-ray flares, indicated by the labels “F1” and “F2”, corresponds in the upper panel to the huge increase in the CBM density that departs from the roughly power-law decrease observed. In the lower panel, the X-ray flares produce an increase of the emitting area which is not real but due to the lack of a complete three-dimensional treatment of the interaction between the fireshell and the CBM (green dashed line). In fact if we adopt a bi-dimensional structure for the CBM clumps responsible for the flares we obtain a constant value (red solid line, see section 4.3). The blue vertical dashed-dotted line sets the limit of our analysis (for details see section 4.4).
4. Analysis of GRB060607A: prompt emission, X-ray flares and late afterglow phase

arrival time are emitted at different comoving time, hence with different temperatures. This calculation produces a non-thermal instantaneous spectrum in the observer frame.

This effect is enhanced if we calculate the time-integrated spectrum: we perform two different integrations, one on the observation time and one on the EQTSs, and what we get is a non-thermal spectrum which is compatible with the one observed by BAT in different time intervals covering the whole prompt emission (see fig. 4.3).

As said in chapter 2, a departure from the picture here described even in the BAT energy range has been found in some cases (see section 5.1): these are all highly energetic and very structured GRBs with an isotropic equivalent energy $E_{\text{iso}} \gtrsim 10^{54}$ ergs, like e.g. GRB050904 (Sugita et al., 2009) and GRB080319B (Golenetskii et al., 2008).

4.3 The X-ray flares

The last step to the understanding of the prompt emission of GRB060607A is the analysis of the X-ray flares observed by Swift XRT (0.2 – 10 keV) in the early part of the decaying phase of the afterglow. As already mentioned, in the current literature this emission is associated to the prompt emission due to their temporal and spectral behavior (Chincarini et al., 2007; Falcone et al., 2007). Even GRB060607A reveals a close correlation between the light curve observed by BAT and the X-ray flares observed by XRT (Ziaeepour et al., 2008).

Accordingly with these observations, the fireshell model predicts that these flares have the same origin of the prompt emission, namely they are produced by the interaction of the fireshell with overdense CBM, and their appearance in the X-ray energy band depends on the hard-to-soft evolution of the GRB spectrum. Initially, the result obtained is not compatible with the observations (we choose as an example the second flare labeled as “F2”, see fig. 4.4, upper panel). This discrepancy is due to the simple “radial ap-
4.3. The X-ray flares

Figure 4.3: Theoretically predicted time-integrated photon number spectrum $N(E)$ corresponding to the $0 − 15$ s (upper panel), $15 − 50$ s (middle panel), and to the whole duration ($T_{90} = 100$ s, lower panel) of the prompt emission (solid lines) compared with the observed spectra integrated in the same intervals.
proximation” adopted, namely the CBM inhomogeneities are assumed to be in spherical shells (Ruffini et al., 2002). Clearly this approximation fails when the visible area of the fireshell is comparable with the size of the CBM clump. This is indeed the case of the X-ray flares, since at those times the fireshell visible area (defined by the condition $\cos \theta \geq v/c$, where $\theta$ is the angle between the emitted photon and the line of sight, and $v$ is the fireshell velocity) is much larger than during the prompt phase (for F2 see fig. 4.4, lower panel).

Following the results obtained for GRB011121 (Caito et al., 2008), we simulated the flare F2 light curve assuming a bi-dimensional structure of the CBM clump along the line of sight. We integrate over the emitting surface only up to a certain angle $\theta_c$ from the line of sight, corresponding to the transverse dimension of the CBM clump which in this case results $d_c = 2r_c \sin \theta_c \sim 10^{15}$ cm $\sim 0.5d_f$, where $d_c$ and $d_f$ are respectively the CBM clump and the fireshell transverse dimensions, and $r_c$ is the distance of the clump from the source. We obtain in this way a flare whose duration $\Delta t/t$ is compatible with the observation (see fig. 4.4, middle panel).

It is worth to observe that with this procedure the value of $\mathcal{R}$ remains constant during the flare (see fig. 4.2, lower panel, red solid line). Hence the increase in $\mathcal{R}$ that we obtained in our previous analysis (see fig. 4.2, lower panel, green dashed line) was only a compensation for the failure of the radial approximation.

At this preliminary stage this procedure does not affect the dynamics of the fireshell in the proper way since the “cut” at $\theta_c$ is performed only on the emitted flux, but the fireshell is supposed to interact still with a spherical CBM shell. Nevertheless, it is a confirmation that it is possible to obtain arbitrarily short flares when a fully three dimensional code for the CBM is implemented.

In section 5.4.3 we will show another case in which with this bi-dimensional model we can correctly reproduce X-ray flares.
4.3. The X-ray flares

Figure 4.4: Upper panel: Swift XRT (0.2–10 keV) light curve of flare F2 compared with the theoretical one obtained under the “radial approximation” assumption (red line). Middle panel: Swift XRT (0.2–10 keV) light curve of flare F2 compared with the theoretical one obtained imposing a finite transverse dimension for the CBM clump (red line). Lower panel: the boundaries of the fireshell visible area ($\cos \theta = v/c$) compared with the dimensions of two CBM clumps corresponding to the first spike of the prompt emission (A) and of the flare (B). It is manifest how similar clumps produce different observational results depending on the evolution of the fireshell visible area (see text).
4. Analysis of GRB060607A: prompt emission, X-ray flares and late afterglow phase

4.4 The decaying afterglow regime

The NIR emission shows no significant evidence for correlation with the prompt emission (Ziaeepour et al., 2008): while the common gamma and X-ray light curves appear to be very close and correlate (Ziaeepour et al., 2008), in the NIR band the flaring activity, if any, is much weaker (Molinari et al., 2007). Even if in GRB060607A it is not possible to perform a comparison between the NIR light curve and the X-ray one after the plateau phase (Molinari et al., 2007), generally they show similar trends (Rykoff et al., 2009).

We propose a possible scenario in which the plateau phase and the following decaying phase of the X-ray afterglow originate from a delayed emission produced by a “second component” of the fireshell. We present here a general description of such a new approach, a more detailed description is presented in appendix A.

This component can be identified with the trailing edge of the matter distribution inside the fireshell that, being slower than the main shell, engulfs it at later times. Depending on the spreading of the Lorentz factor, this collision can occur even in the prompt emission phase producing a sort of “gamma-ray plateau” which can be eventually hidden by a superimposed temporal variability produced by the “internal shocks” among these highly relativistic shells. This could be indeed the case of GRB080319B, where it has been observed a sudden change in the variability during the prompt emission (see section 5.3 and Margutti et al., 2008). On the contrary, when the collisions occurs at later times, the low Lorentz factor suppresses the variability and we observe, as in GRB060607A, the usual X-ray plateau.

In both conditions we tried to analyze the dynamics of this second shell and we found that its Lorentz factor is correlated with the one of the fireshell at the moment of transparency, thus confirming that the second shell in not erratic, emitted in a distinct episode of the engine activity, but it is intertwined with the main fireshell until the transparency (for a complete description of the analysis of the sample and of the implications of this result
4.5 Conclusions

The “canonical GRB” scenario (see section 2.5 and Ruffini et al., 2001b, 2007) implies that the GRB prompt emission is formed by two different components: the P-GRB and the peak of the extended afterglow. Therefore the analysis of the prompt emission light curve and spectrum is necessary and sufficient in order to fix the initial parameters which characterize the dynamics of the fireshell both in the optically thick and in the optically thin regimes through the equations of motion that we explicitly derived. This is one major difference between our approach and the ones in the current literature, where emphasis is usually given to the latest parts of the GRB emission (Optical, NIR) and the dynamics of the early phases is just extrapolated backwards by power-laws (Molinari et al., 2007; Sari & Piran, 1999).

We explicitly show in the case of GRB060607A how the comparison between the numerical simulations and the observed prompt emission spectra and luminosities in different energy bands allows to determine the total energy of the electron positron pairs \( E_{\text{tot}}^{\pm} = 2.5 \times 10^{53} \text{ erg} \). The engine powering the GRB is the gravitational collapse to a black hole and from our analysis this energetics points to a binary neutron star merger as the progenitor...
4. Analysis of GRB060607A: prompt emission, X-ray flares and late afterglow phase

(Ruffini et al., 2009; Cherubini et al., 2009; Caito et al., 2009). The complete numerical modeling of the fireshell dynamics allows to calculate also the exact value of its Lorentz gamma factor at the transparency: $\gamma_{\circ} = 328$, which is lower than the value $\Gamma_{\circ} \sim 400$ inferred from the NIR data analysis.

According to the fireshell model the X-ray flares have the same nature than the peaks observed in the prompt emission, namely they are produced by the interaction of the fireshell with different CBM clumps. What is peculiar in the late afterglow phases is that the typical dimensions of the clumps become smaller than the visible area of the fireshell. Under these conditions, we propose a bi-dimensional model for the CBM clump along the line of sight, the emission being limited to a small fraction of the entire EQTS, to substitute the assumption of spherical symmetry and to take into due account the structure of the clumps. We show that in this condition it is possible to obtain flares with $\Delta t/t_{tot}$ compatible with the observations. In order to fully describe all the X-ray flares properties a three dimensional description will be necessary.

We stop our analysis at the starting of the X-ray plateau. We propose a possible scenario in which this phase arises from the injection of slower material into the fireshell. This phase is characterized by high level of instabilities and may account for both the observed X-ray and NIR emission. Clearly its dynamics does not follow a simple ballistic law. Therefore, any backward extrapolation based on power-law equations of motion approximating a ballistic dynamics may lead to unphysical results.
Chapter 5

The modified thermal spectrum*

5.1 Introduction

The large amount of data obtained with the Swift satellite has improved our understanding of gamma ray bursts and firmly established that GRB after-

5. The modified thermal spectrum

glow emission is produced when the relativistic ejecta is decelerated by an external medium (Gehrels et al., 2009, and references therein). Nevertheless, there are different fundamental questions that remain unanswered, for example on the mechanism for the prompt emission observed in the hard-X / soft-γ energy bands.

In general, the behaviour of the N(E) spectrum of the prompt emission can be phenomenologically fit by the Band function: two power laws joined smoothly at a given break energy (see section 1.3 and Band et al., 1993). The non-thermal character of the spectrum suggests Synchrotron emission from relativistic electrons accelerated by either internal shocks or external shocks as one possible radiation mechanism (see sections 3.1.2 and 3.1.3). Kumar & McMahon (2008) have shown the inconsistency of the observational data with the overall Synchrotron model. In fact, they found that for GRBs to be produced via the Synchrotron process, unphysical conditions are required: the distance of the source from the center of the explosion $R_\gamma$ must be larger than $10^{17}$ cm and the source Lorentz factor $\geq 10^3$; for such an high Lorentz factor the deceleration radius $R_d^1$ is less than $R_\gamma$ and this is in contradiction with the early X-ray and optical afterglow data, showing that γ-rays precede the afterglow flux that is produced by a decelerating forward shock. Another viable mechanism for the prompt γ-ray emission is the Inverse Compton (IC) scattering between relativistic electrons and low energy photons (from IR to UV) produced, for example, by Synchrotron (in this case the emission mechanism is called Synchrotron Self-Compton, SSC; see section 3.1.3). For a typical GRB, IC has to amplify the total energy of a low energy seed photon flux by a factor of $\sim 1000$ to produce the observed prompt γ-ray flux. The same relativistic electrons will, however, continue to upscatter the γ-ray photons to very high energies in the TeV range. In many cases, this second-generation IC will be in the Klein-Nishina regime (i.e. the photon energy

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1The deceleration radius $R_d$ is the radius at which the external medium starts to decelerate the GRB outflow; it is defined as $R_d \sim \left( \frac{3E_f}{4\pi \rho_{ext} \Gamma_f^2} \right)^{1/3}$, where $E_f$ is the energy of the outflow, $\Gamma_f$ is its boundary Lorentz factor and $\rho_{ext}$ is the CBM density (Mészáros & Rees, 1993; Rees & Mészáros, 1992).
will be larger than the electron’s rest mass in the electron’s rest frame). This will somewhat suppress the efficiency of conversion of γ-rays to higher energies, however it would not stop it altogether. Piran et al. (2009) have shown that under general conservative assumption the IC mechanism suffers from an “energy crisis”: IC will require a low energy seed that is more energetic than the prompt γ-rays; in particular, if the low energy photons are emitted in the optical, SSC model cannot explain the prompt emission unless the prompt optical emission is very high (e.g. $\lesssim 14^{\text{th}}$ magnitude for $z \sim 2$, see Kumar & McMahon, 2008; Piran et al., 2009). Zou et al. (2009) applied the SSC model to GRB 080319B, characterized by a peak visual magnitude of 5.3 (Racusin et al., 2008), but even in this extreme case SSC could not be responsible for the prompt γ-rays. In fact, the very strong optical emission put the origin of the optical emission at a very large radius, almost inconsistent with internal shocks; alternatively, the Lorentz factor of electrons must be very large, but typical internal shocks involve modest relativistic collisions in which such high Lorentz factors are not common (Kobayashi & Sari, 2001). Alternatively to the very energetic low energy photons, IC will overproduce a very high energy component that would carry much more energy than the observed prompt γ-rays. Observations with the Fermi satellite (see section 1.2) should put much stronger limit to (or verify) this possibility.

A different approach to interpret GRB emission is given by the fireshell model. One of the fundamental point of this model is the assumption of a spectral energy distribution (SED) of the photons in the comoving frame of the fireshell. This approach represents an attempt to identify the underlying physical process responsible of the energy emission. Each observed instantaneous spectrum is obtained as the convolution over the corresponding Equitemporal Surface (EQTS, Bianco & Ruffini, 2005) of thousands of comoving spectra, duly weighted by their corresponding Lorentz and Doppler factors (Ruffini et al., 2004). Time integrated observed GRB spectra are then obtained by the convolution of such instantaneous spectra over the observation time (Bernardini et al., 2005b). The fireshell model assumes that the
5. The modified thermal spectrum

The modified thermal spectrum emission process is thermal in the comoving frame of the fireshell and with this assumption it has been possible to interpret the observational properties of various low energetic GRBs (bursts with an isotropic energy $E_{\text{iso}} \lesssim 10^{53}$ ergs) such as, for example, GRB 031203 (see section 2.4 and Bernardini et al., 2005b) and GRB 060607A (see chapter 4). Nevertheless, the analysis of very energetic bursts ($E_{\text{iso}} \gtrsim 10^{54}$ ergs) such as, for example, GRB 080319B, has revealed some discrepancies between the theoretical predicted spectra and the observed ones, especially when looking at spectra integrated over a few seconds. This problematic induced us to investigate the possibility of better reproducing the spectral properties of GRB prompt emission starting from a different spectral energy distribution of the photons in the comoving frame of the fireshell. We introduce a phenomenologically “modified” thermal spectrum: a spectrum characterized by a different asymptotic low energy slope with respect to the thermal one, fixed by the free parameter $\alpha$. We obtain all the equations involved in the determination of the source luminosity by considering this new spectrum, then we test the model by comparing the numerical simulations with the observed light curves and spectra of several GRBs. We present, as a specific example, the case of GRB 080319B, that is characterized by one of the highest $\gamma$-ray fluences ever recorded (its 15 - 150 keV fluence is $8.1 \times 10^{-5}$ ergs, Cummings et al., 2008) and therefore allows us to analyse also spectra integrated over few seconds. Within the same framework, following the results obtained for GRB 080319B, we analyse GRB 050904. This burst is not as bright as GRB 080319B (its 15 - 150 keV fluence is $5.4 \times 10^{-6}$ ergs, Sakamoto et al., 2005), but presents many similarities with it, as will be clarified in the next sections: in fact, both the sources are very energetic, with an isotropic energy of the order of $10^{54}$ ergs and show a very intense optical emission; furthermore, their light curves present a similar structure.
5.2. The modified thermal spectrum

Let’s briefly recall some general equations needed to determine the source luminosity within the fireshell model (see section 2.4). The source luminosity at the detector arrival time \( t_d \) per unit solid angle \( d\Omega \) and in the energy band \([\nu_1, \nu_2]\) is given by (Ruffini et al., 2004)

\[
\frac{dE^{[\nu_1,\nu_2]}}{dt_d^4d\Omega} = \int_{\text{EQTS}} \Delta \epsilon v \cos \theta \Lambda^4 \frac{dt}{dt_d^4} W(\nu_1, \nu_2, T_{\text{arr}}) d\Sigma,
\]

where \( \Delta \epsilon = \frac{\Delta E_{\text{em}}}{\nu} \) is the emitted energy density released in the interaction of the accelerated baryons with the CBM measured in the comoving frame, \( \Lambda = \{ \gamma[1 - (v/c)\cos\theta]\}^{-1} \) is the Doppler factor, \( W(\nu_1, \nu_2, T_{\text{arr}}) \) is an “effective weight” required to evaluate only the contributions in the energy band \([\nu_1, \nu_2]\), \( d\Sigma \) is the surface element of the EQTS at detector arrival time \( t_d \) on which the integration is performed and \( T_{\text{arr}} \) is the observed temperature of the radiation emitted from \( d\Sigma \).

In eq. (5.1), there is only one term depending on the comoving spectral energy distribution of the photons: the “effective weight” \( W(\nu_1, \nu_2, T_{\text{arr}}) \), defined as ratio between the energy density emitted in a given energy band \([\nu_1, \nu_2]\) and the bolometric energy density:

\[
W(\nu_1, \nu_2, T_{\text{arr}}) = \frac{\int_{\nu_1}^{\nu_2} \left( \frac{dN}{d\nu d\epsilon} \right) \epsilon d\epsilon}{\int_{0}^{\infty} \left( \frac{dN}{d\nu d\epsilon} \right) \epsilon d\epsilon},
\]  

(5.2)

where \( \frac{dN}{d\nu d\epsilon} \) is the number density of photons per unit of energy. With the assumption of thermal spectrum in the comoving frame of the fireshell

\[
\frac{dN}{d\nu d\epsilon} = \left( \frac{8\pi}{h^3c^3} \right) \epsilon^2 \exp\left( \frac{\epsilon}{k_B T_{\text{arr}}} \right) - 1
\]

(5.3)

\( (h \) is the Planck constant, \( c \) is the speed of light and \( k_B \) is the Boltzmann constant) we have

\[
W(\nu_1, \nu_2, T_{\text{arr}}) = \frac{\int_{\nu_1}^{\nu_2} \left( \frac{dN}{d\nu d\epsilon} \right) \epsilon d\epsilon}{aT^4},
\]  

(5.4)
5. The modified thermal spectrum

where $a$ is the radiation constant and $T$ is the temperature in the comoving frame.

In general, the temperature in the comoving frame can be evaluated starting from the following relation:

$$\frac{\Delta E_{\text{int}}}{\Delta \tau} = \pi r^2 c \mathcal{R} \int_0^\infty \left( \frac{dN_\gamma}{dVd\epsilon} \right) \epsilon d\epsilon, \quad (5.5)$$

where $\Delta \tau$ is the time interval in which the energy $\Delta E_{\text{int}}$ is emitted\(^2\) and $\mathcal{R}$ is the surface filling factor. By inserting eq. (5.3) in eq. (5.5) we obtain

$$T = \left( \frac{\Delta E_{\text{int}}}{4\pi r^2 \sigma \mathcal{R}} \right)^{1/4}, \quad (5.6)$$

with $\sigma$ the Stefan-Boltzmann constant.

We now present the new spectral energy distribution for the radiation emitted in the comoving frame of the fireshell: the modified thermal spectrum. It is a spectrum characterized by a different asymptotic power-law index in the low energy region with respect to the thermal one. This index is represented by a free parameter $\alpha$, so that the pure thermal spectrum corresponds to the case $\alpha = 0$:

$$\frac{dN_\gamma}{dVd\epsilon} = \left( \frac{8\pi}{\hbar^3 c^3} \right) \left( \frac{\epsilon}{k_B T} \right)^\alpha \frac{\epsilon^2}{\exp(\frac{\epsilon}{k_B T}) - 1}. \quad (5.7)$$

By using the eq. (5.7) and introducing the variable $y = \frac{\epsilon}{k_B T}$, we obtain the following expression for the “effective weight”:

$$W(\nu_1, \nu_2, T_{\text{arr}}) = \frac{\int_{y_1}^{y_2} \frac{y^{\alpha+3}}{\exp(y)-1} dy}{\Gamma(4+\alpha) Li_{4+\alpha}(1)} \quad (5.8)$$

where $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ is the Gamma function and $Li_{4+\alpha}(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^{4+\alpha}}$ is the Jonqui\'ère's function.

In analogy with the thermal case, we can define and “effective temperature”

\(^2\)We recall that we assume that the energy released in the collision between the accelerated baryons and the CBM is emitted instantaneously (fully radiative condition, see section 2.3).
5.2. The modified thermal spectrum

for the modified thermal spectrum; by inserting eq. (5.7) in eq. (5.5) we obtain:

\[ T = \left[ \frac{\Delta E_{\text{int}}}{\Delta \tau} \left( \frac{h^3 c^2}{(4\pi r^2) 2\pi R k_B^4 \Gamma(4 + \alpha) L_{4+\alpha}} \right) \right]^{1/4}. \]  

(5.9)

It can be easily seen that, for \( \alpha = 0 \), we obtain eq. (5.6).

In fig. 5.1 are shown several instantaneous spectra for different values of the index \( \alpha \). It can be seen that the main effect of varying the value of \( \alpha \) is a change in the low energy slope of the spectral energy distribution. In particular, by decreasing \( \alpha \) the low energy emission increases. Around the peak energy \( E_{\text{peak}} \) the spectrum is instead only weakly dependent on the value of \( \alpha \).

![Instantaneous spectra](image)

Figure 5.1: Theoretically predicted instantaneous spectra for \( t_a^d = 5s \) characterized by the same temperature and different values of the index \( \alpha \). The curve with \( \alpha = 0.0 \) corresponds to the pure thermal spectrum case. All the spectra have been obtained by assuming \( E_{e^+e^-}^{\text{tot}} = 1.0 \times 10^{54} \) ergs, \( B = 2.5 \times 10^{-3} \) and \( n_{cbm} = 1 \) part cm\(^{-3} \).
5. The modified thermal spectrum

5.3 GRB 080319B

GRB 080319B was discovered by the Burst Alert Telescope (BAT) on board of the Swift satellite on March 19, 2008 (Racusin et al., 2008). It is the most luminous GRB observed in the optical energy band: its prompt optical emission peaked at a visual magnitude of 5.3, making it briefly visible with the naked eye (Racusin et al., 2008). It is characterized by a redshift $z = 0.937$ (Vreeswijk et al., 2008) and an isotropic energy release of $E_\gamma = 1.32 \times 10^{54}$ ergs (Golenetskii et al., 2008).

5.3.1 Characteristics of the prompt emission

The prompt emission of GRB 080319B lasts approximately 57 s (Racusin et al., 2008). Its light curve presents a very high temporal variability (see fig. 5.2) and there are several evidences that it can be separated into two main episodes, partitioned at about 28 s. In fact, Margutti et al. (2008) analysed

![Figure 5.2: The Konus-Wind background-subtracted $\gamma$-ray light curve (black), together with optical data from Pi-of-the-sky (blue) and TORTORA (red; TORTORA is a camera mounted on top of REM telescope). Figure from Racusin et al. (2008).](image-url)
the variability time-scale \( t_{\text{var}} \) of the \( \gamma \)-ray prompt emission, finding that the first part of the light curve (up to \( \sim 28 \) s) is dominated by \( t_{\text{var}} \sim 0.1 \) s, while the last part shows a much longer characteristic time-scale (\( t_{\text{var}} \sim 0.7 \) s). Moreover, it is also apparent that there is not a continuous shift between the two time-scales. Finally, an energy resolved analysis of the same temporal profile reveals that the presence of two distinct time scales is peculiar of the highest energy band (100-150 keV), the lowest one (15-25 keV) showing a unique variability time-scale \( t_{\text{var}} \sim 0.1 \) s. This could be due to different physical processes responsible for the two parts of the light curve.

Further evidences of the possibility of identifying two distinct episodes comes from the analysis performed by Stamatikos et al. (2009): they found that the arrival offset between the Swift-BAT 15-25 keV and 50-100 keV energy band (\( \gamma \)-ray spectral lag) is maximum at \( t \gtrsim 28 \) s and it appears to be anti-correlated with the arrival offset between prompt 15-350 keV \( \gamma \)-rays and the optical emission observed by TORTORA (optical/\( \gamma \)-ray spectral lag), maximum at \( t \lesssim 28 \) s.

### 5.3.2 Analysis of BAT data within the fireshell model

We analysed the prompt emission light curve and spectrum of this burst within the fireshell model, using the “modified” thermal spectrum. According to the canonical GRB scenario, we identify the first 8 s of emission with the Proper-GRB (P-GRB); the theoretically estimated total isotropic energy emitted in the P-GRB is \( E_{\text{iso}}^{P-\text{GRB}} = 1.82 \times 10^{52} \) ergs. The remaining part of the first episode (ending at about 28 s) is interpreted as the peak of the extended afterglow, whose temporal variability is produced by the interaction with the CBM (see section 2.5). The numerical simulation that best reproduce the light curve (fig. 5.3) and the time-integrated spectrum (fig. 5.4) of this first episode (\( 3s \leq t_a^d \leq 28s \)) is obtained with the following parameters: \( E_{\text{tot}}^{e^+e^-} = 1.0 \times 10^{54} \) ergs, \( B = 2.5 \times 10^{-3} \) and \( \alpha = -1.8 \); the Lorentz gamma factor at the transparency point, occurring at \( r_0 = 2.6 \times 10^{14} \) cm, is \( \gamma_0 = 394 \). We consider a mean number density \( \langle n_{\text{cbm}} \rangle \sim 6 \) particles cm\(^{-3} \) and
5. The modified thermal spectrum

Figure 5.3: Theoretically simulated light curve of GRB 080319B prompt emission in the 15-150 keV energy band (black solid curve) is compared with the data observed by BAT (red points); the P-GRB is marked with a magenta circle. The vertical dotted line marks the begin of the second part of the prompt emission.

\[ R = 3.5 \times 10^{-10}. \]

This burst is very bright, so it was possible to analyse also spectra integrated over smaller intervals of time. In fig 5.5 is shown, as an example, the spectrum for \( 3s \leq t_a \leq 13s \): it can be seen that with the modified thermal spectrum we can correctly reproduce also this spectrum; on the contrary, by assuming a thermal spectrum there are several discrepancies between the theoretical prediction and the observational data, especially at the lower energies.

Concerning the second episode, lasting from 28 s to the end of the prompt emission, we propose that it is a “gamma-ray plateau” that originates from a delayed emission produced by a “second component” of the fireshell, which is the same physical process responsible for the plateau phase and the following decaying phase observed in GRB 060607A (see section 4.4). More details will be given in appendix A.
5.4. GRB 050904

GRB 050904 was discovered by BAT on September 4, 2005 (Cummings et al., 2005). It is a very long burst, with $T_{90} = 225 \pm 10$ s (Sakamoto et al., 2005); it is characterized by a very high redshift: $z = 6.29$ (Kawai et al., 2005) and an isotropic energy release of $E_{iso} = 1.04 \times 10^{54}$ ergs (Sugita et al., 2009). Furthermore, a bright optical flare was detected by TAROT\(^3\) near the end of the prompt emission (Boër et al., 2006).

5.4.1 Characteristics of the light curve

In fig. 5.6 it is shown the light curve of GRB 050904 as observed by BAT and XRT. One of the most interesting features of this burst is a long-lasting variability of the X-ray emission, showing several X-ray flares (Watson et al., 2006; Cusumano et al., 2007). In particular, the light curve shows a bright

\(^3\)TAROT (Telescope Action Rapide pour les Objets Transitoires - Rapid Action Telescope for Transients Objects) is a robotic telescope working in the X-ray and UV/Optical energy bands (Boër et al., 1999).
5. The modified thermal spectrum

Figure 5.5: Theoretically simulated spectra of GRB 080319B integrated over the time interval $3s \leq t_d \leq 13s$ with $\alpha = -1.8$ (black solid line) and $\alpha = 0.0$ (thermal case, green solid line) are compared with the data observed by BAT (red points). It can be seen that with the “modified” thermal spectrum we can correctly reproduce the observed spectrum, contrarily to what happens with the thermal spectrum.

flare at $(443 \pm 3) \text{ s}$; this is similar to the one observed in other GRBs at early times (Burrows et al., 2005), but the light curve does not settle into a power-law decay, continuing to be dominated by large variability (see also fig. 5.7, Watson et al., 2006).

5.4.2 Analysis of BAT data within the fireshell model

We analysed the light curve (fig. 5.8) and the spectrum (fig. 5.9) of the prompt emission within the fireshell model, applying the “modified” thermal spectrum. We identify this emission with the peak of the extended afterglow. In this case the P-GRB has not been observed. In fact, we have estimated $E_{iso}^{P-GRB} = 1.82 \times 10^{52} \text{ ergs}$, which for $z = 6.29$ corresponds to $\sim 5.77 \times 10^{-9} \text{ erg cm}^{-2}$. If we assume an observed duration $\Delta t_{P-GRB} \gtrsim 1 \text{ s}$, the P-GRB flux is under the BAT threshold. The numerical simulation that best reproduce the observational data is obtained with the same values of $E_{e^+e^-}^{tot}$ and $B$.
Figure 5.6: Light curve of GRB 050904 as observed by BAT and XRT. Inset: linear blow-up of the data from 0 to 230 s. Figure from Cusumano et al. (2007).

found for GRB 080319B: $E_{e^+e^-}^{\text{tot}} = 1.0 \times 10^{54}$ ergs and $B = 2.5 \times 10^{-3}$, with a Lorentz gamma factor at the transparency point $\gamma_0 = 394$. This represents a first similarity between the two sources and could be an indication of a similar progenitor. In the numerical simulation we assumed a mean number density $\langle n_{e^+e^-} \rangle \sim 10^{-1}$ particles cm$^{-3}$ and $R = 2 \times 10^{-11}$; these values are different from the ones obtained for GRB 080319B and this could be an indication of the fact that the two bursts occurred in different environments. Also in this case we obtain the best fit of the observational data by assuming a “modified” thermal spectrum with $\alpha = -1.8$; in this way we can correctly reproduce also spectra integrated over intervals of time much less than the $T_{90}$ of the source (in fig. 5.10 is shown, as an example, the fit of the BAT spectrum integrated over the first 50 s).
5. The modified thermal spectrum

Figure 5.7: Swift XRT light curve of GRB 050904. Inset: linear blow-up of the data from $\sim10$ to $\sim70$ ks to illustrate the variability of the source at late times. Figure from Watson et al. (2006).

5.4.3 Analysis of XRT data within the fireshell model

We analysed the XRT light curve within the fireshell model. In particular, we identify two different episodes, partitioned at $t \sim 10^3$ s (see fig. 5.11). We propose that the first episode contributes to the extended afterglow emission and the second episode originates from a delayed emission produced by a “second component” of the fireshell, like the second component identified in the prompt emission of GRB 080319B. More details will be given in appendix A.

Concerning the first episode, we can correctly reproduce the first part of the X-ray emission, although there are difficulties in reproducing the steep decay of the flare (see fig. 5.11). As explained in section 4.3, within the fireshell model flares have the same origin of the prompt emission: they are the result of the interaction of the fireshell with the CBM. The observed
Figure 5.8: Theoretically simulated light curve of GRB 050904 prompt emission in the 15-150 keV energy band (black solid curve) is compared with observed data (red points). The black and the blue circles mark the first and the second episode respectively.

Figure 5.9: Theoretically simulated spectrum of GRB 050904 integrated over the time interval $0 \leq t_a \leq 225s$ (black solid line) is compared with the observed data by BAT (red points).
5. The modified thermal spectrum

Figure 5.10: Theoretically simulated spectrum of GRB 050904 integrated over the time interval $0 \leq t_a \leq 50\,s$ (black solid line) is compared with the observed data by BAT (red points).

Discrepancies between the theoretical and the observational data are due to the adopted “radial approximation”: the CBM is assumed to be arranged in spherical shells (see section 2.5). This approximation is valid when the fireshell visible area is smaller than the typical size of the CBM clump, but fails when it becomes comparable to the dimension of the clump: this is the case of the X-ray flares, since at those time the fireshell visible area is much larger than during the prompt phase (see fig. 5.12). As done for GRB 060607A, to solve this problem we simulate the flare light curve assuming a bi-dimensional structure of the CBM clump along the line of sight. We integrate over the emitting surface only up to a certain angle $\theta_c$ from the line of sight, corresponding to the transverse dimension of the CBM clump which in this case results $d_c = 2r_c \sin \theta_c \sim 10^{15}\,\text{cm} \sim 10\,d_f$, where $d_c$ and $d_f$ are respectively the CBM clump and the fireshell transverse dimensions, and $r_c$ is the distance of the clump from the source. We obtain in this way a flare whose duration $\Delta t / t$ is compatible with the observation (see fig. 5.13).

We recall that, at this preliminary stage, this procedure does not affect the dynamics of the fireshell in the proper way since the “cut” at $\theta_c$ is performed.
5.5 Discussion and conclusions

The analysis of various low energetic GRBs ($E_{\text{iso}} \lesssim 10^{53}$ ergs) has shown how the N(E) spectrum of their prompt emission, whose behaviour is usually represented by a “power-law”, can also be interpreted in a satisfactory way by a convolution of thermal spectra as postulated by the fireshell model. Nevertheless, there are high energetic GRBs ($E_{\text{iso}} \gtrsim 10^{54}$ ergs) for which this assumption does not allow to reproduce the observational data. One example is given by GRB 080319B: we analysed in detail its prompt emission spectra integrated over different intervals of time and this revealed some discrepancies between the theoretical predicted spectrum and the observed one.

Figure 5.11: Theoretically simulated light curve of GRB 050904 afterglow emission in the Swift XRT energy band (0.2-10 keV) (black solid curve) is compared with the observed data (red points). The first episode is marked with a black circle, the second one with a blue circle.

only on the emitted flux, but the fireshell is supposed to interact still with a spherical CBM shell. Nevertheless, it confirms that it is possible to obtain arbitrarily short flares with a fully three dimensional code for the CBM.
5. The modified thermal spectrum

Figure 5.12: Boundaries of the fireshell visible area (red solid lines) compared with the dimensions of two CBM clumps, corresponding to the first spike of the prompt emission (A) and the flare (B) respectively.

To better interpret the N(E) spectrum of the prompt emission we have introduced a phenomenologically modified thermal spectrum for photons in the comoving frame of the fireshell: a spectrum with a different asymptotic power-law index in the low energy region with respect to the thermal one, represented by the free parameter $\alpha$. We have obtained all the equations to determine the source luminosity by considering this new spectrum, then we have tested the model by comparing our numerical simulations with the observed light curves and spectra of several GRBs. We have presented, as specific examples, the cases of GRB 080319B and GRB 050904. These two sources have very different redshifts ($z \sim 1$ and $z \sim 6.3$ respectively), but present many similarities: both of them are very energetic sources, with an isotropic energy of the order of $10^{54}$ ergs and show a very intense prompt optical emission; furthermore, their light curves present a similar structure. From this analysis we have found some very interesting results: by applying the modified thermal spectrum with $\alpha = -1.8$ we can successfully reproduce the observed prompt emission light curves and photon number spectra integrated over different intervals of time: from a few seconds up to the whole
5.5. Discussion and conclusions

Figure 5.13: Left: theoretically simulated light curve of GRB 050904 X-ray flare obtained under the “radial approximation” (black solid curve) is compared with the data observed by Swift XRT (red points). Right: theoretically simulated light curve of GRB 050904 X-ray flare obtained by imposing a finite transverse dimension for the CBM clump (black solid curve) is compared with the data observed by Swift XRT (red points). It is clear that relaxing the “radial approximation” it is possible to correctly reproduced the flare.

$T_{90}$ of the source. For both the bursts the numerical simulation that best reproduce the observational data has been obtained by assuming the same values of $E_{tot}^{\pm}$ and $B$ (10^{54} \text{ ergs} and 2.5 \times 10^{-3}$ respectively), but different values of $R$ and $\langle n_{cbm} \rangle$: this could indicate that the two sources have a similar progenitor, but occurred in different environments.

For GRB 050904 we analysed also the XRT data, in particular the X-ray flare. According to the fireshell model the X-ray flares have the same nature than the peaks observed in the prompt emission, namely they are produced by the interaction of the fireshell with different CBM clumps. What is peculiar in the late afterglow phases is that the typical dimensions of the clumps become smaller than the visible area of the fireshell. Under these conditions, the radial approximation for the CBM distribution fails; therefore, we have proposed a bi-dimensional model for the CBM clump along the line of sight, the emission being limited to a small fraction of the entire EQTS,
5. The modified thermal spectrum

to substitute the assumption of spherical symmetry and to take into due account the structure of the clumps. We have shown that in this way it is possible to obtain flares with $\Delta t/t_{tot}$ compatible with the observations.

We are still investigating the reason why for some bursts such as, for example, the ones presented in this chapter we need to introduce a modified thermal spectrum, while for other bursts (see, for example, GRB 060607A, section 4.2) the observational data can be reproduced by assuming a pure thermal spectrum. The most satisfactory explanation is the following one. We have seen in section 5.2 that the main effect of changing $\alpha$ in eq. (5.7) is to modify the low energy slope of the spectral energy distribution, while around the peak energy $E_{\text{peak}}$ the spectrum is only weakly dependent on $\alpha$ (see fig. 5.1). This means that, when looking at data around $E_{\text{peak}}$, we can perform the numerical simulations with both the modified thermal spectrum and the pure thermal one obtaining approximatively the same results in the observer’s frame (see fig. 5.14); therefore, both the spectra allow to correctly reproduce the observational data. On the contrary, if data refer to a lower energy region with respect to $E_{\text{peak}}$, it is not possible to interpret the data with the pure thermal spectrum and the modified thermal one is needed (see fig. 5.14).

A confirmation of this picture comes from the comparison between the properties of the bursts analysed in this work and the ones of e.g. GRB 031203 and GRB 060607A (see table 5.1). These last bursts are two lower energetic GRBs ($E_{\text{iso}} = 10^{50}$ ergs and $1.1 \times 10^{53}$ ergs respectively) whose prompt emission spectra can be interpreted within the fireshell model by assuming a pure thermal spectrum (Bernardini et al., 2005b, 2009). We know from the “Amati” relation (Amati et al., 2002; Amati, 2006) that higher energetic long GRBs are characterized by higher intrinsic peak energies. In this case, GRB 080319B ($E_{\text{iso}} = 1.34 \times 10^{54}$ ergs) and GRB 050904 ($E_{\text{iso}} = 1.04 \times 10^{54}$ ergs) also have higher observed peak energies (see table 5.1) with respect to the other two bursts. In particular, from table 5.1 it is clear that GRB 080319B and GRB 050904 are characterized by a value of $E_{\text{peak}}$ above the energy range
5.5. Discussion and conclusions

Figure 5.14: Two simulated spectra obtained assuming $E_{\text{tot}}^c = 10^{54}$ ergs, $B = 2.5 \times 10^{-3}$, $n_{\text{cmb}} = 1$ part cm$^{-3}$ and a comoving modified thermal spectrum having $\alpha = 0.0$ (pure thermal spectrum, red solid line) and $\alpha = -1.8$ (green solid line). The colored box marked the energy band covered by two hypothetical instruments: A (10 - 20 keV) and B ($10^2 - 2 \times 10^2$ keV). It can be seen that, when interpreting data around $E_{\text{peak}}$ (instrument B), we can perform the numerical simulations with both the modified thermal spectrum and the pure thermal one obtaining approximatively the same results in the observer frame; this in not true when looking at a lower energy region with respect to $E_{\text{peak}}$ (instrument A).
5. The modified thermal spectrum

<table>
<thead>
<tr>
<th>GRB</th>
<th>031203 (a)</th>
<th>060607A (b)</th>
<th>080319B (c)</th>
<th>050904 (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>z</td>
<td>0.106</td>
<td>3.082</td>
<td>0.937</td>
<td>6.29</td>
</tr>
<tr>
<td>$E_{\text{iso}}$ (ergs)</td>
<td>$10^{50}$</td>
<td>$1.1 \times 10^{53}$</td>
<td>$1.34 \times 10^{54}$</td>
<td>$1.04 \times 10^{54}$</td>
</tr>
<tr>
<td>$E_{\text{peak}}^i$ (keV)</td>
<td>158 ± 51</td>
<td>478$^{+314}_{-69}$</td>
<td>1261 ± 65</td>
<td>2291$^{+1263}_{-634}$</td>
</tr>
<tr>
<td>$E_{\text{peak}}^o$ (keV)</td>
<td>144 ± 46</td>
<td>117$^{+77}_{-17}$</td>
<td>675 ± 22</td>
<td>314$^{+173}_{-89}$</td>
</tr>
<tr>
<td>Satellite</td>
<td>INTEGRAL</td>
<td>Swift</td>
<td>Swift</td>
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</tr>
<tr>
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<td>BAT</td>
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<td>BAT</td>
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<tr>
<td>$\Delta E$ (keV)</td>
<td>17 ÷ 500</td>
<td>15 ÷ 150</td>
<td>15 ÷ 150</td>
<td>15 ÷ 150</td>
</tr>
</tbody>
</table>

Table 5.1: Redshift, isotropic energy and peak energy of GRB spectra analysed within the fireshell model. The instruments by which the observational data interpreted within the fireshell model have been taken, together with the energy range they cover, are also reported. References for z: (a) Prochaska et al. (2004); (b) Ledoux et al. (2006); (c) Vreeswijk et al. (2008); (d) Kawai et al. (2005). References for $E_{\text{iso}}$: (a) Amati (2006); (b) L. Amati, private communication; (c) Golenetskii et al. (2008); (d) Sugita et al. (2009). References for $E_{\text{peak}}^i$: (a) Amati (2006); (b) L. Amati, private communication; (c) Golenetskii et al. (2008); (d) Sugita et al. (2009). References for $E_{\text{peak}}^o$: (a) Ulanov et al. (2005); (b) L. Amati, private communication; (c) Racusin et al. (2008); (d) Sugita et al. (2009).

covered by the instrument by which the observational data interpreted within the fireshell model have been taken ($\Delta E$). Therefore, we are in a situation analogous to the case of instrument A in fig. 5.14. On the contrary, the peak energies of GRB 031203 and GRB 060607A are within $\Delta E$. This support our hypothesis.

In conclusion, the modified thermal spectrum introduced in this chapter can be used to interpret the observational properties of both low energetic and high energetic bursts. It is also important to emphasize that this spectrum is defined in the comoving frame of the fireshell: therefore, the comparison of the numerical simulations with the observational data could allow us to put some constraints on the microphysics underlying the emission processes.
Chapter 6

Neutron Stars

6.1 Introduction

The creation of an electron-positron plasma in electric fields whose intensity is close to the critical value $E_c$\(^1\) is of great interest in many fields of physics and astrophysics. For example, it has a central role in the acceleration process in GRBs (Goodman, 1986; Piran, 1999; Ruffini et al., 1999, 2000).

Within the fireshell model, GRBs are related to the expansion of an electron-positron plasma originating from the vacuum polarization process during the formation of a black hole (see chapter 2). Such a black hole with overcritical electromagnetic fields could form by the gravitational collapse of a neutron star (NS) in a binary system (Ruffini et al., 2009; Cherubini et al., 2009; Caito et al., 2009). This induced us to reconsider the standard treatment of NSs in order to investigate the possibility of having systems with an electric field of the order or greater than $E_c$.

In this chapter we present a general approach to Neutron Stars based on the relativistic Thomas-Fermi equation (see section 6.2), amply adopted in the study of superheavy nuclei (Pieper & Greiner, 1969; Müller et al., 1972; Popov, 1971; Zel’dovich & Popov, 1972).

\(^1\)We recall that $E_c$ is the critical value for electron–positron pair creation: $E_c = \frac{m_e^2 c^3}{\hbar}$ (see chapter 2 and Heisenberg & Euler, 1936; Sauter, 1931; Schwinger, 1951, 1954a,b)
6. Neutron Stars

The study of neutral atoms with nuclei having mass number $A \sim 10^2 - 10^6$ is a classic problem of theoretical physics (Zel’dovich & Popov, 1972). The existence of overcritical fields for electron-positron pair creation in these systems has been investigated by different authors (see, for example, Müller et al., 1972; Migdal et al., 1976). The same problem was studied also in the context of the relativistic Thomas-Fermi equation by Ferreirinho et al. (1980); Ruffini & Stella (1981).

In a recent work, Ruffini et al. (2007) used the relativistic Thomas-Fermi equation to extrapolate the treatment of superheavy nuclei to the case of Massive Nuclear Cores. These cores represent the inner part of NSs and are characterized by an atomic number of the order of $10^57$, composed of neutrons, protons and electrons in $\beta$-equilibrium, and expected to be kept at nuclear density by self gravity. Ruffini et al. (2007) found that near the surface of these massive cores it is possible to have an electric field close to the critical value $E_c$, although localized in a very narrow shell of the order of the electron Compton wavelength.

Here we use the same approach of Ruffini et al. (2007) to investigate how the intensity and the shape of the electric field at the core surface changes by assuming various different profiles for the proton number density (see section 6.4). In doing this, we relax the condition $Z \sim A/2$ adopted by Pieper & Greiner (1969) and Popov (1971), as well as the condition $Z = (\frac{4}{2})^{\frac{1}{1+\left(\frac{1}{2\pi}\right)A^{2/3}}}$ adopted by Ferreirinho et al. (1980). Instead, we explicitly impose the $\beta$-equilibrium condition between neutrons, protons and electrons, then we obtain by numerical integrations the relation between $Z$ and $A$ (the charge to mass ratio, see section 6.3), both for nuclei and massive nuclear cores.

We also describe the outer region of neutron stars: the outer crust, whose study is important for the comprehension of various observational phenomena such as, for example, starquakes (Lattimer et al., 2007). We determine the mass and the thickness of this region (see section 6.5). Furthermore, we sketch a possible scenario in which the $B$ parameter of the fireshell model (see section
6.2. The Thomas-Fermi theory

2.2.4) originates from the outer crust of neutron stars: when the mass density increases, the inner part of the star undergoes the process of gravitational collapse to a BH, while the less dense region (the outer crust) is left as a remnant; this remnant represents the baryonic material encountered by the fireshell during its expansion.

6.2 The Thomas-Fermi theory

The Thomas-Fermi model assumes that the electrons of an atom constitute a fully degenerate gas of fermions, confined in a spherical region of space by the Coulomb potential of the point-like nucleus of charge $Ze$. Every electron interacts with the potential of the nucleus, as well as with the other electrons.

6.2.1 The classical Thomas-Fermi model

We describe now the classical Thomas-Fermi model of atoms (Thomas, 1927; Fermi, 1927). Consider a point-like nucleus surrounded by a fully degenerate gas of electrons. The electron number density is given by

$$n_e(r) = \left[ \frac{p_{Fe}^r(r)}{3\pi^2\hbar^3} \right]^3,$$  \hspace{1cm} (6.1)

where $p_{Fe}^r(r)$ is the electron Fermi momentum and $\hbar$ is the Planck constant. The equilibrium condition for an electron inside the atom is expressed by the following relation:

$$\epsilon_{Fe}^e(r) = \frac{[p_{Fe}^r(r)]^2}{2m_e} - eV(r),$$  \hspace{1cm} (6.2)

where $\epsilon_{Fe}^e(r)$ and $m_e$ are the Fermi energy and the mass of the electron respectively, $V(r)$ is the Coulomb potential and $c$ is the light velocity. Based on the Gauss law, $V(r)$ satisfies the Poisson equation

$$\nabla^2 V(r) = 4\pi n_e(r).$$  \hspace{1cm} (6.3)

Let’s introduce the function $\phi(r)$, defined as

$$\phi(r) = V(r) + \frac{\epsilon_{Fe}^e}{e}.$$  \hspace{1cm} (6.4)
6. Neutron Stars

By inserting eq. (6.4) in eq. (6.2) we obtain

\[ p_e^F(r) = \left[ 2m_e e \phi(r) \right]^{1/2}. \] (6.5)

Putting

\[ \phi(r) = Ze \chi(r) \] (6.6)

and introducing the new independent variable \( x \), related to the radial coordinate \( r \) by the relation \( r = bx \), with

\[ b = \frac{(3\pi)^{2/3}}{m_e e^2 \frac{\hbar^2}{2\gamma^3} Z^{1/3}} \],

(6.7)

eq. (6.3) becomes

\[ \frac{d^2 \chi(x)}{dx^2} = \frac{\chi(x)^{3/2}}{x^{1/2}}. \] (6.8)

Eq. (6.8) is the classical Thomas-Fermi equation (Thomas, 1927; Fermi, 1927), which can be integrated with suitable boundary conditions. The first boundary condition comes from the request that, approaching the nucleus, one gets the ordinary Coulomb potential \( Ze/r \), therefore \( \chi(r) \to \infty \) as \( r \to 0 \). The second boundary condition comes from the fact that

\[ N_e = \int_0^{r_0} 4\pi n_e(r)r^2 dr, \] (6.9)

where \( r_0 = bx_0 \) is the atom size. From eq. (6.9) we obtain the following condition:

\[ 1 - \frac{N}{Z} = \chi(x_0) - x_0 \chi'(x_0). \] (6.10)

Depending on the value of \( \chi'(x_0) \), we can have three families of solutions:

(i) \( \chi'(x_0) < -1.588807 \): in this case \( \chi(x) \) vanishes for finite values of \( x_0 \) and \( N < Z \); the solutions correspond to positive ionized atoms.

(ii) \( \chi'(x_0) = -1.588807 \): in this case \( \chi(x) \) vanishes at infinite and \( N = Z \); the unique solution corresponds to a neutral atom.

(iii) \( \chi'(x_0) > -1.588807 \): in this case \( \chi(x) \) attains an infinite value when \( x \to \infty \) and the condition \( N = Z \) is reached for a value of \( x_0 \) such that \( \chi(x_0) = x_0 \chi'(x_0) \); the solutions correspond to compressed atoms.
6.2. The Thomas-Fermi theory

6.2.2 The relativistic Thomas-Fermi model

We now present the relativistic extension of the classical Thomas-Fermi model (Ferreirinho et al., 1980). In this case the finite size of the nucleus must be taken into account to avoid the central singularity. The nucleus is assumed to be spherically symmetric, with a radius \( r_0 = 1.2 A^{1/3} \) fm.

Consider a nucleus with \( Z \) protons and \( A \) nucleons interacting with a fully degenerate gas of electrons. The Poisson equation can be written as

\[
\nabla^2 V(r) = 4\pi [n_e(r) - n_p(r)],
\]

where \( n_p(r) \) is the proton number density inside the nucleus. The equilibrium condition for an electron in now expressed as

\[
\epsilon_e^F(r) = \sqrt{[p_e^F(r)e^2/c^2 + m_e^2 c^4 - m_e c^2 - eV(r)],}
\]

By inserting eq. (6.4) in eq. (6.12) we obtain

\[
p_e^F = \left[ \frac{\phi(r)^2 e^2}{c^2} + 2m_e e\phi(r) \right]^{1/2}.
\]

By introducing the function \( \chi(r) \) (see eq. 6.6) and the independent variable \( x \) the number density of electron can be written as

\[
n_e(x) = \frac{Z}{4\pi b^3} \left[ \frac{\chi(x)}{x} \right]^{3/2} \left[ 1 + \left( \frac{Z}{Z_c} \right)^{4/3} \right]^{3/2}
\]

with \( Z_c = \left( \frac{3\pi}{4} \right)^{1/2} \left( \frac{\hbar^2}{me} \right)^{3/2} \).

The number density of protons inside this nucleus is:

\[
n_p(x) = \frac{3Z}{4\pi r_0^3} \theta(x_0 - x),
\]

with \( r_0 = bx_0 \).

By inserting eqs. (6.14) and (6.15) in eq. (6.11), the Poisson equation can be written in the form

\[
\frac{d^2 \chi(x)}{dx^2} = \frac{\chi(x)^{3/2}}{x^{1/2}} \left[ 1 + \left( \frac{Z}{Z_c} \right)^{4/3} \frac{\chi(x)}{x} \right]^{3/2} - \frac{3x}{x_0^3} \theta(x_0 - x),
\]
6. Neutron Stars

which is called the generalized adimensional Thomas-Fermi equation (Ferreirinho et al., 1980).

The first boundary condition for this equation follows from the fact that \( \chi(r) \to 0 \) as \( r \to 0 \). The second boundary condition comes from eq. (6.9), which also in this case gives the eq. (6.10). Also in this case there are three families of solutions.

6.3 On the charge to mass ratio: from nuclei to massive nuclear cores*

It is well known that stable nuclei are located, in the \( N_n-N_p \) plane (where \( N_n \) and \( N_p \) are the total number of neutrons and protons respectively), in a region that, for small values of \( N_p \), is almost a line well described by the relation \( N_n = N_p \) (e.g. \( N_p = A/2 \), with \( A = N_n + N_p \)).

In the past, several efforts have been made to explain theoretically this property, for example with the liquid drop model of atoms. The liquid drop model approximates the nucleus as a sphere composed of protons and neutrons (and not electrons) and takes into account the Coulombian repulsion between protons and the strong nuclear force. Another important characteristic of this model is that it is based on the property that the mass densities of nuclei are approximately the same, independently from \( A \) (Eisberg & Resnick, 1985). In fact, from scattering experiments it was found the following expression for the nuclear radius \( R_N \):

\[
R_N = r_0 A^{1/3},
\]

with \( r_0 = 1.2 \) fm. Using eq. (6.17) the nuclear density can be write as follows:

\[
\rho_N = \frac{Am_N}{V} = \frac{3Am_N}{4\pi r_0^3 A} = \frac{3m_N}{4\pi r_0^3} = 2.314 \times 10^{14} \text{ g/cm}^3,
\]

where $m_N$ is the nucleon mass. From eq. (6.18) it is clear that nuclear density is independent from $A$, so it is constant for all nuclei.

By using this theory, as well as empirical measurements, Weizsäcker (1935) formulated the well known semi-empirical mass formula, by minimizing which it is possible to obtain the semi-empirical relation

$$N_p = \left(\frac{A}{2}\right) \cdot \frac{1}{1 + \left(\frac{3}{400}\right) \cdot A^{2/3}}.$$  \hspace{1cm} (6.19)

In the limit of low $A$, eq. (6.19) gives the well known relation $N_p = A/2$ (Segrè, 1977).

Here we present a different approach to derive numerically the relation between $N_p$ and $A$: we develop a unified treatment extending from nuclei to massive nuclear cores. In particular, following the work of Ruffini et al. (2007), we extend the relativistic Thomas-Fermi model for nuclei (see section 6.2.2) to systems having $A$ up to $\sim 10^{57}$, that is a typical value for neutron stars.

### 6.3.1 The theoretical model

Following the work of Ruffini et al. (2007), we describe nuclei and massive nuclear cores as spherically symmetric systems composed of degenerate protons, electrons and neutrons and impose the condition of global charge neutrality ($N_e = N_p$).

We assume that the proton’s number density $n_p(r)$ is constant inside the core ($r \leq R_C$) and vanishes outside the core ($r > R_C$):

$$n_p(r) = \left(\frac{3N_p}{4\pi R_C^3}\right) \theta(R_C - r),$$  \hspace{1cm} (6.20)

where $N_p$ is the total number of protons and $R_C$ is the core-radius, parametrized as:

$$R_C = \Delta \lambda_\pi N_p^{1/3},$$  \hspace{1cm} (6.21)

with $\Delta$ a free parameter and $\lambda_\pi$ the pion Compton wavelength. We choose $\Delta$ in order to have $\rho \sim \rho_N$, where $\rho$ and $\rho_N$ are the mass density of the system.
6. Neutron Stars

and the nuclear density respectively.

The electron number density \( n_e(r) \) is given by:

\[
n_e(r) = \frac{1}{3\pi^2\hbar^3} \left[ p_e^F (r) \right]^3, \tag{6.22}
\]

where \( p_e^F (r) \) is the electron Fermi momentum. It can be calculated from the condition of equilibrium of Fermi degenerate electrons, that implies the null value of their Fermi energy \( \epsilon_e^F (r) \):

\[
\epsilon_e^F (r) = \sqrt{\left[ p_e^F (r)c \right]^2 + m_e^2c^4} - m_e c^2 + V_c (r) = 0, \tag{6.23}
\]

where \( V_c (r) \) and \( m_e \) are the Coulomb potential energy and the mass of electrons respectively. From this condition we obtain:

\[
p_e^F (r) = \frac{1}{c} \sqrt{V_c^2 (r) - 2m_e c^2 V_c (r)}, \tag{6.24}
\]

hence the electron number density is:

\[
n_e(r) = \frac{1}{3\pi^2\hbar^3 c^3} \left[ V_c^2 (r) - 2m_e c^2 V_c (r) \right]^{3/2}. \tag{6.25}
\]

The Coulomb potential energy of electrons, necessary to derive \( n_e(r) \), can be determined as follows. Based on the Gauss law, \( V_c (r) \) obeys the Poisson equation

\[
\nabla^2 V_c (r) = -4\pi e [n_e (r) - n_p (r)], \tag{6.26}
\]

with the boundary conditions \( V_c (\infty) = 0, V_c (0) = finite \). Introducing the dimensionless function \( \chi (r) \), defined by the relation:

\[
V_c (r) = -\frac{\hbar c \chi (r)}{r}, \tag{6.27}
\]

and the new variable \( x = r/b = r/\lambda_\pi \), from eq. (6.26) we obtain the relativistic Thomas-Fermi equation:

\[
\frac{1}{3x} \frac{d^2 \chi (x)}{dx^2} = -\alpha \left\{ \frac{1}{\Delta^3} \theta (x_c - x) - \frac{4}{9\pi} \left[ \frac{\chi^2 (x)}{x^2} + 2 \frac{m_e \chi (x)}{m_\pi} \right] \right\}^{3/2}. \tag{6.28}
\]

The boundary conditions for the function \( \chi (x) \) are:

\[
\chi (0) = 0, \quad \chi (\infty) = 0, \tag{6.29}
\]
6.3. On the charge to mass ratio: from nuclei to massive nuclear cores

as well as the continuity of $\chi(x)$ and its first derivative $\chi'(x)$ at the boundary of the core.

The number density of neutrons $n_n(r)$ is:

$$n_n(r) = \frac{1}{3\pi^2 \hbar^3} \left[ p_n^F(r) \right]^3,$$

(6.30)

where $p_n^F(r)$ is the neutron Fermi momentum. It can be calculated with the $\beta$-equilibrium condition, which is the condition of equilibrium between the processes

$$\begin{cases}
e^- + p \rightarrow n + \nu_e; \\
n \rightarrow p + e^- + \bar{\nu}_e.
\end{cases}$$

Assuming that neutrinos escape from the core as soon as they are produced, the $\beta$-equilibrium condition is

$$\epsilon^F_e(r) + \epsilon^F_p(r) = \epsilon^F_n(r).$$

(6.31)

Eq. (6.31) can be explicitly written as:

$$\sqrt{[p_p^F(r)c]^2 + m_p^2 c^4} - m_p c^2 - V_c(r) = \sqrt{[p_n^F(r)c]^2 + m_n^2 c^4} - m_n c^2.$$  

(6.32)

From eq. (6.32) we obtain $p_n^F(r)$ and therefore the number density of neutrons.

6.3.2 Results of the numerical integrations

Using the previous equations (section 6.3.1), we derive $n_e(r)$, $n_n(r)$ and $n_p(r)$ and, by integrating them, we obtain the $N_e$, $N_n$ and $N_p$. We also derive a theoretical relation between $N_p$ and $A$ and compare it with the data of the Periodic Table and with the semi-empirical relation (see eq. 6.19).

In doing this we consider, like in the liquid drop model of atoms, a constant density for all nuclei: for each $N_p$ (e.g. for each system) we change the value of the parameter $\Delta$ (see eq. 6.21) in order to have a mean mass density

$$\langle \rho \rangle = \frac{3A m_N}{4\pi R_C^3} = \left( \frac{3A m_N}{4\pi \lambda^3} \right) \times \frac{1}{\Delta^3 N_p} = \rho_N.$$

(6.33)

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Figure 6.1: The $N_p - A$ relation obtained with our model and with the semi-empirical mass formula, the $N_p = A/2$ relation and the data of the Periodic Table; relations are plotted for values of $A$ from 0 to 200.

In Table (6.3.2) are listed some values of $A$ obtained with our model and with the semi-empirical mass formula, as well as the ones of the Periodic Table; in fig. 6.1 and 6.2 it is shown the comparison between the various $N_p - A$ relations. It is clear that there is a good agreement between all the relations for values of $A$ typical of nuclei, with differences of the order of a few percent. Our relation and the semi-empirical one are in agreement up to $A \sim 10^4$; for higher values, we find that the two relations differ. We interpret these differences as due to the effects of penetration of electrons inside the core. In our model we consider a system composed of degenerate protons, neutrons and electrons. For the smallest values of $A$, all the electrons are in a shell outside the core. By increasing $A$, they progressively penetrate into the core (see fig. 6.3 and Ruffini et al., 2007). This produce some modifications on the Coulomb potential and therefore on the number density of neutrons (see eq. 6.32). These effects, which need the relativistic approach introduced in Ruffini et al. (2007), are not taken into account in the semi-empirical mass formula.
6.3. On the charge to mass ratio: from nuclei to massive nuclear cores

Figure 6.2: The $N_p - A$ relation obtained with our model and with the semi-empirical mass formula and the $N_p = A/2$ relation; relations are plotted for values of $A$ from $0$ to $10^8$. It is clear how the semi-empirical relation and the one obtained with our model are in good agreement up to values of $A$ of the order of $10^4$; for greater values of $A$ the two relation differ because our model takes into account the penetration of electrons inside the core, which is not considered in the semi-empirical mass formula.

The charge to mass ratio becomes constant for $A$ greater that $10^7$; in particular, it reaches the asymptotic value of 0.026 (see fig. 6.4).

6.3.3 Conclusions on the $N_p$ versus $A$ relation

We have derived theoretically a universal relation between the total number of protons $N_p$ and the mass number $A$ for nuclear matter in bulk within the model proposed by Ruffini et al. (2007), considering values of $A$ ranging from few units (nuclei) up to $10^{57}$ (massive nuclear cores).

We have considered spherically symmetric systems composed of degenerate electrons, protons and neutrons having global charge neutrality. By integrating the relativistic Thomas-Fermi equation and using the equation of $\beta$-equilibrium, we have determined the total number of protons, electrons and neutrons in the system and hence a theoretical relation between $N_p$ and
Figure 6.3: The electron number in units of the total proton number $N_p$ as function of the radial distance in units of the core radius $R_C$, for different values of $A$. It is clear that, by increasing the value of $A$, the penetration of electrons inside the core increases. Figure from Ruffini et al. (2007).

Figure 6.4: The $N_p - A$ relation obtained with our model and the asymptotic limit $N_p = 0.026A$
6.3. On the charge to mass ratio: from nuclei to massive nuclear cores

<table>
<thead>
<tr>
<th>$N_p$</th>
<th>$A_M$</th>
<th>$A_{PT}$</th>
<th>$A_{SE}$</th>
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</tr>
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<td>32.28</td>
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Table 6.1: Different values of $N_p$ (column 1) and corresponding values of $A$ from our model ($A_M$, column 2), the Periodic Table ($A_{PT}$, column 3) and the semi-empirical mass formula ($A_{SE}$, column 4).
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A.

We have compared the values of \( N_p \) and \( A \) obtained with our model with the ones of the Periodic Table and with the ones calculated with the semi-empirical mass formula; this comparison has been done considering systems with the same mass densities \( (\rho \sim \rho_N) \).

We have shown that there’s a good agreement between all the relations for values of \( A \) typical of nuclei, with differences of the order of per cent. Our relation and the semi-empirical one are in agreement up to \( A \sim 10^4 \); for higher values, we find that the two relations differ. We interpret the different behaviour of our theoretical relation as a result of the penetration of electrons (initially confined in an external shell) inside the core (see fig. 6.3), that becomes more and more important by increasing \( A \); these effects, which need the relativistic approach introduced in Ruffini et al. (2007), are not taken into account in the semi-empirical mass-formula.

6.4 The Extended Nuclear Matter Model with Smooth Transition Surface∗

In their work, Ruffini et al. (2007) have shown the existence of electric fields close to the critical value for pair creation by vacuum polarization effect \( E_c \) in massive nuclear cores. In particular, these overcritical electric fields develop within a region of \( 10^2 \) electron Compton wavelength near the core surface, with a sharp peak at the core radius.

In their treatment, Ruffini et al. (2007) have assumed that proton’s number density is given by a step function centered on the surface of the core, as we have done in the previous section. Here we relax the sharp profile of the

proton distribution: we assume a monotonically decreasing proton distribution function fulfilling a Wood-Saxon dependence, in analogy with nuclear models describing the electric charge distribution inside nuclei.

### 6.4.1 The Relativistic Thomas–Fermi Equation

Let us introduce the proton distribution function \( f_p(x) \) by means of

\[
n_p(x) = n_p^c f_p(x),
\]

where \( n_p^c \) is the central number density of protons. We use the dimensionless unit \( x = (r - R_c)/a \), with \( R_c \) the point where initial conditions are given \((x = 0)\) and

\[
a^{-1} = \sqrt{4\pi\alpha\lambda_e n_p^c},
\]

where \( \alpha \) is the fine structure constant and \( \lambda_e \) is the electron Compton wavelength.

Let us also introduce the function

\[
\xi_e(x) = \sqrt{1 + x_e(x)^2},
\]

where \( x_e(x) = \frac{p_F(x)}{m_e c} \) is the normalized fermi momentum of electrons.

The number density of electrons (see eq. 6.22) can be written as a function of \( \xi_e(x) \):

\[
n_e(x) = \frac{m_e^3 c^3}{3\pi^2 \hbar^3} [\xi_e(x)^2 - 1]^3
\]

By inserting eq. (6.34) and (6.37) in eq. (6.26) and considering the equilibrium condition for electrons (see eq. 6.23), we obtain the following expression for the relativistic Thomas–Fermi equation:

\[
\xi_e''(x) + \left( \frac{2}{x + R_c/a} \right) \xi_e'(x) - \frac{[\xi_e^2(x) - 1]^{3/2}}{\mu} + f_p(x) = 0,
\]

where \( \mu = 3\pi^2\lambda_e^3 n_p^c \).

For a given distribution function \( f_p(x) \) and a central number density of protons \( n_p^c \), eq. (6.38) can be integrated numerically with the boundary conditions

\[
\xi_e(0) = \sqrt{1 + [\mu \delta f_p(0)]^{2/3}}, \quad \xi_e'(0) < 0,
\]

(6.39)
6. Neutron Stars

![Proton distribution function](image)

Figure 6.5: Proton distribution function for $\gamma = 1.5$, $\beta \approx 0.0586749$.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\xi'_n(0)$</th>
<th>$n_p^\beta(cm^{-3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9662053</td>
<td>-0.8680512263367902</td>
<td>$1.38 \times 10^{36}$</td>
</tr>
</tbody>
</table>

Table 6.2: Set of initial conditions

where $\delta \equiv n_e(0)/n_p(0)$. After integrating the Thomas-Fermi equation, we can calculate the neutron number density using the $\beta$-equilibrium condition (eq. 6.32). To do this, in analogy with nuclear models describing the electric charge distribution inside nuclei, we assume a monotonically decreasing proton distribution function fulfilling a Woods–Saxon dependence

$$f_p(x) = \frac{\gamma}{\gamma + e^{-\beta x}},$$

(6.40)

where $\gamma > 0$ and $\beta > 0$. In fig. 6.5 we show the proton distribution function for a particular set of parameters.

### 6.4.2 Results of the numerical integration

We have integrated numerically eq. (6.38) for several sets of parameters and initial conditions. Here we present, as an example, the results obtained starting from the set of initial conditions in tab. 6.2. The values of the physical quantities corresponding to these initial conditions are listed in tab. 6.3.
6.4. The Extended Nuclear Matter Model with Smooth Transition Surface

<table>
<thead>
<tr>
<th>$N_e = N_p$</th>
<th>$A$</th>
<th>$E_{peak}/E_c$</th>
<th>$R_e(km)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{54}$</td>
<td>$1.61 \times 10^{56}$</td>
<td>95</td>
<td>5.56</td>
</tr>
</tbody>
</table>

Table 6.3: Physical quantities for the set of parameters in table 6.2

Figure 6.6: Electron and proton number density for the set of parameters in table 6.2

In fig. 6.6 we show the electron and proton number density. In order to see more clearly the difference between the electron and the proton profiles, we introduce the charge separation function

$$\Delta(x) = \frac{n_p(x) - n_e(x)}{n_p^c}, \quad (6.41)$$

shown in fig. 6.7.

It can be seen that there is no local charge neutrality because of the small difference between the electron and proton profiles. In particular, we see two well defined zones with opposite charge. In the first zone we have $n_p > n_e$, so we have a positive charged shell; in the second one we have $n_p < n_e$ and a negative charged shell develops. We can also see how the system reaches indeed global charge neutrality in a very small scale; this scale is of the order of $\lambda_{\pi}/\sqrt{\alpha}$.

We have plotted in fig. 6.8 the electric field in units of the critical field.
6. Neutron Stars

Figure 6.7: Charge separation for the set of parameters in table 6.2

\[ E_c = \frac{m^2 c^3}{e \hbar}, \text{ namely} \]

\[ \frac{E}{E_c} = -\frac{\lambda_e}{a} \xi_e'(x). \quad (6.42) \]

We can see that the electric field is well above the critical field \( E_c \). The maximum of the electric field occurs at the point where the transition from the positive charged shell to the negative charged one takes place, i.e., where \( n_e = n_p \). Of course, having assumed a smoother proton profile, we find also a smoother electric field near the maximum with respect to the results found by Ruffini et al. (2007). We recall that in the case of sharp proton profile the first derivative of the electric field has a discontinuity in the point of charge inversion.

6.4.3 Conclusions on the Extended Nuclear Matter Model with Smooth Transition Surface

We have shown that in the smooth transition surface of a massive nuclear core overcritical electric fields can exist. In particular, we have evaluated the electric field \( E \) by assuming a monotonically decreasing proton distribution function fulfilling a Woods–Saxon dependence, finding that \( E \) is greater
that the critical value for electron positron pair creation. Having assumed a smoother proton profile, we have found also a smoother electric field near the maximum with respect to the results found by Ruffini et al. (2007).

6.5 The outer crust of Neutron Stars

In the previous sections we have analysed the core, that is the innermost physical region of NSs. The outer layer is the crust, in which we can identify two different physical regions: the inner crust and the outer crust. The inner crust is a region characterized by a mass density $\rho_N < \rho < \rho_{\text{drip}}$, where $\rho_{\text{drip}} = 4.3 \cdot 10^{14} \text{ g cm}^{-3}$ is the neutron drip density (Baym et al., 1971); it is composed of free neutrons and electrons and nuclei. The outer crust is a

---

6. Neutron Stars

physical region characterized by a mass density $\rho < \rho_{\text{drip}}$; it is composed of white dwarf-like material: fully ionized nuclei and free electrons (Yakovlev & Pethick, 2004; Chamel et al., 2008).

In this section we describe some general properties of the outer crust; in particular, we use a general relativistic model to determine the mass and the thickness of the outer crust for different sets of initial conditions. This could be important, for example, for the study of Quasi Periodic Oscillations (QPOs) of giant flares emitted by neutron stars (Lattimer et al., 2007, and references therein).

We also investigate a possible tie between the mass of the outer crust and the $B$ parameter considered in the fireshell model, which is is a measure of the amount of baryonic material left over from the gravitational collapse of the star progenitor (see section 2.2.4).

6.5.1 The General Relativistic Model

The internal structure of the outer crust of NSs is described by the Tolman-Oppenheimer-Volkoff (TOV) equation

$$\frac{dP}{dr} = -\frac{G \left( \rho + \frac{P}{c^2} \right) \left( m + \frac{4\pi r^3 P}{c^2} \right)}{r^2 \left( 1 - \frac{2Gm}{rc^2} \right)}, \quad (6.43)$$

together with the equation

$$\frac{dm}{dr} = 4\pi r^2 \rho, \quad (6.44)$$

where $P$ and $m$ are the pressure and the mass respectively. $M_{\text{crust}}$ and $\Delta R_{\text{crust}}$ can be calculated by integrating eq. (6.43) and (6.44) from an initial radius $r_{\text{in}} = R_{\text{is}}$, where $R_{\text{is}}$ is the radius of the inner part of the star (the base of the outer crust). To perform the integration, the expressions of $P$ and $\rho$ in the outer crust are needed. $P$ is well approximated by the degeneracy pressure of electrons $P_e$, given by (Shapiro & Teukolsky, 2004)

$$P \approx P_e = k_e \phi_e, \quad (6.45)$$
6.5. The outer crust of Neutron Stars

where

$$k_e = \frac{m_e c^2}{8\pi^2 \lambda_e^3}, \quad \phi_e = \xi_e \left(\frac{2}{3} \xi_e^2 - 1\right) \sqrt{\xi_e^2 - 1} + \log \left(\xi_e + \sqrt{\xi_e^2 - 1}\right)$$ (6.46)

with $\lambda_e$ the Compton wavelength of electrons, $\xi_e = \sqrt{1 + x_e^2}$ and $x_e$ the normalized Fermi momentum of electrons ($x_e = \frac{p_F}{m_e c}$).

$\rho$ is well approximated by the mass density of nuclei; by assuming that the system has local charge neutrality it can be expressed as

$$\rho \approx \mu_e m_n n_e,$$ (6.47)

where $\mu_e$ is the mean molecular weight per electron (for a completely ionized element of atomic weight $A$ and number $Z$, $\mu_e = A/Z$), $m_n$ is the mass of neutrons and $n_e$ is the number density of electrons $n_e = \frac{x_e^3}{8\pi^2 \lambda_e^3}$.

### 6.5.2 The mass and the thickness of the crust

We have integrated eq. (6.43) and (6.44) for different sets of initial conditions (initial mass $M_{is}$, radius $R_{is}$ and pressure $P_{is}$); for simplicity we have assumed $\mu_e = 2$.

In fig. 6.9 are shown the results obtained with

$$1 M_\odot \leq M_{is} \leq 3 M_\odot,$$

$$10 \text{ km} \leq R_{is} \leq 20 \text{ km},$$

and $P_{is} = 1.6 \cdot 10^{30} \text{ dyne} \cdot \text{ cm}^{-2}$, that corresponds to $\rho = \rho_{\text{drip}}$.

It can be seen that both $M_{\text{crust}}$ and $\Delta R_{\text{crust}}$ increase by increasing $R_{is}$ and decreasing $M_{is}$ (see fig. 6.9 and 6.10). In particular, we have obtained the following ranges of values:

$$3.0 \cdot 10^{-6} M_\odot \leq M_{\text{crust}} \leq 1.2 \cdot 10^{-3} M_\odot,$$ (6.48)

$$0.04 \text{ km} \leq \Delta R_{\text{crust}} \leq 4.63 \text{ km}.$$
6. Neutron Stars

Figure 6.9: Values of $M_{\text{crust}}$ in units of solar masses, as function of $R_{1s}$, for different values of $M_{1s}$.

Figure 6.10: Values of $\Delta R_{\text{crust}}$ in km, as function of $R_{1s}$, for different values of $M_{1s}$.
6.5.3 A comparison with $M_B$

As seen in chapter 2, within the Fireshell Model GRBs are generated by the gravitational collapse of the star progenitor to a black hole (BH); the star progenitor could be a neutron star (NS) in a binary system (Ruffini et al., 2009; Cherubini et al., 2009; Caito et al., 2009). The electron–positron plasma created in the process of BH formation expands as a spherically symmetric “fireshell”. It evolves and encounters the baryonic remnant of the star progenitor of the newly formed BH, then is loaded with baryons and expands until the transparency condition is reached and the P-GRB is emitted. The extended afterglow emission starts due to the collision between the remaining optically thin fireshell and the CircumBurst Medium. It is important to note that, from numerical simulations, it results that the expansion of the pulse after the baryon loading is quite insensitive to the position of the baryonic remnants (see section 2.2.4); the only important thing is the mass of the baryonic material engulfed $M_B$.

The baryon loading of the electron–positron plasma is measured by the dimensionless quantity

$$B = \frac{M_B c^2}{E_{e^+e^-}^{\text{tot}}},$$

(6.49)

where $E_{e^+e^-}^{\text{tot}}$ is the energy of the dyadosphere, the region outside the horizon of a BH where the electric field is of the order of the critical value for electron–positron pair creation $E_c$. $B$ and $E_{e^+e^-}^{\text{tot}}$ are the two free parameters that describe the optically thick fireshell phase (see section 2.2.4).

We sketch a possible scenario in which $B$ originates from the outer crust of neutron stars: when the mass density increases, the inner part of the NS undergoes the process of gravitational collapse to a BH, while the less dense region (the outer crust) is left as a remnant; this remnant represents the baryonic material encountered by the fireshell during its expansion. With the values of $B$ and $E_{e^+e^-}^{\text{tot}}$ constrained by the comparison between the numerical simulations and the observational data of various GRBs and eq. (6.49), we have obtained the correspondent values of $M_B$ (see Table 6.4), then we have
6. Neutron Stars

<table>
<thead>
<tr>
<th>GRB</th>
<th>$M_B/M_\odot$</th>
</tr>
</thead>
<tbody>
<tr>
<td>970228</td>
<td>$3.9 \times 10^{-3}$</td>
</tr>
<tr>
<td>050315</td>
<td>$3.7 \times 10^{-3}$</td>
</tr>
<tr>
<td>991216</td>
<td>$7.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>011121</td>
<td>$9.4 \times 10^{-5}$</td>
</tr>
<tr>
<td>030329</td>
<td>$5.7 \times 10^{-5}$</td>
</tr>
<tr>
<td>060614</td>
<td>$4.6 \times 10^{-6}$</td>
</tr>
<tr>
<td>060218</td>
<td>$1.3 \times 10^{-6}$</td>
</tr>
<tr>
<td>060607A</td>
<td>$3.9 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 6.4: GRBs and correspondent values of $M_B$ used to reproduce the observed data within the Fireshell Model (Bernardini et al., 2007; Ruffini et al., 2006, 2001b; Caito et al., 2008; Bernardini et al., 2005a; Caito et al., 2009; Dainotti et al., 2007; Bernardini et al., 2009), in units of solar masses.

compared them with the values of $M_{\text{crust}}$ previously obtained (we don’t consider in this comparison the thickness of the outer crust because, as already said, the only important thing for GRB phenomena is the mass of the baryonic material). It can be seen that these values are compatible with the ones of $M_{\text{crust}}$.

6.5.4 Conclusions on the outer crust of NS

We have studied the outer crust of neutron stars; in particular, we have used a general relativistic model to calculated its mass and its thickness for different sets of initial conditions, finding that the outer crust is smaller in mass and in radial extension for more compact objects. Furthermore, we have investigated a possible tie between these results and the $B$ parameter considered in the fireshell model, which is is a measure of the amount of baryonic material left over from the gravitational collapse of the star progenitor. In particular, we have sketched a possible scenario in which $B$ originates from the outer crust of neutron stars: when the mass density increases, the inner part of the
The outer crust of Neutron Stars

star undergoes the process of gravitational collapse to a BH, while the less dense region (the outer crust) is left as a remnant; this remnant represents the baryonic material encountered by the fireshell during its expansion. With the values of $B$ and $E_{\text{e+e-}}^{\text{tot}}$ constrained by the comparison between the numerical simulations and the observational data of various GRBs and eq. (6.49), we have obtained the correspondent values of $M_B$ (see Table 6.4), then we have compared them with the values of $M_{\text{crust}}$ (without taking into account the thickness of the outer crust as it is of no relevance for GRB phenomena within the fireshell model), finding that they are compatible.

The results presented here are very preliminary, as we have used a very simplified model, but they could pave the way for putting some constraints on the $B$ parameter considered in the fireshell model.
6. Neutron Stars
Appendix A

The correlation between $\gamma_0$ and $\gamma_2^*$

In general, the fireshell model explains satisfactorily the GRB features concerning the prompt emission (Bernardini et al., 2005b; Ruffini et al., 2006) and the initial phase of the afterglow (Ruffini et al., 2006; Dainotti et al., 2007; Caiò et al., 2009). The shallow decay phase has been reproduced in several examples by means of a huge increase in the effective emitting area (Dainotti et al., 2007; Caiò et al., 2009). Since there is no evident reason for that, in the framework of the fireshell model we propose as an alternative explanation, initially proposed by Rees & Mészáros (1998): we assume that the material stored inside the fireshell does not move as a unique shell, instead it moves with a range of Lorentz factors. During the prompt emission phase, the fastest material that is in front is slowed by the CircumBurst Medium (CBM) so that at the end of the prompt emission the slower material will catch up with it. Therefore, the plateau phase is the result of this injection, that produces a modification both in the dynamics of the fireshell and in the spectrum of the emitted radiation. Depending on the spreading of

the Lorentz factor, the collision between the fastest and the slower material can occur even in the prompt emission phase producing a sort of “gamma-ray plateau” which can be eventually hidden by a superimposed temporal variability produced by the “internal shocks” among these highly relativistic shells. This could be indeed the case of GRB 080319B (see section 5.3.2). On the contrary, when the collision occurs at later times, the low Lorentz factor suppresses the variability and we observe as in GRB 060607A the usual X-ray plateau (see section 5.4.3).

We postulate that the spread in the fireshell Lorentz factor occurs when the fireshell becomes transparent, and some hints on this directions have been provided by studies of the Proper-GRB (P-GRB) structure with different baryon loadings (see de Barros et al. (submitted to A&A)). This hypothesis is compatible with the basic idea of the fireshell model that all GRBs originate from the gravitational collapse to a black hole that provides the initial energy of the system, but all the following evolution is unconnected from the progenitor’s details and do not depend on a prolonged activity of the central engine. The novelty in this approach is that while the prompt phase has an “external” origin, being the result of the interaction of the fireshell, the shallow decaying phase has an “internal” origin.

A.1 Dynamics of the Collision and determination of $\gamma_2$

Suppose that the fireshell moves with an initial Lorentz factor $\gamma_0$ that decreases due to the interaction with the CBM, and that the injecting material is contained in a second shell with constant Lorentz factor $\gamma_2 \leq \gamma_0$ (see fig. A.1). The second shell will catch up with the fireshell at a certain radius $r^*$ and laboratory time $t^*$ such that:

$$r^* = \beta_2 ct^*, \quad (A.1)$$
A.1. Dynamics of the Collision and determination of $\gamma_2$

Figure A.1: The fireshell (red shell) moves with an initial Lorentz factor $\gamma_0$ that decreases due to the interaction with the CBM, and the injecting material, represented by the second shell (pink shell), moves with constant Lorentz factor $\gamma_2 \leq \gamma_0$. The second shell will catch up with the fireshell at a certain radius $r^*$.

where $\beta_2 = \sqrt{1 - 1/\gamma_2^2}$ is the velocity of the second shell, and $r^*$ and $t^*$ are fixed by the dynamics of the fireshell.

If we assume that the X-ray plateau is the result of the collision between the fireshell and the second shell, we can identify its onset with the moment at which the collision occurs. We determine this time in the observer frame $t^*_{ob}$ by fitting the X-ray afterglow with a broken power law:

$$F(t_{ob}) = N \left[ \left( \frac{t_{ob}}{t^*_{ob}} \right)^a + \left( \frac{t_{ob}}{t^*_{ob}} \right)^{b} \right].$$

(A.2)

Since the second shell expands with constant velocity, the corresponding laboratory time at which a photon is emitted along the line of sight is (neglecting
A. The correlation between $\gamma_0$ and $\gamma_2$

the initial time that corresponds to the time at which the transparency occurs $t_o << t^*$):

$$t^* = 2\gamma_2^2 \frac{t_{ob}^*}{1+z},$$  \hspace{1cm} (A.3)

where we accounted also for the cosmological redshift of the source $z$.

The second step is to identify the radius $r^*$. It is well known that for relativistic expanding emitters, the photons observed at the same time are emitted by the source at different radii, and they belong to a surface called EQuiTemporal Surface (see section 2.3.2). Therefore, it is not obvious to establish in the evolution of the fireshell which is the collision radius, i.e. which radius corresponds to the $t_{ob}^*$ determined above. From the analysis of the luminosity emitted by a fixed EQTS it results that at observed times corresponding to the onset of the plateau the most luminous region is the boundary of the visible region, $\theta_{max} \sim 1/\gamma$, where $\gamma$ is the Lorentz factor of the fireshell at the moment of the collision and depends strongly on the details of the CBM adopted. The most reasonable choice is to estimate tentatively $r^*$ as the radius corresponding to the boundary of the EQTS visible area associated to $t_{ob}^*$ (dashed line in fig. A.2), which is approximately smaller than the head on radius by a factor 2 for a constant speed motion.

We can now evaluate the Lorentz factor that the second shell should have in order to produce a collision observed at $t_{ob}^*$ with the fireshell:

$$\gamma_2 = \sqrt{\frac{r^*(1+z)}{ct_{ob}^*} + \frac{1}{2}},$$  \hspace{1cm} (A.4)

We select several GRBs among those examined within the fireshell model that show in the X-ray afterglow a behavior compatible with the X-ray plateau. We include in the set GRB 050904, for which we guess that the X-ray plateau corresponds to the second episode of the XRT light curve (see section 5.4.3), as well as GRB 080319B (see section A.2).

In tab. A.1 the results obtained for the GRBs of the sample are reported.
A.2 $\gamma_2$-$\gamma_0$ correlation

The result of this analysis reveals (see fig. A.3) the existence of a correlation between the Lorentz factor of the second shell $\gamma_2$ and the maximum Lorentz factor of the fireshell $\gamma_0$, $\gamma_2 \propto \gamma_0^{0.9}$, with $\chi^2/dof = 0.67$ and a Spearman rank coefficient $r_s = 0.91$ (null hypothesis probability $nhp = 0.0083$).

This correlation is not trivial since the dynamics of the fireshell after the transparency is determined uniquely by the CBM distribution that is different for each GRB. It reveals indeed that the second shell in not erratic, emitted in a second episode of the engine activity, but it is intertwined with the main fireshell until the transparency. Therefore, the correlation provides some hints on the origin of the second shell.

The correlation between $\gamma_2$ and $\gamma_0$ can also be used to “predict” the X-ray...
A. The correlation between $\gamma_0$ and $\gamma_2$

Table A.1: Table of the GRBs of our sample including also GRB 080319B (see section A.2). For these GRBs there are reported the observed time with error and radius of the collision ($t^\ast_{ob}$ and $r^\ast$), the Lorentz gamma factor of the fireshell at $r^\ast$ ($\gamma_1$) and at the transparency ($\gamma_0$), the redshift $z$, the value adopted for the free parameters of the fireshell model ($E_{e^{\pm}}^{\text{tot}}$ and $B$) and the Lorentz gamma factor of the second shell derived $\gamma_2$ with error.

<table>
<thead>
<tr>
<th>GRB</th>
<th>060218</th>
<th>050315</th>
<th>060607A</th>
<th>060614</th>
<th>080319B</th>
<th>090423</th>
<th>050904</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t^\ast_{ob}$ (s)</td>
<td>1.05E+004</td>
<td>4.64E+002</td>
<td>1.18E+003</td>
<td>1.20E+003</td>
<td>3.00E+001</td>
<td>3.94E+002</td>
<td>6.46E+002</td>
</tr>
<tr>
<td>$\Delta t^\ast_{ob}$ (s)</td>
<td>1.57E+003</td>
<td>2.98E+001</td>
<td>5.41E+002</td>
<td>4.60E+001</td>
<td>2.00E+001</td>
<td>5.13E+001</td>
<td>4.54E+001</td>
</tr>
<tr>
<td>$r^\ast$ (cm)</td>
<td>1.12E+018</td>
<td>7.30E+016</td>
<td>3.87E+017</td>
<td>9.40E+017</td>
<td>5.65E+016</td>
<td>4.26E+017</td>
<td>1.68E+017</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>5.36E+001</td>
<td>1.02E+002</td>
<td>1.96E+002</td>
<td>1.48E+002</td>
<td>3.35E+002</td>
<td>5.48E+002</td>
<td>2.01E+002</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>9.93E+001</td>
<td>2.20E+002</td>
<td>3.28E+002</td>
<td>3.45E+002</td>
<td>3.94E+002</td>
<td>1.19E+003</td>
<td>4.89E+002</td>
</tr>
<tr>
<td>$z$</td>
<td>0.03</td>
<td>1.95</td>
<td>3.08</td>
<td>0.13</td>
<td>0.94</td>
<td>8.1</td>
<td>6.29</td>
</tr>
<tr>
<td>$E_{e^{\pm}}^{\text{tot}}$ (erg)</td>
<td>2.32E+050</td>
<td>1.46E+054</td>
<td>2.50E+053</td>
<td>2.94E+051</td>
<td>1.00E+054</td>
<td>1.20E+053</td>
<td>1.00E+054</td>
</tr>
<tr>
<td>$B$</td>
<td>1.00E-002</td>
<td>4.55E-003</td>
<td>3.00E-003</td>
<td>2.80E-003</td>
<td>2.50E-003</td>
<td>8.00E-004</td>
<td>2.50E-003</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>6.66E+001</td>
<td>1.24E+002</td>
<td>2.11E+002</td>
<td>1.71E+002</td>
<td>3.49E+002</td>
<td>5.72E+002</td>
<td>2.51E+002</td>
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<tr>
<td>$\Delta \gamma_2$</td>
<td>6.15E+000</td>
<td>3.70E+000</td>
<td>4.31E+001</td>
<td>1.15E+001</td>
<td>1.11E+002</td>
<td>3.57E+001</td>
<td>4.70E+001</td>
</tr>
</tbody>
</table>

Figure A.3: Lorentz gamma factor of the second shell $\gamma_2$ versus the maximum Lorentz factor of the fireshell $\gamma_0$ for the 6 GRBs of the sample. The correlation between the two quantities is manifest.
Figure A.4: Same as fig. A.3 with in addition the values for GRB 080319B, assuming that the plateau phase occurs during the prompt emission. A correlation is still present.

plateau when it is not observed. We use as an example GRB 080319B (see section 5.3). This GRB exhibits an X-ray afterglow well monitored from 51 s (Racusin et al., 2008) that decays as a simple power-law. Moreover, we recall that it has been pointed out (see section 5.3.1 and Margutti et al., 2008; Stamatikos et al., 2009) that the prompt emission can be divided in two different episodes on the basis of different properties of the temporal variability. We guessed therefore that the plateau phase could be occurred during the prompt emission as a “γ-ray plateau”, coincident with the second episode. With this assumption we performed the analysis described before and we found that the γ₂ obtained is only marginally correlated with γ₀, thus neither confirming nor excluding this possibility (see fig. A.4).

The values of γ₂ and γ₀ determined for GRB 050904 are within the correlation, supporting the idea that the X-ray plateau corresponds to the second episode of the XRT light curve (see section 5.4.3).
A. The correlation between $\gamma_0$ and $\gamma_2$
Conclusions

In this work I have tested the validity of the various assumptions of the fireshell model of GRBs, proposed by Ruffini and collaborators, by comparing the numerical simulations with the observed GRB light curves and spectra. From this analysis it has emerged that some assumptions must be relaxed. In particular:

- There are high energetic bursts ($E_{iso} \gtrsim 10^{54}$ ergs) for which the assumption of thermal spectrum in the comoving frame of the fireshell does not allow to correctly reproduce the observed spectra.

- With the assumption of spherical symmetry for the CircumBurst Medium (CBM) it is not possible to correctly reproduce the X-ray flares observed in GRB light curves, as it does not take into due account the structure of the CBM clumps.

In this work I have faced with these problems in the following way:

- I have investigated the possibility of better reproducing the spectral properties of GRBs starting from a different spectral energy distribution of photons in the comoving frame of the fireshell. In particular, I have introduced a phenomenologically “modified” thermal spectrum: a spectrum with a different asymptotic power-law index in the low energy region with respect to the thermal one; this index is represented by the free parameter $\alpha$. I have obtained all the equations involved in the determination of the source luminosity and spectrum, then I have tested the model by comparing the numerical simulations with the observed
light curves and spectra of two high energetic bursts ($E_{iso} \sim 10^{54}$ ergs) whose observational properties cannot be interpreted by assuming a pure thermal spectrum: GRB 080319B and GRB 050904. From this analysis it has emerged that it is possible to correctly reproduce the observed prompt emission light curves and photon number spectra integrated over different intervals of time (from a few seconds to the whole $T_{90}$ of the source) by applying the modified thermal spectrum with $\alpha = -1.8$.

- I have introduced a simplified bi-dimensional model describing the CBM structure along the line of sight to reproduce the observed characteristics of X-ray flares. In particular, I have modified the numerical code simulating the GRB emission in such a way that the integration of the emitted flux over the emitting surface is performed only up to a certain angle $\theta_c$ from the line of sight, corresponding to the transverse dimension of the CBM clump producing the flare. I have tested this model by comparing the numerical simulations with the observed X-ray flares of GRB 060607A and GRB 050904. From this analysis it has emerged that the bi-dimensional model allows to obtain flares with $\Delta t/t_{tot}$ compatible with the observations.

In this work I have also faced with another drawback of the fireshell model: the lack of a complete theory describing how the Reissner-Nordstøm black hole forms from the star progenitor. This star could be, for example, a neutron star (NS) in a binary system (Ruffini et al., 2009; Cherubini et al., 2009; Caito et al., 2009); therefore, I have studied the electrodynamics of NSs in order to investigate the possibility of having systems with an electric field of the order or greater than $E_c$. To do this, I have introduced a generalization of the relativistic Thomas-Fermi equation to study the inner region of NSs: the core, described as a system composed of electrons, protons and neutrons in $\beta$-equilibrium, then I have investigated how the electric field at the core surface changes by assuming various different profiles for the proton number density. From this study it has emerged that overcritical electric fields can exist...
Conclusions

at the core surface of neutron stars. I have also presented a general relativistic model to study the outer region of NSs: the crust. In particular, I have investigated the possibility that the baryonic material encountered by the fireshell during its expansion is the crust, left as a remnant when the NS underwent the process of gravitational collapse to a black hole. To verify this possibility, I have determined the mass of the crust, then I have compared it with the one of the baryonic remnant estimated for various GRBs, finding that they are compatible.

It is clear that the results presented in this work need further investigations. In particular, concerning the modified thermal spectrum the extension of the analysis to other GRBs is needed to confirm the results found. Concerning the flares, a three dimensional description of the CBM clumps is needed in order to fully describe all the X-ray flares properties, as with the bi-dimensional model introduced the fireshell is supposed to interact still with a spherical CBM shell. Finally, the study of NSs here presented is still preliminary. Nevertheless, this work has given a fundamental contribution to the improvement of fireshell model and could pave the way for further developments. In particular, it is important to emphasize that the modified thermal spectrum is defined in the comoving frame of the fireshell: this will allow us to put some constraints on the microphysics underlying the emission processes of GRBs. Concerning the X-ray flares, the results obtained with the bi-dimensional model have given us a confirmation that it is possible to obtain arbitrarily short flares when a fully three dimensional code for the CBM is implemented. Finally, the simplified model here introduced to describe NSs represents the starting point for the development of a more detailed model for GRB progenitors that takes into account, for example, the extension of the NS core treatment to the case of general relativity.
Attachments
Attachment 1
Prompt emission and X-ray flares: the case of GRB 060607A.

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Abstract. GRB 060607A is a very distant and energetic event. Its main peculiarity is that the peak of the near-infrared (NIR) afterglow has been observed with the REM robotic telescope, allowing to estimate the initial Lorentz gamma factor within the fireball forward shock model. We analyze GRB 060607A within the fireball model. The initial Lorentz gamma factor of the fireshell can be obtained adopting the exact solutions of its equations of motion, dealing only with the BAT and XRT observations, that are the basic contribution to the afterglow emission, up to a distance from the progenitor \( r \sim 10^{18} \) cm. According to the “canonical GRB” scenario we interpret the whole prompt emission as the peak of the afterglow emission, and we show that the observed temporal variability of the prompt emission can be produced by the interaction of the fireshell with overdense CircumBurst Medium (CBM) clumps. This is indeed the case also of the X-ray flares which are present in the early phases of the afterglow light curve.

Keywords: gamma rays: bursts — black hole physics

PACS: 98.70.Rz

INTRODUCTION

GRB 060607A is a very distant \( [z = 3.082, \text{ see } 1] \) and energetic event \( [E_{\text{iso}} \sim 10^{53} \text{ erg}, \text{ see } 2] \). Its BAT light curve shows a double-peaked structure with a duration of \( T_{\text{dur}} = (100 \pm 5) \text{ s} [3] \). The XRT light curve shows a prominent flaring activity (at least three flares) superimposed to the normal afterglow decay [4].

GRB 060607A main peculiarity is that the peak of the near-infrared (NIR) afterglow has been observed with the REM robotic telescope [2]. Interpreting the NIR light curve as corresponding to the afterglow onset as predicted by the fireball forward shock model [5, 6], it is possible to infer the initial Lorentz gamma factor of the emitting system that results to be \( \Gamma_{\text{f}} \sim 400 [2, 7, 8] \). Moreover, these measurements seem to be consistent with an interstellar medium environment, ruling out the wind-like medium [2, 8].

We analyze GRB 060607A within the fireball model [9, 10, 11]. The initial Lorentz gamma factor of the fireshell can be obtained adopting the exact solutions of its equations of motion [12]. In our analysis we deal only with the BAT and XRT observations, that are the basic contribution to the afterglow emission, up to a distance from the progenitor \( r \sim 10^{18} \) cm. We do not deal with the infrared emission that, on the contrary, is used in the current literature to estimate the dynamical quantities of the fireball in the forward external shock regime. Nevertheless, the initial value of Lorentz gamma factor we predict is compatible with the one deduced from the REM observations even under very different assumptions [2, 7, 8].

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According to the “canonical GRB” scenario [10, 11] we interpret the whole prompt emission as the joined contribution of both the P-GRB and the peak of the afterglow, and the remaining part of the light curve as the decaying tail of the afterglow. We show that, while the variability of the P-GRB is probably linked to the collapse mechanism of the progenitor [13], the observed temporal variability of the peak of the afterglow can be produced by the interaction of the fireshell with overdense CircumBurst Medium (CBM) clumps. This is indeed the case also of the X-ray flares that are present in the early phases of the afterglow light curve.

**GRB 060607A PROMPT EMISSION AND X-RAY FLARES**

It is commonly believed that the observed temporal variability of the prompt emission and the X-ray flares are related to the activity of a “central engine” and originate from internal rather than external shocks [14, 15, 16, 17, 18, 19]. This interpretation claims that it can account for both the duration (Δt/t ≪ 1) and the observed spectral properties of such a variability [20, 19, 21].

Within the fireshell model we proposed a different scenario [22] in which the interaction of the fireshell with the CBM can produce the observed variability of both the prompt emission and the X-ray flares. In both cases the GRB spectrum is postulated to be thermal in the comoving frame of the fireshell, and its non-thermal behavior is recovered by integrating over the EQuTemporal Surfaces [23, 24].

In Fig. 1 we present the theoretical fit of *Swift* BAT light curves in different energy bands (15–25 keV, 25–50 keV, 50–100 keV, 100–150 keV) of GRB 060607A.
prompt emission. According to the “canonical GRB” scenario [10, 11] we interpret the whole prompt emission as the peak of the afterglow emission, and the remaining part of the light curve as the decaying tail of the afterglow. We therefore obtain for the two parameters characterizing the source in our model $E^{\text{int}} = 2.5 \times 10^{53}$ erg and $B = 3.0 \times 10^{-3}$. The theoretically estimated total isotropic energy emitted in the P-GRB is $E_{\text{P-GRB}} = 1.9\% E^{\text{int}} = 4.7 \times 10^{51}$ erg, hence the P-GRB results to be under the instrumental threshold if we assume a duration $\Delta t_{\text{P-GRB}} \gtrsim 10$ s. The theoretical light curves reproduce correctly both the observed temporal variability and the emitted flux in each energy channel.

We then analyzed the X-ray flares observed by Swift XRT (0.2 – 10 keV) in the early part of the decaying phase of the afterglow. According to the fireshell model these flares have the same origin of the peaks observed in the prompt emission, namely they are produced by the interaction of the fireshell with overdense CBM as the whole afterglow emission. We found initially that our theoretical light curve is not compatible with the observations. This discrepancy is due to the simple modeling adopted, namely the CBM is arranged in spherical shells [22]. This approximation fails when the visible area of the fireshell is comparable with the size of the CBM clumps. This is indeed the case of the X-ray flares, since at those times the fireshell visible area (defined by the condition $\cos \theta \geq v/c$ where $\theta$ is the angle between the emitted photon and the line of sight, and $v$ is the fireshell velocity) is much larger than during the prompt phase (see Fig. 2, right).

Following the results obtained for GRB 011121 [25], we simulated the flare light curve accounting for the three-dimensional structure of the CBM clumps. We “cut” the emission at a certain angle $\theta_i$ from the line of sight corresponding to the transverse dimension of the CBM clump which results $d_c = 2r_c \sin \theta_i \sim 10^{15}$ cm $\sim 0.5 d_f$, where $d_c$ and $d_f$ are respectively the CMB clump and the fireshell transverse dimensions, and $r_c$ is the distance of the clump from the source. We obtain in this way a flare whose duration $\Delta t/t$ is compatible with the observation (see Fig. 2, left) and it is indeed a confirmation that it is possible to obtain arbitrarily short flares by the interaction with the CBM when the filamentary structure is radially directed and with arbitrarily small transverse dimension.

We are currently performing a similar analysis of GRB 060418, which show many similarities with GRB 060607A [26].

FIGURE 2. Left: Swift XRT (0.2–10 keV) light curve compared with the theoretical one obtained imposing a finite transverse dimension for the CBM clump (solid line). Right: the boundaries of the fireshell visible area $(\cos \theta = v/c)$ compared with the dimensions of two CBM clumps corresponding to the first spike of the prompt emission (A) and of the flare (B). It is manifest how similar clumps produce different observational results depending on the evolution of the fireshell visible area (see text).
REFERENCES

Attachment 2
ON THE CHARGE TO MASS RATIO OF NEUTRON CORES AND HEAVY NUCLEI

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Abstract. We determine theoretically the relation between the total number of protons Np and the mass number A (the charge to mass ratio) of nuclei and neutron cores with the model recently proposed by Ruffini et al. (2007) and we compare it with other Np versus A relations: the empirical one, related to the Periodic Table, and the semi-empirical relation, obtained by minimizing the Weizsäcker mass formula. We find that there is a very good agreement between all the relations for values of A typical of nuclei, with differences of the order of per cent. Our relation and the semi-empirical one are in agreement up to A ∼ 10²⁶; for higher values, we find that the two relations differ. We interpret the different behaviour of our theoretical relation as a result of the penetration of electrons (initially confined in an external shell) inside the core, that becomes more and more important by increasing A; these effects are not taken into account in the semi-empirical mass formula.

Keywords: Nuclei; Neutron cores; Thomas-Fermi equation; Mass to charge ratio.

PACS: 26.60.-c, 26.60.Dd, 32.10.-f

INTRODUCTION

It is well known that stable nuclei are located, in the Np-Nc plane (where Np and Nc are the total number of neutrons and protons respectively), in a region that, for small values of Np, is almost a line well described by the relation Nc = Np. In the past, several efforts have been made to explain theoretically this property, for example with the liquid drop model of atoms, that is based on two properties common to all nuclei: their mass densities and their binding energies for nucleons are almost independent from the mass number A = Np + Nc [4]. This model takes into account the strong nuclear force and the Coulombian repulsion between protons and explains different properties of nuclei, for example the relation between Np and A (the charge to mass ratio).

In this work we derive theoretically the charge to mass ratio of nuclei and extend it to neutron cores (characterized by higher values of A) with the model of Ruffini et al. [2], [3]. We consider systems composed of degenerate neutrons, protons and electrons and we use the relativistic Thomas-Fermi equation and the equation of β-equilibrium to determine the number density and the total number of these particles, from which we obtain the relation between Np and A.

THE THEORETICAL MODEL

Following the work of Ruffini et al. [2], [3], we describe nuclei and neutron cores as spherically symmetric systems composed of degenerate protons, electrons and neutrons and impose the condition of global charge neutrality.

We assume that the proton’s number density n_p(r) is constant inside the core (r ≤ R_C) and vanishes outside the core (r > R_C):

n_p(r) = \left( \frac{3N_p}{4\pi R_C^3} \right) \theta(R_C - r),

where N_p is the total number of protons and R_C is the core-radius, parametrized as:

R_C = \frac{\rho}{m_p N^{1/3}}.

We choose ∆ in order to have ρ ∼ ρ_N, where ρ and ρ_N are the mass density of the system and the nuclear density respectively (ρ_N = 2.314 · 10¹⁴ g/cm⁻³).

The electron number density n_e(r) is given by:

n_e(r) = \frac{1}{3\pi \hbar^2} \left| p_F^e(r) \right|^3,

where p_F^e(r) is the electron Fermi momentum. It can be calculated from the condition of equilibrium of Fermi degenerate electrons, that implies the null value of their Fermi energy ε_F^e(r):

ε_F^e(r) = \sqrt{\left| p_F^e(r) c \right|^2 + m_e^2 c^4} - m_e c^2 + V(r) = 0,
where \( V_c(r) \) is the Coulomb potential energy of electrons. From this condition we obtain:

\[
p_p^F(r) = \frac{1}{e} \sqrt{V_c^2(r) - 2m_e c^2 V_c(r)},
\]

hence the electron number density is:

\[
n_e(r) = \frac{1}{3 \pi^2 \hbar^3 c^3} \left[ V_c^2(r) - 2m_e c^2 V_c(r) \right]^{3/2}.
\]

The Coulomb potential energy of electrons, necessary to derive \( n_e(r) \), can be determined as follows. Based on the Gauss law, \( V_c(r) \) obeys the following Poisson equation:

\[
\frac{\nabla^2 V_c(r)}{\pi} = 4 \pi e [n_e(r) - n_p(r)],
\]

with the boundary conditions \( V_c(\infty) = 0 \), \( V_c(0) = \text{finite} \).

Introducing the dimensionless function \( \chi(r) \), defined by the relation:

\[
V_c(r) = -\hbar c \frac{\chi(r)}{r},
\]

and the new variable \( x = rb^{-1} = r \left( \frac{\hbar}{m} \right)^{-1} \), from eq. (7) we obtain the relativistic Thomas-Fermi equation:

\[
\frac{1}{x^4} \frac{d}{dx} \left( x^4 \frac{d\chi}{dx} \right) = -\alpha \left\{ \frac{1}{16} \theta(x_e - x) - \frac{1}{16} \left[ \frac{x_1^2}{x^3} + 2 \frac{\pi}{3} \frac{x_1^2}{x^3} \right]^{3/2} \right\}, \tag{9}
\]

The boundary conditions for the function \( \chi(x) \) are:

\[
\chi(0) = 0, \quad \chi(\infty) = 0, \tag{10}
\]

as well as the continuity of \( \chi(x) \) and its first derivative \( \chi'(x) \) at the boundary of the core.

The number density of neutrons \( n_n(r) \) is:

\[
n_n(r) = \frac{1}{3 \pi^2 \hbar^3} \left[ p_p^F(r) \right]^3, \tag{11}
\]

where \( p_p^F(r) \) is the neutron Fermi momentum. It can be calculated with the following considerations. In a system composed of electrons, protons and neutrons, there are these weak interactions:

\[
\begin{align*}
e^- + p &\rightarrow n + \nu_e; \tag{12} \\
n &\rightarrow p + e^- + \nu_e. \tag{13}
\end{align*}
\]

In order to maintain stability, these two processes must be at equilibrium, that is:

\[
\epsilon^p_e(i) + \epsilon^p_p(i) = \epsilon^p_n(i), \tag{14}
\]

(we assume that neutrinos escape from the core as soon as they are produced).

Eq. (14) is the energetic equation of \( \beta \)-equilibrium; it can be explicitly written as:

\[
\sqrt{[p_p^F(r)]^2 + m_p^2 c^2} + m_e c^2 - V_c(r) = \sqrt{[p_n^F(r)]^2 + m_n^2 c^2} + m_e c^2 \tag{15}
\]

where \( V_c(r) \) is the Coulomb potential energy of electrons.

From this condition we obtain:

\[
p_p^F(r) = \frac{1}{e} \sqrt{V_c^2(r) - 2m_e c^2 V_c(r)},
\]

hence the electron number density is:

\[
n_e(r) = \frac{1}{3 \pi^2 \hbar^3 c^3} \left[ V_c^2(r) - 2m_e c^2 V_c(r) \right]^{3/2}.
\]

We interpret these differences as due to the effects of penetration of electrons inside the core [see fig. (3)]; in our model we consider a system composed of degenerate

**Np VERSUS A RELATION**

Using the previous equations, we derive \( n_e(r) \), \( n_n(r) \) and \( n_p(r) \) and, by integrating these, we obtain the \( N_e \), \( N_n \) and \( N_p \). We also derive a theoretical relation between \( N_p \) and \( A \) and we compare it with the data of the Periodic Table and with the semi-empirical relation:

\[
N_p = \left( \frac{A}{2} \right) \frac{1}{1 + \left( \frac{N_p}{N_e} \right) A^{2/3}} \tag{16}
\]

that, in the limit of low \( A \), gives the well known relation \( N_p = A/2 \) [4].

Eq. (16) can be obtained by minimizing the semi-empirical mass formula, that was first formulated by Weizäcker in 1935 and is based on empirical measurements and on theory (the liquid drop model of atoms). The liquid drop model approximates the nucleus as a sphere composed of protons and neutrons (and not electrons) and takes into account the Coulombian repulsion between protons and the strong nuclear force. Another important characteristic of this model is that it is based on the property that the mass densities of nuclei are approximately the same, independently from \( A \) [1]. In fact, from scattering experiments it was found the following expression for the nuclear radius \( R_N \):

\[
R_N = r_0 A^{1/3}, \tag{17}
\]

with \( r_0 = 1.2 \text{ fm} \). Using eq. (17) the nuclear density can be written as follows:

\[
\rho_N = \frac{4 \pi N_p}{3} = \frac{3 \rho_N}{4 A^{1/3}}, \tag{18}
\]

where \( m_N \) is the nucleon mass. From eq. (18) it is clear that nuclear density is independent from \( A \), so it is constant for all nuclei.

The property of constant density for all nuclei is a common point with our model: in fact, we choose \( A \) in order to have the same mass density for every value of \( A \), in particular we consider the case \( \rho \sim \rho_N \), as previously said.

In table (1) are listed some values of \( A \) obtained with our model and the semi-empirical mass formula, as well as the data of the Periodic Table; in fig. (1) and (2) it is shown the comparison between the various \( Np - A \) relations.

It is clear that there is a good agreement between all the relations for values of \( A \) typical of nuclei, with differences of the order of per cent. Our relation and the semi-empirical one are in agreement up to \( A \sim 10^4 \), for higher values, we find that the two relations differ.
protons, neutrons and electrons. For the smallest values of A, all the electrons are in a shell outside the core; by increasing A, they progressively penetrate into the core [2]. These effects, which need the relativistic approach introduced in [2], are not taken into account in the semi-empirical mass formula.

We also note that the charge to mass ratio becomes constant for A greater than $10^7$; in particular, it is well approximated by the relation $N_p = 0.026A$ (see fig. (4)).

FIGURE 1. The Np-A relation obtained with our model and with the semi-empirical mass formula, the $N_p = A/2$ relation and the data of the Periodic Table; relations are plotted for values of A from 0 to 200.

FIGURE 2. The Np-A relation obtained with our model and with the semi-empirical mass formula and the $N_p = A/2$ relation; relations are plotted for values of A from 0 to $10^8$. It is clear that, by increasing the value of A, the penetration of electrons inside the core increases. Figure from R. Ruffini, M. Rotondo and S. S. Xue [2].

FIGURE 3. The electron number in units of the total proton number $N_p$ as function of the radial distance in units of the core radius $R_C$, for different values of A. It is clear that, by increasing the value of A, the penetration of electrons inside the core increases. Figure from R. Ruffini, M. Rotondo and S. S. Xue [2].

FIGURE 4. The Np-A relation obtained with our model and the asymptotic limit $N_p = 0.026A$.

CONCLUSIONS

In this work we have derived theoretically a relation between the total number of protons $N_p$ and the mass number A for nuclei and neutron cores with the model recently proposed by Ruffini et al. [2], [3]). We have considered spherically symmetric systems composed of degenerate electrons, protons and neutrons having global charge neutrality and the same mass densities ($\rho \sim \rho_0$). By integrating the relativistic Thomas–Fermi equation and using the equation of $\beta$-equilibrium, we have determined the total number of protons, electrons and neutrons in the system and hence a theoretical relation between $N_p$ and A.

We have compared this relation with the empirical data of the Periodic Table and with the semi-empirical relation, obtained by minimizing the Weizsäcker mass for-
We have shown that there’s a good agreement between all the relations for values of $A$ typical of nuclei, with differences of the order of per cent. Our relation and the semi-empirical one are in agreement up to $A \sim 10^4$; for higher values, we find that the two relations differ. We interpret the different behaviour of our theoretical relation as a result of the penetration of electrons (initially confined in an external shell) inside the core [see fig. (3)], that becomes more and more important by increasing $A$; these effects, which need the relativistic approach introduced in [2], are not taken into account in the semi-empirical mass-formula.

### TABLE 1. Different values of $N_p$ (column 1) and corresponding values of $A$ from our model ($A_M$, column 2), the Periodic Table ($A_{PT}$, column 3) and the semi-empirical mass formula ($A_{SE}$, column 4).

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<tr>
<td>$10^{10}$</td>
<td>3.9 $10^{11}$</td>
<td>3.37 $10^{12}$</td>
<td>3.37 $10^{12}$</td>
</tr>
</tbody>
</table>

### REFERENCES

Attachment 3
The Extended Nuclear Matter Model with Smooth Transition Surface

Jorge A. Rueda H.1,2, B. Patricelli1,2, M. Rotondo1,2, R. Ruffini1,2,3 and S-S. Xue2

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Summary. The existence of electric fields close to their critical value $E_c = \frac{m^2}{e^2} \epsilon_3 e^3 \hbar$ has been proved for massive cores of $10^7$ up to $10^{57}$ nucleons using a proton distribution of constant density and a sharp step function at its boundary [1, 2, 3]. We explore the modifications of this effect by considering a smoother density profile with a proton distribution fulfilling a Woods–Saxon dependence. The occurrence of a critical field has been confirmed. We discuss how the location of the maximum of the electric field as well as its magnitude is modified by the smoother distribution.

1 Introduction

One of the most active field of research has been to analyze a general approach to Neutron Stars based on the Thomas-Fermi ultrarelativistic equations amply adopted in the study
of superheavy nuclei. The aim is to have a unified approach both to superheavy nuclei, up to atomic numbers of the order of $10^5$ to $10^6$, and to what we have called "Massive Nuclear Cores". These cores are characterized by atomic number of the order of $10^{10}$, composed by neutrons, protons and electrons in $\beta$-equilibrium, and expected to be kept at nuclear density by self gravity.

The analysis of superheavy nuclei has historically represented a major field of research, guided by Prof. V. Popov and Prof. W. Greiner and their schools. This same problem was studied in the context of the relativistic Thomas-Fermi equation also by R. Ruffini and L. Stella, already in the 80s. A more recent approach [2, 3] has shown the possibility to extrapolate this treatment of superheavy nuclei to the case of Massive Nuclear Cores.

The very unexpected result has been that also around these massive cores there is the distinct possibility of having an electric field close to the critical value $E_0 = \frac{4\pi}{3}\lambda n_0^e$, although localized in a very narrow shell of the order of the electron Compton wavelength.

In all the mentioned works has been assumed a sharp profile for the proton distribution and analyze the changes that it produce on the general properties of the system. In this work we model the transition surface in a smoother way by relaxing the sharp profile of the proton distribution and analyze the changes that it produce on the general properties of the system.

2 The Relativistic Thomas–Fermi Equation

Let us to introduce the proton distribution function $f_p(x)$ by mean of $n_p(x) = n_p^c f_p(x)$, where $n_p^c$ is the central number density of protons. We use the dimensionless unit $x = (r - R_c)/a$, with $a^{-1} = \sqrt{4\pi\alpha\lambda n_p^e\lambda}$, $\lambda$ is the electron Compton wavelength, $R_c$ the point where initial conditions are given ($x = 0$) and $a$ is the fine structure constant.

Using the Poisson’s equation and the equilibrium condition for the gas of electrons

$$E_p = m_p c^2 \sqrt{1 - x^2} - m_e c^2 - eV = 0,$$

where $e$ is the fundamental charge, $x$, the normalized electron Fermi momentum and $V$ the electrostatic potential, we obtain the relativistic Thomas–Fermi equation

$$\frac{\mu}{x + R_c/a} \xi''(x) + \frac{2}{x + R_c/a} \xi'(x) - \frac{(\xi'(x) - 1)^{3/2}}{\mu} + f_p(x) = 0,$$

where $\mu = 3\pi^2 \lambda^2 n_p^e$ and we have introduced the normalized electron chemical potential in absence of any field $\xi = \sqrt{1 + x^2}$. For a given distribution function $f_p(x)$ and a central number density of protons $n_p^c$, the above equation can be integrated numerically with the boundary conditions

$$\xi(0) = \sqrt{1 + [\mu \delta f_p(0)]^{2/3}}, \quad \xi'(0) < 0,$$

where $\delta \equiv n_e(0)/n_p(0)$.

After integrating the TF equation, we can to calculate the neutron number density using the equilibrium condition of the direct and inverse $\beta$ decay

$$n \rightarrow e^- + p + \nu, \quad e^- + p \rightarrow n + \nu,$$

which results in

$$m_n c^2 \xi_n - m_p c^2 = m_p c^2 \xi_p - m_e c^2 + eV$$

The electrostatic potential $V$ is calculated using the equilibrium condition (1).
3 The Woods-Saxon–like Proton Distribution Function

We propose a monotonically decreasing proton distribution function fulfilling a Woods–Saxon dependence

\[ f_p(x) = \frac{\gamma}{\gamma + e^{\beta x}}, \]

where \( \gamma > 0 \) and \( \beta > 0 \). In fig. 1 we show the proton distribution function for a particular set of parameters.

![Proton distribution function for \( \gamma = 1.5, \beta \approx 0.0585749 \).](attachment:3)

4 Results of the Numerical Integration

We have integrated numerically the eq.(2) for several sets of parameters and initial conditions. To show the general behaviour of the system we show here the following two samples of initial conditions.

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( \zeta_0(0) )</th>
<th>( n_e'(cm^{-3}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9662053</td>
<td>-0.868051226367902</td>
<td>( 1.38 \times 10^{8} )</td>
</tr>
<tr>
<td>0.978293547</td>
<td>-0.8992012365573692</td>
<td>( 2.76 \times 10^{8} )</td>
</tr>
</tbody>
</table>

**Table 1** Sets of initial conditions

for the which we obtain respectively the physical quantities

<table>
<thead>
<tr>
<th>( N_e = N_p )</th>
<th>( A )</th>
<th>( E_{\text{peak}}/E_c )</th>
<th>( R_c(km) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{34} )</td>
<td>( 1.61 \times 10^{38} )</td>
<td>95</td>
<td>5.56</td>
</tr>
<tr>
<td>( 2 \times 10^{34} )</td>
<td>( 2.35 \times 10^{38} )</td>
<td>125</td>
<td>5.56</td>
</tr>
</tbody>
</table>

**Table 2** Physical quantities for the sets of parameters in table 1
4.1 Number Densities

In fig.2 we show the electron and proton number density together. We can see here that there is no local charge neutrality because the small difference between the two profiles. Nevertheless, this small difference creates a charge separation (see fig.3) enough to produce huge electric fields (see fig.4).

![Fig. 2](image1)

**Fig. 2** Electron and Proton Number Density for the sets of parameters in table 1

We can also see how the system reaches indeed global charge neutrality in a very small scale as noted by Migdal et al. [1] in their classical paper. This scale have been calculated to be of the order of $\lambda_\pi/\sqrt{\alpha}$.

4.2 The Charge Separation

In order to see more clearly the difference between the electron and the proton profiles we have plotted in fig.3 the charge separation function given by

\[ \Delta(x) = \frac{n_p(x) - n_e(x)}{n_p} \]  

(6)

![Fig. 3](image2)

**Fig. 3** Charge separation for the sets of parameters in table 1

We see two well defined zones with opposite charge. In the first zone we have $n_p > n_e$ so we have a positive charged shell while in the second one we have $n_p < n_e$ and a negative charged shell develops. At the point $n_e = n_p$ we have a maximum of the electric field, which is screened by the negative charged shell until reach global charge neutrality (see fig.4).
4.3 The Electric Field

We have plotted in fig. 4 the electric field in units of the critical field $E_c = \frac{m^2e^3}{\sqrt{\pi}\hbar}$, namely

$$\frac{E}{E_c} = -\frac{\lambda}{\alpha} \zeta'(x).$$

We see that the electric field is overcritical but smaller in respect to the case of a sharp step proton distribution as used in [1, 2, 3]. Nevertheless, it is yet well above the critical field $E_c$.

The maximum of the electric field occur at the point where the transition from the positive charged shell to the negative charged one takes place, i.e., where $n_e = n_p$ (see fig.3). Of course because we have assumed a smoother proton profile we find also a smoother electric field about the maximum. We recall that in the case of sharp proton profile the first derivative of the electric field has a discontinuity on the point of charge inversion.

5 Conclusions

We confirm the existence of overcritical electric fields in the smooth transition surface of a massive nuclear core. The intensity of the electric field depends on the proton density mainly by two factors: the first one is the value of $n_p$ about the surface and the second one is how it changes about the surface (sharpness). In this line we note that the first factor depends strongly on a precise value of the so called ‘melting density’ and the correct value of the charge to mass ratio ($Z/A$) as given by $\beta$–equilibrium, while for the second factor could be very important the surface tension as given for instance by the strong interaction.

References

1. A. B. Migdal, D. N. Voekresenskii and V. S. Popov, JETP letters, 24 186 (1976)
Attachments
Attachment 4
On the Crust of Neutron Stars: a progress report

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Abstract. We study the characteristics of the Outer Crust of neutron stars, that is the region of neutron stars characterized by a mass density less than the "neutron drip" density and composed by white dwarf - like material (fully ionized nuclei and free electrons). In particular, we calculate its mass and its thickness \(M_{\text{crust}}\) and \(\Delta R_{\text{crust}}\) respectively with a general relativistic model, finding that the Outer Crust is smaller in mass and in radial extension for more compact objects. We also propose a correlation with the Fireshell Model of GRBs, that assumes that GRBs originates from the gravitational collapse to a black hole. One of the parameters used in this model is the baryon loading \(B\) of the electron - positron plasma, related to the mass of the baryonic remnant of the star progenitor \(M_B\). We propose that \(B\) originates from the Outer Crust of neutron stars and compare \(M_{\text{crust}}\) with the values of \(M_B\) used to reproduce the observational data of several GRBs, finding that they are compatible.

Keywords: Neutron Stars, Neutron Star Crust, Gamma Ray Bursts

PACS: 97.60.Jd, 26.60.Gj, 98.70.Rz

INTRODUCTION

Neutron stars are one of the most fascinating systems in the Universe: they are involved in many observational phenomena, for example X–ray bursts and provide a unique laboratory to study a variety of phenomena of great interest, such as the behaviour of matter at very high density.

In this work we study the characteristics of the outer layer of neutron stars: the Outer Crust, which is composed by ions and free electrons. In particular, we use general relativistic equations to calculate its mass \(M_{\text{crust}}\) and its thickness \(\Delta R_{\text{crust}}\), as shown in [1]. We also study a possible correlation between the Outer Crust and the Fireshell Model of GRBs. This model assumes that GRBs originates from the gravitational collapse to a black hole and one of its parameters is the baryon loading \(B\) of the electron - positron plasma, which is related to the mass of the baryonic remnant of the star progenitor \(M_B\). We propose that \(B\) originates from the Outer Crust of neutron stars and test this idea by comparing \(M_{\text{crust}}\) with the values of \(M_B\) used to reproduce the observational data of several GRBs.

THE GENERAL RELATIVISTIC MODEL

We describe the Outer Crust of neutron stars as a spherically symmetric region characterized by a mass density \(\rho\) less than the neutron drip density \(\rho_{\text{drip}} = 4.3 \cdot 10^{11} \text{g} \cdot \text{cm}^{-3}\)
and composed by white dwarf-like material (fully ionized nuclei and free electrons). It’s internal structure is described by the Tolman-Oppenheimer-Volkoff (TOV) equation

\[ \frac{dP}{dr} = -\frac{G}{r^2} \left( \rho + \frac{P}{c^2} \right) \left( m + \frac{4\pi r^3 P}{c^2} \right) \left( 1 - \frac{2Gm}{rc^2} \right), \]

(1)
together with the equation

\[ \frac{dm}{dr} = 4\pi r^2 \rho, \]

(2)
where \( P \) and \( m \) are the pressure and the mass respectively. We have calculated \( M_{\text{crust}} \) and \( \Delta R_{\text{crust}} \) by integrating eq. (1) and (2) from an initial radius \( r_i = R_{\text{is}} \), where \( R_{\text{is}} \) is the radius of the inner part of the star (the base of the Outer Crust).

The integration has been done with a specific expression for \( P \) and \( \rho \). The pressure of the system is dominated by the degeneracy pressure of electrons \( P_e \):

\[ P \approx P_e = k_e \phi_e, \]

(3)
where

\[ k_e = \frac{m_e c^2}{8\pi^2 \lambda_e^3}, \quad \phi_e = \xi_e \left( \frac{2}{3} \xi_e^2 - 1 \right) \sqrt{\xi_e^2 - 1} + \log \left( \xi_e + \sqrt{\xi_e^2 - 1} \right) \]

(4)
with \( \lambda_e \) the Compton wavelength of electrons, \( \xi_e = \sqrt{1 + x_e^2} \) and \( x_e \) the normalized Fermi momentum of electrons \( (x_e = p_e/m_e c) \).

The mass density of the system is approximately the mass density of nuclei; by assuming the local charge neutrality condition it can be expressed as

\[ \rho \approx \mu_e m_n n_e, \]

(5)
where \( \mu_e \) is the mean molecular weight per electron that, for a completely ionized element of atomic weight \( A \) and number \( Z \), is equal to \( A/Z \) (for simplicity, we have assumed \( \mu_e = 2 \)); \( m_n \) is the mass of neutrons and \( n_e \) is the number density of electrons

\[ n_e = \frac{\xi_e^2}{3\pi^2 \lambda_e^3}. \]

**THE MASS AND THE THICKNESS OF THE CRUST**

We have integrated eq. (1) and (2) for different sets of initial conditions (initial mass \( M_{\text{is}} \), radius \( R_{\text{is}} \) and pressure \( P_{\text{is}} \)); in fig. 1 are shown the results obtained assuming

\[ 1M_\odot \leq M_{\text{is}} \leq 3M_\odot, \]
\[ 10\text{km} \leq R_{\text{is}} \leq 20\text{km} \]
and \( P_{\text{is}} = 1.6 \times 10^{30} \text{ dyn} \cdot \text{cm}^{-2} \), that corresponds to a mass density equal to \( \rho_{\text{drip}} \).

It can be seen that both \( M_{\text{crust}} \) and \( \Delta R_{\text{crust}} \) increases by increasing \( R_{\text{is}} \) and decreasing \( M_{\text{is}} \) (see fig. 1). In particular, we have obtained the following ranges of values:
FIGURE 1. Left: values of $M_{\text{crust}}$ in units of solar masses, as function of $R_{\text{is}}$, for different values of $M_{\text{is}}$. Right: values of $\Delta R_{\text{crust}}$ in km, as function of $R_{\text{is}}$, for different values of $M_{\text{is}}$.

\[
3.0 \cdot 10^{-6} M_{\odot} \lesssim M_{\text{crust}} \lesssim 1.2 \cdot 10^{-3} M_{\odot},
\]

\[
0.04 \text{km} \lesssim \Delta R_{\text{crust}} \lesssim 4.63 \text{km}.
\]

The values of $M_{\text{crust}}$ can be compared with the mass of the baryonic remnant $M_B$ considered in the Fireshell model of GRBs.

**A COMPARISON WITH $M_B$**

In the Fireshell Model [3] GRBs are generated by the gravitational collapse of the star progenitor to a charged black hole (BH). The electron–positron plasma created in the process of BH formation expands as a spherically symmetric “fireshell”. It evolves and encounters the baryonic remnant of the star progenitor of the newly formed BH, then is loaded with baryons and expands until the transparency condition is reached and the Proper - GRB is emitted. The afterglow emission starts due to the collision between the remaining optically thin fireshell and the CircumBurst Medium.

The baryon loading is measured by the dimensionless quantity

\[
B = \frac{M_B c^2}{E_{\text{dya}}},
\]

where $M_B$ is the mass of the baryonic remnant and $E_{\text{dya}}$ is the energy of the dyadosphere, the region outside the horizon of a BH where the electric field is of the order of the critical value $E_c = \frac{m_e^2 c^3}{\epsilon R_{\text{eff}}}$ [12, 13, 14, 15, 16]. $B$ and $E_{\text{dya}}$ are the two free parameters of the model.

We propose that $B$ originates from the Crust of neutron stars: when the mass density increases, the inner part of the neutron star undergoes the process of gravitational collapse to a black hole and the Outer Crust (the less dense region of the star) is left as a remnant; this remnant represents the baryonic material encountered by the fireshell during its expansion.

With the values of $B$ and $E_{\text{dya}}$ used to reproduce the observational data of several GRBs
TABLE 1. GRBs and correspondent values of $M_B$ used to reproduce the observed data within the Fireshell Model [4, 5, 6, 7, 8, 9, 10, 11], in units of solar masses.

<table>
<thead>
<tr>
<th>GRB</th>
<th>$M_B/M_\odot$</th>
</tr>
</thead>
<tbody>
<tr>
<td>970228</td>
<td>3.90 x 10^{-3}</td>
</tr>
<tr>
<td>050315</td>
<td>3.70 x 10^{-3}</td>
</tr>
<tr>
<td>061007</td>
<td>1.310 x 10^{-3}</td>
</tr>
<tr>
<td>991216</td>
<td>7.3 x 10^{-4}</td>
</tr>
<tr>
<td>011121</td>
<td>9.4 x 10^{-5}</td>
</tr>
<tr>
<td>030329</td>
<td>5.7 x 10^{-5}</td>
</tr>
<tr>
<td>060614</td>
<td>4.61 x 10^{-6}</td>
</tr>
<tr>
<td>060218</td>
<td>1.31 x 10^{-6}</td>
</tr>
</tbody>
</table>

and eq. (6), we have obtained the correspondent values of $M_B$ (see table 1). It can be seen that these values are compatible with the ones of $M_{crust}$, supporting then the idea that $B$ originates from the Outer Crust of neutron stars.

CONCLUSIONS

We have studied the Outer Crust of neutron stars with a general relativistic model. In particular, we have calculated its mass and its thickness for different sets of initial conditions, finding that the Outer Crust is smaller in mass and in radial extension for more compact objects. We have also proposed a correlation between $M_{crust}$ and the Fireshell Model of GRBs, that assumes that GRBs originate from the gravitational collapse to a black hole. One of the parameters used in this model is the baryon loading $B$ of the electron - positron plasma, related to the mass of the baryonic remnant of the star progenitor $M_B$. We have proposed that $B$ originates from the Outer Crust of neutron star; we have compared $M_{crust}$ with $M_B$ and found that the values are compatible, supporting the idea that $B$ originates from the Outer Crust of neutron stars.

REFERENCES

6. L. Izzo et al., in preparation
13. F. Sauter Z Phys., 69, 742 (1931)
Attachment 5
On the Crust of Neutron Stars

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Summary. We study the characteristics of the Outer Crust of neutron stars, that is the region of neutron stars characterized by a mass density less than the “neutron drip” density and composed by fully ionized nuclei and free electrons (white dwarf - like material). In particular, we use a general relativistic model to calculate the mass and the thickness of this region ($M_{\text{crust}}$ and $\Delta R_{\text{crust}}$ respectively), finding that the Outer Crust is smaller in mass and in radial extension for more compact objects. We also propose a correlation with the Fireshell Model of GRBs, that assumes that GRBs originates from the gravitational collapse to a black hole. One of the parameters of this model is the baryon loading of the electron - positron plasma $B$, which is related to the mass of the baryonic remnant of the star progenitor $M_B$. We propose that $B$ originates from the Outer Crust and compare $M_{\text{crust}}$ with the values of $M_B$ that allow the best fit of the observational data of various GRBs, finding that they are compatible.

1 Introduction

Neutron stars are one of the most fascinating systems in the Universe: they are involved in many observational phenomena, for example X-ray bursts and provide a unique laboratory to study a variety of phenomena of great interest, such as the behaviour of matter at very high density [1].

In this work we study the outer region of neutron stars: the Outer Crust, which is composed
by white dwarf - like material (fully ionized nuclei and free electrons) [2, 3]. We use general relativistic equations to calculate the mass $M_{\text{crust}}$ and the thickness $\Delta R_{\text{crust}}$ of this region, as shown in [4, 5]. We also study a possible correlation between the Outer Crust and the Fireshell Model of GRBs. This model assumes that GRBs originates from the gravitational collapse to a black hole and one of its parameters is the baryon loading $B$ of the electron - positron plasma, which is related to the mass of the baryonic remnant of the star progenitor $M_B$. We propose that $B$ originates from the Outer Crust of neutron stars and test this idea by comparing $M_{\text{crust}}$ with the values of $M_B$ used to reproduce the observational data of several GRBs.

2 The General Relativistic Model

We describe the Outer Crust of neutron stars as a spherically symmetric system having a mass density $\rho$ less than the neutron drip density $\rho_{\text{drip}} = 4.3 \cdot 10^{11} \text{ g cm}^{-3}$ [6] and composed by white dwarf-like material. Its structure is described by the Tolman-Oppenheimer-Volkoff (TOV) equation

$$\frac{dP}{dr} = -\frac{G (\rho + \frac{P}{c^2}) (m + \frac{4\pi \rho c^2}{\lambda_e})}{r^2 \left(1 - \frac{2m}{r}\right)},$$

(1)

together with the equation

$$\frac{dm}{dr} = 4\pi r^2 \rho,$$

(2)

where $P$ and $m$ are the pressure and the mass respectively.

$M_{\text{crust}}$ and $\Delta R_{\text{crust}}$ can be calculated by integrating eq. (1) and (2) from an initial radius $r_i$; where $R_0$ is the radius of the inner part of the star (the base of the Outer Crust).

To make the integration the expressions of $P$ and $\rho$ are needed. $P$ is well approximated by the degeneracy pressure of electrons $P_e$, given by [7]

$$P \approx P_e = k_e \phi_e,$$

(3)

with $\lambda_e$, the Compton wavelength of electrons, $\phi_e = \xi_e \left(\frac{2}{3} \xi_e^2 - 1\right) \sqrt{\xi_e - 1} + \log \left(\xi_e + \sqrt{\xi_e - 1}\right)$

(4)

where $\lambda_e$ is the Compton wavelength of electrons, $\xi_e = \sqrt{1 + x_e^2}$ and $x_e$ the normalized Fermi momentum of electrons ($x_e = \frac{p_F}{m_e c}$).

$\rho$ is well approximated by the mass density of nuclei; by assuming that the system has local charge neutrality it can be expressed as

$$\rho \approx \mu_e m_n n_e,$$

(5)

where $\mu_e$ is the mean molecular weight per electron (for a completely ionized element of atomic weight $A$ and number $Z$, $\mu_e = A/Z$), $m_n$ is the mass of neutrons and $n_e$ is the number density of electrons $n_e = \frac{x_e^3}{\lambda_e} n_i$.

3 The mass and the thickness of the crust

We have integrated eq. (1) and (2) for different sets of initial conditions (initial mass $M_i$, radius $R_i$ and pressure $P_i$); for simplicity we have assumed $\mu_e = 2$.

In fig. 1 are shown the results obtained with
1 On the Crust of Neutron Stars

\( 1 \text{M}_\odot \leq M_{\text{crust}} \leq 3 \text{M}_\odot \), \hspace{1cm} (6)

\( 10 \text{km} \leq R_{\text{is}} \leq 20 \text{km} \), \hspace{1cm} (7)

and \( P_{\text{is}} = 1.6 \times 10^{30} \text{dyne cm}^{-2} \), that corresponds to \( \rho = \rho_{\text{drip}} \).

It can be seen that both \( M_{\text{crust}} \) and \( \Delta R_{\text{crust}} \) increases by increasing \( R_{\text{is}} \) and decreasing \( M_{\text{is}} \) (see fig. 1).

Fig. 1 Left: values of \( M_{\text{crust}} \) in units of solar masses, as function of \( R_{\text{is}} \), for different values of \( M_{\text{is}} \). Right: values of \( \Delta R_{\text{crust}} \) in km, as function of \( R_{\text{is}} \), for different values of \( M_{\text{is}} \).

In particular, we have obtained the following ranges of values:

\( 3.0 \times 10^{-6} \text{M}_\odot \leq M_{\text{crust}} \leq 1.2 \times 10^{-3} \text{M}_\odot \), \hspace{1cm} \hspace{1cm} \hspace{1cm} (8)

\( 0.04 \text{km} \leq \Delta R_{\text{crust}} \leq 4.63 \text{km} \). \hspace{1cm} (9)

The values of \( M_{\text{crust}} \) can be compared with the mass of the baryonic remnant \( M_B \) considered in the Fireshell model of GRBs.

4 A comparison with \( M_B \)

In the Fireshell Model [8], schematized in fig. 2, GRBs are generated by the gravitational collapse of the star progenitor to a black hole (BH). The electron–positron plasma created in the process of BH formation expands as a spherically symmetric “fireshell”. It evolves and encounters the baryonic remnant of the star progenitor of the newly formed BH, then is loaded with baryons and expands until the transparency condition is reached and the Proper - GRB is emitted. The afterglow emission starts due to the collision between the remaining optically thin fireshell and the CircumBurst Medium.

The baryon loading of the electron–positron plasma is measured by the dimensionless quantity

\[ B = \frac{M_B c^2}{E_{\text{dy}}}, \] \hspace{1cm} (10)

where \( M_B \) is the mass of the baryonic remnant and \( E_{\text{dy}} \) is the energy of the dyadosphere, the region outside the horizon of a BH where the electric field is of the order of the critical value for electron–positron pair creation \( \tilde{E}_c = \frac{m_e^2 c^3}{\bar{e} h} \) [9, 10, 11, 12, 13]. \( B \) and \( E_{\text{dy}} \) are the two free parameters of the model.
We propose that $B$ originates from the Outer Crust of neutron stars: when the mass density increases, the inner part of the star undergoes the process of gravitational collapse to a BH, while the less dense region (the Outer Crust) is left as a remnant; this remnant represents the baryonic material encountered by the fireshell during its expansion.

With the values of $B$ and $E_{\text{dyas}}$ constrained by the best fit of the observational data of various GRBs and eq. (10), we have obtained the correspondent values of $M_B$ (see table 1). It can be seen that these values are compatible with the ones of $M_{\text{crust}}$, supporting then the idea that $B$ originates from the Outer Crust of neutron stars.

5 Conclusions

We have studied the Outer Crust of neutron stars; in particular, we have used a general relativistic model to calculated its mass and its thickness for different sets of initial conditions, finding that the Outer Crust is smaller in mass and in radial extension for more compact objects. We have also proposed a correlation between $M_{\text{crust}}$ and the Fireshell Model of GRBs, that assumes that GRBs originates from the gravitational collapse to a black hole. One of the parameters of this model is the baryon loading $B$ of the electron - positron plasma, related to the mass of the baryonic remnant of the star progenitor $M_B$. We have proposed that $B$ originates from the Outer Crust of neutron star; we have compared $M_{\text{crust}}$ with $M_B$ and found that the values are compatible, supporting the idea that $B$ originates from the Outer Crust of neutron stars.

<table>
<thead>
<tr>
<th>GRB</th>
<th>$M_B/M_C$</th>
<th>$M_{\text{crust}}$/M$_\odot$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRB 970228</td>
<td>3.910</td>
<td>3.910</td>
</tr>
<tr>
<td>GRB 050315</td>
<td>3.710</td>
<td>3.710</td>
</tr>
<tr>
<td>GRB 061007</td>
<td>1.310</td>
<td>1.310</td>
</tr>
<tr>
<td>GRB 991216</td>
<td>7.310</td>
<td>7.310</td>
</tr>
<tr>
<td>GRB 011121</td>
<td>9.410</td>
<td>9.410</td>
</tr>
<tr>
<td>GRB 030329</td>
<td>5.710</td>
<td>5.710</td>
</tr>
<tr>
<td>GRB 060614</td>
<td>4.610</td>
<td>4.610</td>
</tr>
<tr>
<td>GRB 060218</td>
<td>1.310</td>
<td>1.310</td>
</tr>
<tr>
<td>GRB 060607A</td>
<td>3.910</td>
<td>3.910</td>
</tr>
</tbody>
</table>

Table 1 GRBs and correspondent values of $M_B$ used to reproduce the observed data within the Fireshell Model [14, 15, 16, 17, 18, 19, 20, 21, 22], in units of solar masses.
References

15. L. Izzo et al., in preparation
Attachments
Attachment 6
The Electrodynamics of the Core and the Crust components in Neutron Stars

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Abstract.
We study the possibility of having a strong electric field (E) in Neutron Stars. We consider a system composed by a core of degenerate relativistic electrons, protons and neutrons, surrounded by an oppositely charged leptonic component and show that at the core surface it is possible to have values of E of the order of the critical value for electron-positron pair creation, depending on the mass density of the system. We also describe Neutron Stars in general relativity, considering a system composed by the core and an additional component: a crust of white dwarf - like material. We propose that, when the mass density of the star increases, the core undergoes the process of gravitational collapse to a black hole, leaving the crust as a remnant; we compare M crust with the mass of the baryonic remnant considered in the fireshell model of GRBs and find that their values are compatible.

Keywords: Neutron Stars; core; Crust; Electric Field

PACS: 26.60.Dd, 26.60.Gj, 03.75.Ss, 95.30.Sf

INTRODUCTION

The study of the electric field in compact stars, for example in Neutron Stars, is of great interest for different aspects, for example in the determination of the critical mass of these systems. The presence of a strong electric field at the surface of compact objects has been considered by different authors. For example, following the work of Alcock et al. [1], Huang & Lu [2] have shown that Quark Stars can develop an electric field of the order of 10^17 V cm^{-1} in the layer between the core surface (with the core composed by quarks bounded by strong interactions) and the thin layer of electrons, that screen the net positive charge of the core. We recall that also in Neutron Stars it is possible to have a strong electric field at the core surface [3], [4], [5] of the order or greater than the critical value

\[ E_c = \frac{m_e^2 c^3}{\hbar} \approx 10^{16} \text{V cm}^{-1} \]  

(1)

for electron–positron pair creation [6], depending on the mass density of the system. We also describe Neutron Stars in general relativity, considering a system composed by the core and an additional component: a crust of white dwarf - like material. We study the characteristics of the crust, in particular we calculate its mass M crust. We propose that, when the mass density of the star increases, the core undergoes the process of gravitational collapse to a black hole, leaving the crust as a remnant; we compare M crust with the mass of the baryonic remnant considered in the fireshell model of GRBs and find that their values are compatible.

THE THEORETICAL MODEL

We assume that Neutron Stars have a core composed by a plasma of degenerate relativistic electrons, protons and neutrons, surrounded by an oppositely charged leptonic component [3], [4], [5]. The number density of particles is given by

\[ n_i = \frac{(p_i^F)^3}{3\pi^2\hbar^3} \quad (i = e, p, n), \]  

(2)

where \( p_i^F \) is the Fermi momentum of particles \( i \). We assume that the proton number density is constant inside the core \( (r < R_c) \) and vanishes outside the core \( (r > R_c) \):

\[ n_p(r) = \left( \frac{3N_p}{4\pi R_c^3} \right) \theta(R_c - r), \quad R_c = \Delta \rho N_p^{1/3}, \]  

(3)

where \( N_p \) is the total number of protons, \( \Delta \rho \) is the pion Compton wavelength and \( \Delta \) is a free parameter that fixes the density; for example, for \( \Delta \approx 1 \) the mean mass density of the core is about nuclear density, \( \rho \approx \rho_0 = 2.314 \cdot 10^{14} \text{g cm}^{-3} \), by increasing \( \Delta \) the density decreases.

The electron number density is determined by the Fermi energy condition

\[ \varepsilon_e^F = \left( (p_e^F c)^2 + m_e^2 c^4 \right)^{1/2} - m_e c^2 - eV = 0, \]  

(4)

that is

\[ n_e(r) = \frac{1}{3\pi^2\hbar^3 c} \left[ \varepsilon_e^V(r) + 2m_e c^2 eV(r) \right]^{3/2}, \]  

(5)

where \( \varepsilon_e^V \) is the electron–positron pair creation energy and \( eV \) is the electrostatic potential.
where \( V \) is the Coulomb potential, which can be calculated from Poisson Equation
\[
\nabla^2 V = -4\pi e [n_e(r) - n_i(r)].
\]

Introducing the dimensionless variable \( x = r\lambda^{-1}_c \) and the function \( \chi \), defined as
\[
\frac{\chi}{r} = \frac{eV}{\hbar c}
\]
we obtain the Relativistic Thomas–Fermi equation
\[
\frac{1}{3x} \frac{d^2 \chi}{dx^2} = -\alpha \left\{ \frac{1}{\Delta^2} \theta(x_t - x) - \frac{4}{9\pi^3} \left[ \frac{\chi^2}{x^3} + \frac{2m\chi}{m\pi x} \right]^{3/2} \right\}
\]
where \( \alpha = e^2 / (\hbar c) \) and \( \chi(0) = 0, \chi(\infty) = 0 \). If we consider the case of ultrarelativistic electrons, introducing the new function \( \hat{\phi} \) and the variables \( \hat{x} \) and \( \xi \), defined by
\[
\hat{\phi} = \Delta \left[ \frac{4}{9\pi^3} \right]^{1/3} \frac{\chi}{\hat{x}},
\]
\[
\hat{x} = (12/\pi)^{1/6} \sqrt{\Delta} \xi x, \quad \xi = \hat{x} - \hat{x}_c
\]
eq (8) becomes
\[
\frac{d^2 \hat{\phi}(\xi)}{d\xi^2} = -\theta(-\xi) + \hat{\phi}(\xi)^3,
\]
which has the analytical solution
\[
\begin{align*}
1 - 3 \left[ 1 + 2^{-1/2} \sinh(a - \sqrt{3}b) \right]^{-1}, & \quad \xi < 0, \\
\sqrt{3}b, & \quad \xi > 0,
\end{align*}
\]
with \( \sinh a = 11 \sqrt{3}, \quad b = (4/3) \sqrt{3} \).

The neutron number density can be obtained from the condition of \( \beta \)-equilibrium between direct and inverse \( \beta \) decay
\[
e^x_e(r) + e^x_p(r) = e^x_e(r),
\]
which can be explicitly written as
\[
\sqrt{(p_e^x c)^2 + m_e^2 c^4} - m_n c^2 = \sqrt{(p_p^x c)^2 + m_n^2 c^4} - m_p c^2 + eV.
\]
Eq. (14) is related to the proton and neutron number densities by eq. (2).

**The electric field**

Using eq. (12) we obtain an analytical expression for the electric field [4]:
\[
E(\xi) = \left( \frac{3\pi^4}{4} \right)^{1/6} \left( \frac{m_e^2 c^4}{\Delta x} \right)^{3/2} \hat{\phi}(\xi).
\]

The maximum value of the electric field \( E_{\text{max}} \), reached at the surface of the core, has the following expression:
\[
E_{\text{max}} \approx 0.95 \sqrt{\frac{1}{\alpha \Delta} \frac{m_e^2 c^3}{e\hbar}}.
\]

The value of \( E_{\text{max}} \) is function only of the pion mass \( m_\pi \), the parameter \( \Delta \) and the fine structure constant; in particular it increases by decreasing \( \Delta \), that means by increasing the mass density of the system (see fig. 1).

**THE GENERAL RELATIVISTIC MODEL**

We describe Neutron Stars within general relativity considering a system composed by the core and an additional component surrounding it: a crust of white-dwarf like material. The internal structure of the system is described by the Tolman-Oppenheimer-Volkoff (TOV) equation
\[
\frac{dP}{dr} = -G \left( \rho + \frac{P}{\gamma} \right) \left( m + \frac{4\pi r^3 P}{3} \right),
\]
which can be explicitly written as
\[
\frac{dm}{dr} = 4\pi r^2 \rho.
\]
To determine \( M_{\text{rms}} \) we integrate eq. (17) and (18) from
$r = R_c$, where $R_c$ is the core radius, together with the equations

$$P \approx P_e,$$  \hspace{1cm} (19)

$$\rho \approx \mu_e \rho_n n_e.$$  \hspace{1cm} (20)

$P_e$ is the pressure of electrons, given by [10]

$$P_e = \frac{n_e e^2}{\pi \hbar^2},$$  \hspace{1cm} (21)

where

$$\xi_e = \sqrt{1+x_e^2}$$  \hspace{1cm} (22)

$$\phi_e = \frac{2}{3} \xi_e^2 - 1 \sqrt{\xi^2_e - 1} + \log \left( \xi_e + \sqrt{\xi^2 - 1} \right),$$  \hspace{1cm} (23)

$$P_e = k_e \phi_e,$$  \hspace{1cm} (24)

with $\xi_e = \sqrt{1+x_e^2}$ and $x_e$ the Fermi momentum of electrons normalized to $(m_e c)$. $\mu_e$ is the mean molecular weight per electron that, for a completely ionized gas, is equal to $A/Z$ (for simplicity, we assume $\mu_e = 2$), $m_n$ is the mass of neutrons and $n_e$ is the number density of electrons (eq. 2). In eq. (20) we have assumed the local charge neutrality of the system.

**RESULTS - THE MASS OF THE CRUST**

We have integrated eq. (17) and (18) for different sets of initial conditions; in fig. 2 are shown the results obtained assuming

$$10 \text{ km} \leq R_c \leq 20 \text{ km},$$

$$1 \text{ M}_\odot \leq M_c \leq 3 \text{ M}_\odot$$

and an initial pressure equal to $1.58 \times 10^{30} \text{ dyne cm}^{-2}$, that corresponds to a mass density equal to the neutron drip density $\rho_{\text{drip}} \approx 4.3 \times 10^{13} \text{ g cm}^{-3}$ [10].

It can be seen that $M_{\text{crust}}$ has values ranging from $10^{-6} \text{ M}_\odot$ to $10^{-3} \text{ M}_\odot$, and increases by increasing $R_c$ and decreasing $M_c$. It’s important to note that the values estimated for $M_{\text{crust}}$ strongly depend on the values of $M_c$ and $R_c$ used; in particular, the values of $M_c$ considered are greater than the maximum mass calculated for neutron stars with a core of degenerate relativistic electrons, protons and neutrons in local charge neutrality ($M_{\text{max}} = 0.7 \text{ M}_\odot$ [12]). The outstanding theoretical problem to address is to identify the physical forces influencing such a strong departure; the two obvious candidate are the electromagnetic structure in the core and/or the strong interactions.

**A COMPARISON WITH $M_B$**

We propose that in the process of gravitational collapse of the core the crust is left as a remnant and compare $M_{\text{crust}}$ with the mass of the baryonic remnant considered in the Fireshell Model of GRBs.

**The Fireshell Model of GRBs**

In the Fireshell Model [7] GRBs are generated by the gravitational collapse of the star progenitor to a charged black hole. The electron–positron plasma created in the process of black hole (BH) formation expands as a spherically symmetric “fireshell”. It evolves and encounters the baryonic remnant of the newly formed BH, then is loaded with baryons and expands until the transparency condition is reached and the Proper - GRB is emitted. The afterglow emission starts due to the collision between the remaining optically thin fireshell and the CircumBurst Medium. A schematization of the model is shown in fig. 3. The baryon loading is measured by the dimensionless...
TABLE 1. GRBs and correspondent values of $M_B$ used to reproduce the observed data within the Fireshell Model [11], in units of solar masses.

<table>
<thead>
<tr>
<th>GRB</th>
<th>$M_B/M_\odot$</th>
</tr>
</thead>
<tbody>
<tr>
<td>970228</td>
<td>$5.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>050315</td>
<td>$4.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>061007</td>
<td>$1.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>991216</td>
<td>$7.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>011121</td>
<td>$9.4 \times 10^{-5}$</td>
</tr>
<tr>
<td>030329</td>
<td>$5.7 \times 10^{-5}$</td>
</tr>
<tr>
<td>060614</td>
<td>$4.6 \times 10^{-6}$</td>
</tr>
<tr>
<td>060218</td>
<td>$1.3 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

where $M_B$ is the mass of the baryonic remnant and $E_{dy}$ is the energy of the dyadosphere, the region outside the horizon of a BH where the electric field is of the order of the critical value (eq. 1). $B$ and $E_{dy}$ are the two free parameters of the model.

The mass of the crust and $M_B$

Using the values of $B$ and $E_{dy}$ constrained by the observational data of several GRBs and eq. (25), we have obtained the correspondent values of $M_B$ (see table 1, [11]). It can be seen that these values are compatible with the ones of $M_{crust}$.

CONCLUSIONS

We have studied the possibility of having a strong electric field in Neutron Stars considering a system composed by a core of degenerate relativistic electrons, protons and neutrons, surrounded by an oppositely charged leptonic component; we have shown that at the core surface it is possible to have values of $E$ of the order of the critical value for electron-positron pair creation, depending on the mass density of the system. We have also described Neutron Stars in general relativity, considering a system composed by the core and an additional component: a crust of white dwarf like material. We have studied the characteristics of the crust, in particular we have calculated its mass. We have proposed that, when the mass density increases, the core undergoes the process of gravitational collapse to a black hole, leaving the crust as a remnant; we have compared $M_{crust}$ with the mass of the baryonic remnant considered in the Fireshell Model of GRBs [7], finding that their values are compatible.

REFERENCES

4. R. Ruffini, M. Rotondo and S. S. Xue, Submitted to PRL
7. R. Ruffini et al., to appear on the Proceedings of the Eleventh Marcel Grossmann Meeting, Berlin (Germany), July 2006
11. M. G. Dainotti et al., in preparation
Attachment 7
On Gamma-Ray Bursts

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We show by example how the uncoding of Gamma-Ray Bursts (GRBs) offers unprecedented possibilities to foster new knowledge in fundamental physics and in astrophysics.

After recalling some of the classic work on vacuum polarization in uniform electric fields by Klein, Sauter, Heisenberg, Euler and Schwinger, we summarize some of the efforts to observe these effects in heavy ions and high energy ion collisions. We then turn to the theory of vacuum polarization around a Kerr-Newman black hole, leading to the extraction of the blackhole’s energy, to the concept of dyadosphere and dyadotorus, and to the creation of an electron-positron-photon plasma. We then present a new theoretical approach encompassing the physics of neutral stars and heavy nuclei. It is shown that configurations of nuclear matter in bulk with global charge neutrality can exist on macroscopic scales and with electric fields close to the critical value near their surfaces. These configurations may represent an initial condition for the process of gravitational collapse, leading to the creation of an electron-positron-photon plasma: the basic self-accelerating system explaining both the energetics and the high energy Lorentz factor observed in GRBs.

We then turn to the special role of the baryon loading in discriminating between “genuine” short and long or “fake” short GRBs and to the special role of the GRB-Supernova Time Sequence (GSTS) paradigm: the concept of induced gravitational collapse. We illustrate this paradigm by the systems GRB 980425 / SN 1998bw, GRB 030329 / SN 2003dh, GRB 031203 / SN 2003lw, GRB 060218 / SN 2006aj, and we present the enigma of the URCA sources. We then present some general conclusions.
1. Introduction

After almost a century of possible observational evidences of general relativistic effects, all very weak and almost marginal to the field of physics, the direct observation of gravitational collapse and of black hole formation promises to bring the field of general relativity into the mainstream of fundamental physics, testing a vast arena of unexplored regimes and leading to the explanation of a large number of yet unsolved astrophysical problems.

There are two alternative procedures for observing the process of gravitational collapse: either by gravitational waves or by joint observation of electromagnetic radiation and high energy particles. The gravitational wave observations may lead to the understanding of the global properties of the gravitational collapse process, inferred from the time-varying component of the global quadrupole moment of the system. Their observation is also made difficult and at times marginal due to the weak coupling of gravitational waves with the detectors. The observation of the electromagnetic radiation in the X, γ, optical and radio bands, and of the associated high energy particle component, is carried out by an unprecedented observational effort in space, ground and underground observatories. This effort is offering the possibility of giving for the first time a detailed description of the gravitational collapse process all the way to the formation of the black hole horizon.

The Grossmann Meetings have been dedicated to foster the mathematical and physical developments of Einstein theories. They have grown in recent years due to remarkable progress both in fundamental physics and in astrophysics. In this sense, I will present here some highlights of recent progress on the theoretical understanding of Gamma-Ray Bursts (GRBs, for a review see e.g. Ref. 1 and references therein), which nicely represents two complementary aspects of the problem: progress in probing fundamental theories in yet unexplored regimes, as well as understanding the astrophysical scenario underlying novel astrophysical phenomena.

It is particularly inspiring that this MG11 takes place in Dahlem, close to where many of the fundamental breakthroughs and discoveries of modern physics have indeed occurred. A few hundred meters from here, in Ehrenbergstrasse 33, Albert Einstein once lived while introducing and developing the theory of general relativity (see Fig. 1).

A few hundred meters from here there is also the Kaiser-Wilhelm-Institut, where Lise Meitner, Otto Hahn and Fritz Strassmann (see Fig. 2) continued the work on neutron capture by Uranium initiated a few years before by Fermi.\(^2\) This work led to the revolutionary evidence for Uranium fission.\(^3,4\) There is no need to stress the enormous consequences of this discovery following the work of Fermi,\(^5\) Feynman, Metropolis & Teller,\(^6\) Oppenheimer,\(^7\) Wigner,\(^8\) etc., and, just to keep symmetry, the work of Kurchatov, Sakharov, and Zel’dovich (see e.g. Ref. 9). The message from the nuclear and thermonuclear work leads to nuclear reactors and to explosive events typically of \(\sim 10^{22}\) ergs/pulse (see Fig. 2).

What I would like to stress in this lecture is the possible role of a theoretical
work, coeval to the work of Otto Hahn and Lise Meitner, developed close to Dahlem, at the University of Leipzig. Such a theoretical work may very well lead in the future to the understanding of yet bigger explosions and have an equally, if not more, fundamental role on the existence and the dynamics of our Universe. We refer here to the work pioneered by Klein, Sauter, Euler and Heisenberg, Weisskopf (see Fig. 3). The work deals with the creation of electron-positron pairs out of the vacuum generated by an overcritical electric field. In the following we give evidences that this process is indeed essential to the extraction of energy from the black hole and occurs in the GRBs. The characteristic energy of these sources is typically on the order of $10^{49} - 10^{54}$ ergs/pulse. I am giving here, as examples of these GRBs, the light curves of GRB 980425 (with a total energy of $\sim 10^{48}$ ergs) and GRB 050315 (with a total energy of $\sim 10^{53}$ ergs).

So much for fundamental science. From the astrophysical point of view, GRBs offers an equally rich scenario, being linked to processes like supernovae, coalescence of binary neutron stars, black holes and binaries in globular clusters, and possibly intermediate mass black holes. After reviewing some of the fundamental work on vacuum polarization, I will focus on a new class of 4 particularly weak GRBs which promises to clarify the special connection between GRBs and supernovae. I will then
conclude on the very surprising aspect that GRBs can indeed originate from a very wide variety of different astrophysical sources, but their features can be explained within a unified theoretical model, which applies in the above-mentioned enormous range of energies. The reason of this “uniqueness” of the GRBs is strictly linked to the late phases of gravitational collapse leading to the formation of the black hole and to its theoretically expected “uniqueness”: the black hole is uniquely characterized by mass, charge and angular momentum. The GRB phenomenon originates in the late phases of the process of gravitational collapse, when the formation of the horizon of the black hole is being reached. The phenomenon is therefore quite independent of the different astrophysical settings giving origin to the black hole formation.

2. Vacuum polarization in a uniform electric field

We recall some early work on pair creation following the introduction by Dirac of the relativistic field equation for the electron and leading to the classical results of Klein, Heisenberg & Euler, Schwinger. 

2.1. Klein and Sauter work

It is well known that every relativistic wave equation of a free relativistic particle of mass $m_e$, momentum $p$ and energy $E$, admits symmetrically “positive energy”
and "negative energy" solutions. Namely the wave-function
\[
\psi^\pm(x, t) \sim e^{i(k \cdot x - \xi \pm t)}
\]
describes a relativistic particle, whose energy, mass and momentum must satisfy,
\[
\xi^2 = m^2c^4 + c^2|p|^2; \quad \xi = \pm \sqrt{m^2c^4 + c^2|p|^2},
\]
this gives rise to the familiar positive and negative energy spectrum (\(\xi^\pm\)) of positive and negative energy states (\(\psi^\pm(x, t)\)) of the relativistic particle, as represented in Fig. 4. In such free particle situation (flat space, no external field), all the quantum states are stable; that is, there is no possibility of “positive” (“negative”) energy states decaying into a “negative” (“positive”) energy states, since all negative energy states are fully filled and there is an energy gap 2mc^2 separating the negative energy spectrum from the positive energy spectrum. This is the view of Dirac theory on the spectrum of a relativistic particle.\(^{20,21}\)

Klein studied a relativistic particle moving in an external constant potential \(V\) and in this case Eq. (2) is modified as
\[
(\xi - V)^2 = m^2c^4 + c^2|p|^2; \quad \xi = V \pm \sqrt{m^2c^4 + c^2|p|^2}.
\]
He solved this relativistic wave equation by considering an incident free relativistic wave of positive energy states scattered by the constant potential $V$, leading to reflected and transmitted waves. He found a paradox that in the case $V \geq \mathcal{E} + m_e c^2$, the reflected flux is larger than the incident flux $j_{\text{ref}} > j_{\text{inc}}$, although the total flux is conserved, i.e., $j_{\text{inc}} = j_{\text{ref}} + j_{\text{tran}}$. This was known as the Klein paradox (see Ref. 10). This implies that negative energy states have contributions to both the transmitted flux $j_{\text{tran}}$ and reflected flux $j_{\text{ref}}$.

Fig. 4. The mass-gap $2m_e c^2$ that separates the positive continuum spectrum $\mathcal{E}_+$ from the negative continuum spectrum $\mathcal{E}_-$. 

Sauter studied this problem by considering an electric potential of an external constant electric field $E$ in the $\hat{z}$ direction.\textsuperscript{11} In this case the energy $\mathcal{E}$ is shifted by the amount $V(z) = -eEz$, where $e$ is the electron charge. In the case of the electric field $E$ uniform between $z_1$ and $z_2$ and null outside, Fig. 5 represents the
corresponding sketch of allowed states. The key point now, which is the essence of the Klein paradox, is that the above mentioned stability of the “positive energy” states is lost for sufficiently strong electric fields. The same is true for “negative energy” states. Some “positive energy” and “negative energy” states have the same energy-levels, i.e., the crossing of energy-levels occurs. Thus, these “negative energy” waves incident from the left will be both reflected back by the electric field and partly transmitted to the right as a “positive energy” wave, as shown in Fig. 5. This transmission is nothing else but a quantum tunneling of the wave function through the electric potential barrier, where classical states are forbidden. This is the same as the so-called the Gamow tunneling of the wave function through nuclear potential barrier (Gamow-wall).

Sauter first solved the relativistic Dirac equation in the presence of the constant electric field by the ansatz,

\[ \psi_s(x, t) = e^{i(k_xx + k_yy - E_z t)} \chi_{s3}(z) \]  

Where the spinor function \( \chi_{s3}(z) \) obeys the following equation (\( \gamma_0, \gamma_i \) are Dirac matrices)

\[ \left[ i\hbar c \gamma_i \frac{d}{dz} + \gamma_0 (V(z) - E_{\pm}) + (mc^2 + ic\gamma_2 p_y + ic\gamma_1 p_x) \right] \chi_{s3}(z) = 0, \]
and the solution $\chi_s(z)$ can be expressed in terms of hypergeometric functions.\textsuperscript{11} Using this wave-function $\psi_s(x,t)$ (4) and the flux $i\epsilon\chi_s^\dagger \gamma_3 \psi_s$, Sauter computed the transmitted flux of positive energy states, the incident and reflected fluxes of negative energy states, as well as exponential decaying flux of classically forbidden states, as indicated in Fig. 5. Using continuous conditions of wave functions and fluxes at boundaries of the potential, Sauter found that the transmission coefficient $|T|^2$ of the wave through the electric potential barrier from the negative energy state to positive energy states:

$$|T|^2 = \frac{\text{transmission flux}}{\text{incident flux}} \sim e^{-\frac{2\pi m^2 e^2 c^3}{\hbar^2 E}}.$$  \hspace{1cm} (6)

This is the probability of negative energy states decaying to positive energy states, caused by an external electric field. The method that Sauter adopted to calculate the transmission coefficient $|T|^2$ is the same as the one Gamow used at that time to calculate quantum tunneling of the wave function through nuclear potential barrier (Gamow-wall), leading to the $\alpha$-particle emission.\textsuperscript{23}

\section*{2.2. Heisenberg-Euler-Weisskopf effective theory}

To be able to explain elastic light-light scattering,\textsuperscript{12} Heisenberg and Euler\textsuperscript{13} and Weisskopf\textsuperscript{14,15} proposed a theory that attributes to the vacuum certain nonlinear electromagnetic properties, as if it were a dielectric and permeable medium.\textsuperscript{13,14}

Let $L$ to be the Lagrangian density of electromagnetic fields $E, B$, a Legendre transformation produces the Hamiltonian density:

$$\mathcal{H} = E_i \delta L/\delta E^i - L.$$ \hspace{1cm} (7)

In Maxwell’s theory, the two densities are given by

$$L_M = \frac{1}{8\pi} (E^2 - B^2), \quad \mathcal{H}_M = \frac{1}{8\pi} (E^2 + B^2).$$ \hspace{1cm} (8)

To quantitatively describe nonlinear electromagnetic properties of the vacuum based on the Dirac theory, the above authors introduced the concept of an effective Lagrangian $L_{\text{eff}}$ of the vacuum state in the presence of electromagnetic fields, and an associated Hamiltonian density

$$L_{\text{eff}} = L_M + \Delta L, \quad \mathcal{H}_{\text{eff}} = \mathcal{H}_M + \Delta \mathcal{H}.$$ \hspace{1cm} (9)

From these one derives induced fields $D, H$ as the derivatives

$$D_i = 4\pi \delta L_{\text{eff}}/\delta E^i, \quad H_i = -4\pi \delta L_{\text{eff}}/\delta B^i.$$ \hspace{1cm} (10)

In Maxwell’s theory, $\Delta L \equiv 0$ in the vacuum, so that $D = E$ and $H = B$. In Dirac’s theory, however, $\Delta L$ is a complex function of $E$ and $B$. Correspondingly, the vacuum behaves as a dielectric and permeable medium\textsuperscript{13,14} in which,

$$D_i = \sum_k \epsilon_{ik} E_k, \quad H_i = \sum_k \mu_{ik} B_k.$$ \hspace{1cm} (11)
where complex $\epsilon_{ik}$ and $\mu_{ik}$ are the field-dependent dielectric and permeability tensors of the vacuum.

The discussions on complex dielectric and permeability tensors ($\epsilon_{ik}$ and $\mu_{ik}$) can be found for example in Ref. 24. The effective Lagrangian and Hamiltonian densities in such a medium is given by,

$$\mathcal{L}_{\text{eff}} = \frac{1}{8\pi}(E \cdot D - B \cdot H), \quad \mathcal{H}_{\text{eff}} = \frac{1}{8\pi}(E \cdot D + B \cdot H). \tag{12}$$

In this medium, the conservation of electromagnetic energy has the form

$$-\text{div} S = \frac{1}{4\pi} \left( E \frac{\partial D}{\partial t} + B \frac{\partial H}{\partial t} \right), \quad S = \frac{c}{4\pi} E \times B, \tag{13}$$

where $S$ is the Poynting vector describing the density of electromagnetic energy flux. By considering electromagnetic fields complex and monochromatic

$$E = E(\omega) \exp(-i\omega t); \quad B = B(\omega) \exp(-i\omega t), \tag{14}$$

of frequency $\omega$, the dielectric and permeability tensors are frequency-dependent, i.e., $\epsilon_{ik}(\omega)$ and $\mu_{ik}(\omega)$, Substituting these fields and tensors into the r.h.s. of Eq. (13), one obtains the dissipation of electromagnetic energy per time into the medium,

$$Q = \frac{\omega}{8\pi} \left\{ \text{Im} \left[ \epsilon_{ik}(\omega) \right] E_i E_k^* + \text{Im} \left[ \mu_{ik}(\omega) \right] B_i B_k^* \right\}. \tag{15}$$

This is nonzero if $\epsilon_{ik}(\omega)$ and $\mu_{ik}(\omega)$ contain an imaginary part. The dissipation of electromagnetic energy is accompanied by heat production. In light of the third thermodynamical law of entropy increase, the energy $Q$ of electromagnetic fields lost in the medium is always positive, i.e., $Q > 0$. As a consequence, $\text{Im}[\epsilon_{ik}(\omega)] > 0$ and $\text{Im}[\mu_{ik}(\omega)] > 0$. The real parts of $\epsilon_{ik}(\omega)$ and $\mu_{ik}(\omega)$ represent an electric and magnetic polarizability of the vacuum and lead, for example, to the refraction of light in an electromagnetic field, or to the elastic scattering of light from light.\(^{12}\) The $n_{ij}(\omega) = \sqrt{\epsilon_{ik}(\omega)\mu_{kj}(\omega)}$ is the reflection index of the medium. The field-dependence of $\epsilon_{ik}$ and $\mu_{ik}$ implies nonlinear electromagnetic properties of the vacuum as a dielectric and permeable medium.

The effective Lagrangian density ($9$) is a relativistically invariant function of the field strengths $E$ and $B$. Since $(E^2 - B^2)$ and $(E \cdot B)^2$ are relativistic invariants, one can formally expand $\Delta \mathcal{L}$ in powers of weak field strengths:

$$\Delta L = \kappa_{20}(E^2 - B^2)^2 + \kappa_{16}(E \cdot B)^2 + \kappa_{16}(E^2 - B^2)^2 + \kappa_{12}(E^2 - B^2)(E \cdot B)^2 + \ldots, \tag{16}$$

where $\kappa_{ij}$ are field-independent constants whose subscripts indicate the powers of $(E^2 - B^2)$ and $E \cdot B$, respectively. Note that the invariant $E \cdot B$ appears only in even powers since it is odd under parity and electromagnetism is parity invariant. The Lagrangian density (16) corresponds, via relation (7), to

$$\Delta H = \kappa_{2,0}(E^2 - B^2)(3E^2 + B^2)^2 + \kappa_{1,2}(E \cdot B)^2$$

$$+ \kappa_{3,0}(E^2 - B^2)^3(3E^2 + B^2)^2 + \kappa_{1,2}(3E^2 - B^2)(E \cdot B)^2 + \ldots. \tag{17}$$
To obtain $\mathcal{H}_{\text{eff}}$ in Dirac’s theory, one has to calculate
\[ \Delta \mathcal{H} = \sum_k \left\{ \psi_k^* \left( \alpha \cdot (-i\hbar \nabla + eA) + \beta m_e c^2 \right) \psi_k \right\}, \] (18)
where $\alpha, \beta$ are Dirac matrices, $A$ is the vector potential, and $\{\psi_k(x)\}$ are the wave functions of the occupied negative-energy states. When performing the sum, one encounters infinities which were removed by Weisskopf.\textsuperscript{14,15} Dirac,\textsuperscript{26} Heisenberg\textsuperscript{26} by a suitable subtraction.

Heisenberg\textsuperscript{26} expressed the Hamiltonian density in terms of the density matrix $\rho(x, x') = \sum_k \psi_k^*(x)\psi_k(x')$.\textsuperscript{25} Euler and Kockel,\textsuperscript{12} and Heisenberg and Euler\textsuperscript{13} calculated the coefficients $\kappa_{ij}$. They did so by solving the Dirac equation in the presence of parallel electric and magnetic fields $E$ and $B$ in a specific direction,
\[ \psi_k(x) \rightarrow \psi_{p_n,n,k} \equiv e^{\frac{i}{\hbar c} (p_n - E)t} u_n(y) \chi_n(x), \quad n = 0, 1, 2, \ldots \] (19)
where $\{u_n(y)\}$ are the Landau states\textsuperscript{27,28} depending on the magnetic field and $\chi_n(x)$ are the spinor functions calculated by Ref. 11. Heisenberg and Euler used the Euler-Maclaurin formula to perform the sum over $n$, and obtained for the additional Lagrangian in (9) the integral representation,
\[ \Delta \mathcal{L}_{\text{eff}} = \frac{e^2}{16\pi^2\hbar c} \int_0^{\infty} e^{-s} s^3 \left[ i s^2 E \bar{B} \frac{\cos(\sqrt{s}E^2 - B^2) + 2i(\bar{E}B)^{1/2}}{\cos(\sqrt{s}E^2 - B^2) + 2i(\bar{E}B)^{1/2}} + \text{c.c.} \right] \]
\[ + \left( \frac{m_e c}{\hbar} \right)^2 \left[ \frac{2}{3} (|B|^2 - |\bar{E}|^2) \right], \] (20)
where $\bar{E}, \bar{B}$ are the dimensionless reduced fields in the unit of the critical field $E_c$,
\[ \bar{E} = |E|/E_c, \quad \bar{B} = |B|/E_c; \quad E_c \equiv \frac{m_e c^3}{\hbar}. \] (21)
Expanding this in powers of $\alpha$ up to $\alpha^3$ yields the following values for the four constants:
\[ \kappa_{2,0} = \frac{\alpha}{90\pi^2 E_c^2}, \quad \kappa_{0,2} = 7\kappa_{2,0}, \quad \kappa_{3,0} = \frac{32\pi\alpha}{315} E_c^{-2}, \quad \kappa_{1,2} = \frac{13}{2} \kappa_{3,0}. \] (22)
Weisskopf\textsuperscript{14} adopted a simpler method. He considered first the special case in which $E = 0, B \neq 0$ and used the Landau states to find $\Delta \mathcal{H}$ of Eq. (17), extracting from this $\kappa_{2,0}$ and $\kappa_{3,0}$. Then he added a weak electric field $E \neq 0$ to calculate perturbatively its contributions to $\Delta \mathcal{H}$ in the Born approximation (see for example Landau and Lifshitz\textsuperscript{27,28}). This led again to the coefficients (22).

The above results receive higher corrections in QED and are correct only up to order $\alpha^2$. Up to this order, the field-dependent dielectric and permeability tensors $\epsilon_{ik}$ and $\mu_{ik}$ (11) have the following real parts for weak fields
\[ \text{Re}(\epsilon_{ik}) = \delta_{ik} + \frac{4\alpha}{45} \left[ 2 (\bar{E}^2 - B^2) \delta_{ik} + 7 E_i B_k \right] + \mathcal{O}(\alpha^2), \]
\[ \text{Re}(\mu_{ik}) = \delta_{ik} + \frac{4\alpha}{45} \left[ 2 (\bar{E}^2 - B^2) \delta_{ik} + 7 E_i B_k \right] + \mathcal{O}(\alpha^2). \] (23)
2.3. Imaginary part of the effective Lagrangian

Heisenberg and Euler\textsuperscript{13} were the first to realize that for \( E \neq 0 \) the powers series expansion (16) is not convergent, due to singularities of the integrand in (20) at \( s = \pi/E, 2\pi/E, \ldots \). They concluded that the powers series expansion (16) does not yield all corrections to the Maxwell Lagrangian, calling for a more careful evaluation of the integral representation, see Eq. (20). Selecting an integration path that avoids these singularities, they found an imaginary term. Motivated by Sauter’s work\textsuperscript{11} on Klein paradox,\textsuperscript{10} Heisenberg and Euler estimated the size of the imaginary term in the effective Lagrangian as

\[
\sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left(-\frac{n\pi E}{E} \right),
\]

and pointed out that it is associated with pair production by the electric field. This imaginary term in the effective Lagrangian is related to the imaginary parts of field-dependent dielectric \( \varepsilon \) and permeability \( \mu \) of the vacuum.

In 1950’s, Schwinger\textsuperscript{17–19} derived the same formula (20) once more within the quantum field theory of Quantum Electromagnetics (QED),

\[
\sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left(-\frac{n\pi E}{E} \right),
\]

and its Lorentz-invariant expression in terms of electromagnetic fields \( E \) and \( B \),

\[
\sum_{n=1}^{\infty} \frac{1}{n^2} \tanh \left(\frac{n\pi \beta}{\varepsilon} \right) \exp \left(-\frac{n\pi E}{E} \right),
\]

where

\[
\left\{ \frac{\varepsilon}{\beta} \right\} \equiv \frac{1}{\sqrt{2}} \sqrt{(E^2 - B^2)^2 + 4(E \cdot B)^2} \approx (E^2 - B^2).
\]

The exponential factor \( e^{-\pi E/E} \) in Eqs. (6) and (24) characterizes the transmission coefficient of quantum tunneling, Ref. 13 introduced the critical field strength \( E_c = m^2 e^2 c^3/h \) (21). They compared it with the field strength \( E_0 \) of an electron at its classical radius, \( E_0 = e/r_e^2 \) where \( r_e = \alpha h/(m_e c) \) and \( \alpha = 1/137 \). They found the field strength \( E_0 \) is 137 time larger than the critical field strength \( E_c \), i.e., \( E_0 = \alpha^{-1} E_c \).

At a critical radius \( r_e \), the field strength of the electron would be equal to the critical field strength \( E_c \).

As shown in Fig. 4, the negative-energy spectrum of solutions of the Dirac equation has energies \( E_- < -m_e c^2 \), and is separated from the positive energy-spectrum \( E_+ > m_e c^2 \) by a gap \( 2m_e c^2 \approx 1.02 \text{MeV} \). The negative-energy states are all filled. The energy gap is by a factor \( 4/\alpha^2 \approx 10^8 \) larger than the typical binding energy of atoms (\( \sim 13.6 \text{eV} \)). In order to create an electron-positron pair, one must spend this large amount of energy. The source of this energy can be an external field.

If an electric field attempts to tear an electron out of the filled state the gap energy must be gained over the distance of two electron radii. The virtual particles
give an electron a radius of the order of the Compton wavelength \( \lambda \equiv \frac{\hbar}{m_e c} \). Thus we expect a significant creation of electron-positron pairs if the work done by the electric field \( E \) over twice the Compton wave length \( \frac{\hbar}{m_e c} \) is larger than \( 2m_e c^2 \)

\[
e E \left( \frac{2\hbar}{m_e c} \right) > 2m_e c^2.
\]

This condition defines a critical electric field

\[
E_c \equiv \frac{m_e^2 c^3}{\alpha \hbar^2} \approx 1.3 \cdot 10^{16} \text{ V/cm}, \tag{28}
\]

above which pair creation becomes abundant. To have an idea how large this critical electric field is, we compare it with the value of the electric field required to ionize a hydrogen atom. There the above inequality holds for twice of the Bohr radius and the Rydberg energy

\[
e E_{\text{ion}} \left( \frac{2\hbar}{m_e c} \right) > \alpha^2 m_e c^2,
\]

so that \( E_c \approx E_{\text{ion}}^m/\alpha^3 \) is about \( 10^6 \) times as large, a value that has so far not been reached in a laboratory on Earth.

3. Pair production in Coulomb potential of nuclei and heavy-ion collisions

By far the major attention to build a critical electric field has occurred in the physics of heavy nuclei and in heavy ion collisions. We recall in the following some of the basic ideas, calculations, as well as experimental attempts to obtain the pair creation process in nuclear physics.

3.1. The \( Z = 137 \) catastrophe

Soon after the Dirac equation for a relativistic electron was discovered,\textsuperscript{29–31} Gordon\textsuperscript{32} (for all \( Z < 137 \)) and Darwin\textsuperscript{33} (for \( Z = 1 \)) found its solution in the point-like Coulomb potential \( V(r) = -Ze/r, \quad 0 < r < \infty \). Solving the differential equations for the Dirac wave function, they obtained the well-known Sommerfeld’s formula\textsuperscript{34} for the energy-spectrum,

\[
E(n,j) = m_e c^2 \left[ 1 + \left( \frac{Z\alpha}{n - |K| + (K^2 - Z^2\alpha^2)^{1/2}} \right)^2 \right]^{-1/2}. \tag{29}
\]

Here the principle quantum number \( n = 1, 2, 3, \ldots \) and

\[
K = \begin{cases} 
-(j + 1/2) = -(l + 1), & j = l + \frac{1}{2}, \quad l \geq 0 \\
(j + 1/2) = l, & j = l - \frac{1}{2}, \quad l \geq 1
\end{cases}
\]

where \( l = 0, 1, 2, \ldots \) is the orbital angular momentum corresponding to the upper component of Dirac bi-spinor, \( j \) is the total angular momentum, and the states with \( K = \pm 1, \pm 2, \pm 3, \cdots, \pm (n - 1) \) are doubly degenerate, while the state \( K = -n \) is
The integer values $n$ and $K$ label bound states whose energies are $E(n,j) \in (0, m_c^2)$. For the example, in the case of the lowest energy states, one has

$$E(1S_{1/2}) = \sqrt{1 - (\alpha Z)^2},$$  \hspace{1cm} (31)

$$E(2S_{1/2}) = E(2P_{1/2}) = \sqrt{\frac{1 + \sqrt{1 - (\alpha Z)^2}}{2}},$$  \hspace{1cm} (32)

$$E(2P_{3/2}) = \sqrt{1 - \frac{1}{4} (\alpha Z)^2}.$$  \hspace{1cm} (33)

For all states of the discrete spectrum, the binding energy $m_c^2 - E(n,j)$ increases as the nuclear charge $Z$ increases, as shown in Fig. 6. When $Z = 137$, $E(1S_{1/2}) = 0$, $E(2S_{1/2}) = E(2P_{1/2}) = 1/\sqrt{2}$ and $E(2S_{3/2}) = \sqrt{3}/2$. Gordon noticed in his pioneer paper that no regular solutions with $n = 1, j = 1/2, l = 0$, and $K = -1$ (the $1S_{1/2}$ ground state) are found beyond $Z = 137$. This phenomenon is the so-called “$Z = 137$ catastrophe” and it is associated with the assumption that the nucleus is point-like in calculating the electronic energy-spectrum.

Fig. 6. Atomic binding energies as function of nuclear charge $Z$. This figure is reproduced from Fig. 1 in Ref. 35.
3.2. Semi-Classical description

In order to have further understanding of this phenomenon, we study it in the semi-classical scenario. Setting the origin of spherical coordinates \((r, \theta, \phi)\) at the point-like charge, we introduce the vector potential \(A_\mu = (A_r, A_0)\), where \(A_r = 0\) and \(A_0\) is the Coulomb potential. The motion of a relativistic “electron” (scalar particle) with mass \(m\) and charge \(e\) is described by its radial momentum \(p_r\), angular momenta \(p_\phi\) and the Hamiltonian,

\[
H_\pm = \pm mc^2 \sqrt{1 + \left(\frac{p_r}{mc}\right)^2 + \left(\frac{p_\phi}{mc}\right)^2} - V(r),
\]

(34)

where the potential energy \(V(r) = eA_0\), and \(\pm\) corresponds for positive and negative solutions. The states corresponding to negative energy solutions are fully occupied. The angular momentum \(p_\phi\) is conserved, for the Hamiltonian is spherically symmetric. For a given angular momentum \(p_\phi\), the Hamiltonian (34) describes electron’s radial motion in the following the effective potential

\[
E_\pm = \pm mc^2 \sqrt{1 + \left(\frac{p_\phi}{mc}\right)^2} - V(r).
\]

(35)

The Coulomb potential energy \(V(r)\) is given by

\[
V(r) = \frac{Ze^2}{r},
\]

(36)

where \(Ze^2 = |Q_e|\).

In the classical scenario, given different values of angular momenta \(p_\phi\), the stable circulating orbits (states) are determined by the minimum of the effective potential \(E_\pm(r)\) (35). Using \(dE_\pm(r) / dr = 0\), we obtain the stable orbit location at the radius \(R_L\) in the unit of the Compton length \(\lambda = \hbar/mc\),

\[
R_L(p_\phi) = Z\alpha\lambda\sqrt{1 - \left(\frac{Z\alpha}{p_\phi/\hbar}\right)^2},
\]

(37)

where \(\alpha = e^2/\hbar c\) and \(p_\phi > Z\alpha\). Substituting Eq. (37) into Eq. (35), we find the energy of the electron at each stable orbit,

\[
\mathcal{E}(p_\phi) \equiv \min(E_+) = mc^2 \sqrt{1 - \left(\frac{Z\alpha}{p_\phi/\hbar}\right)^2}.
\]

(38)

The last stable orbits (minimal energy) are given by

\[
p_\phi \rightarrow Z\alpha\hbar + 0^+, \quad R_L(p_\phi) \rightarrow 0^+, \quad \mathcal{E}(p_\phi) \rightarrow 0^+.
\]

(39)

For stable orbits for \(p_\phi \gg 1\), the radii \(R_L \gg 1\) and energies \(\mathcal{E} \rightarrow mc^2 + 0^-\); electrons in these orbits are critically bound since their banding energy goes to zero. As the energy-spectrum (29) (see Eqs. (31,32,33), Eq. (38) shows, only positive or null energy solutions (states) exist in the presence of a point-like nucleus.
In the semi-classical scenario, the discrete values of angular momentum $p_\phi$ are selected by the Bohr-Sommerfeld quantization rule

$$\int p_\phi \, d\phi \simeq \hbar (l + \frac{1}{2}), \quad \Rightarrow \quad p_\phi(l) \simeq \hbar (l + \frac{1}{2}), \quad l = 0, 1, 2, 3, \ldots \quad (40)$$

describing the semi-classical states of radius and energy

$$R_L(l) \simeq Z\alpha \lambda \sqrt{1 - \left(\frac{2Z\alpha}{2l + 1}\right)^2}, \quad (41)$$

$$E(l) \simeq mc^2 \sqrt{1 - \left(\frac{2Z\alpha}{2l + 1}\right)^2}. \quad (42)$$

Other values of angular momentum $p_\phi$, radius $R_L$ and energy $E$ given by Eqs. (37,38) in the classical scenario are not allowed. When these semi-classical states are not occupied as required by the Pauli Principle, the transition from one state to another with different discrete values $(l_1, l_2$ and $\Delta l = l_2 - l_1 = \pm 1)$ is made by emission or absorption of a spin-1 ($\hbar$) photon. Following the energy and angular-momentum conservations, photon emitted or absorbed in the transition have angular momenta $p_\phi(l_2) - p_\phi(l_1) = \hbar (l_2 - l_1) = \pm \hbar$ and energy $E(l_2) - E(l_1)$. As required by the Heisenberg indeterminacy principle $\Delta \phi \Delta p_\phi \simeq 4\pi \hbar (l \gtrsim \hbar$, the absolute ground state for minimal energy and angular momentum is given by the $l = 0$ state, $p_\phi \sim \hbar/2$, $R_L \sim Z\alpha \lambda \sqrt{1 - \left(2Z\alpha\right)^2} > 0$ and $E \sim mc^2 \sqrt{1 - \left(2Z\alpha\right)^2} > 0$ for $Z\alpha \leq 1/2$. Thus the stability of all semi-classical states $l > 0$ is guaranteed by the Pauli principle. In contrast for $Z\alpha > 1/2$, there is not an absolute ground state in the semi-classical scenario. We see now how the lowest energy states are selected by the quantization rule in the semi-classical scenario out of the last stable orbits (39) in the classical scenario. For the case of $Z\alpha \leq 1/2$, equating Eq. (39) to $p_\phi = \hbar (l + 1/2)$ (40), we find the selected state $l = 0$ is only possible solution so that the ground state $l = 0$ in the semi-classical scenario corresponds to the last stable orbits (39) in the classical scenario. On the other hand for the case $Z\alpha > 1/2$, equating Eq. (39) to $p_\phi = \hbar (l + 1/2)$ (40), we find the selected state $l = \tilde{l} \equiv (Z\alpha - 1)/2 > 0$ in the semi-classical scenario corresponds to the last stable orbits (39) in the classical scenario. This state $l = \tilde{l} > 0$ is not protected by the Heisenberg indeterminacy principle from quantum-mechanically decaying in $\hbar$-steps to the states with lower angular momentum and energy (correspondingly smaller radius $R_L$ (41)) via photon emissions. This clearly shows that the “$Z = 137$-catastrophe” corresponds to $R_L \to 0$, falling to the center of the Coulomb potential and all semi-classical states ($l$) are unstable.

3.3. The critical value of the nuclear charge $Z_{cr} = 173$.

A very different situation is encountered when considering the fact the nucleus is not point-like and has an extended charge distribution.36–44 When doing so, the $Z = 137$ catastrophe disappears and the energy-levels $E(n,j)$ of the bound states
As $Z$ increases to values larger than 137, as shown in Fig. 6. The reason is that the finite size $R$ of the nucleus charge distribution provides a cutoff for the boundary condition at the origin $r \to 0$ and the energy-levels $\mathcal{E}(n,j)$ of the Dirac equation are shifted due to this cutoff. In order to determine the critical value $Z_{\text{cr}}$ when the negative energy continuum ($E < -m_e c^2$) is encountered (see Fig. 6), Zel’dovich and Popov\textsuperscript{40–44} solved the Dirac equation corresponding to a nucleus of finite extended charge distribution, i.e., the Coulomb potential is modified as
\begin{equation}
V(r) = \begin{cases} \frac{-Ze^2}{r}, & r > R, \\
\frac{-Ze^2}{R} f\left(\frac{r}{R}\right), & r < R,
\end{cases}
\end{equation}
where $R \sim 10^{-12}\text{cm}$ is the size of the nucleus. The form of the cutoff function $f(x)$ depends on the distribution of the electric charge over the volume of the nucleus ($x = r/R$, $0 < x < 1$, with $f(1) = 1$). Thus, $f(x) = (3 - x^2)/2$ corresponds to a constant volume density of charge.

Solving the Dirac equation with the modified Coulomb potential (43) and calculating the corresponding perturbative shift $\Delta\mathcal{E}_R$ of the lowest energy level (31) one obtains\textsuperscript{40,44}
\begin{equation}
\Delta\mathcal{E}_R = m_e c^2 \left(\xi^2(2\xi e^{-\Lambda})^{\gamma_2}/\gamma_2(1 + 2\gamma_2) - 1 + 2\gamma_2 \int_0^1 f(x)x^{2\gamma_2} dx\right),
\end{equation}
where $\xi = Z\alpha$, $\gamma_2 = \sqrt{1 - \xi^2}$ and $\Lambda = \ln(h/m_e c R) \gg 1$ is a logarithmic parameter in the problem under consideration. The asymptotic expressions for the $1S_{1/2}$ energy that were obtained are\textsuperscript{43,44}
\begin{equation}
\mathcal{E}(1S_{1/2}) = m_e c^2 \begin{cases} \sqrt{1 - \xi^2} \coth(\Lambda\sqrt{1 - \xi^2}), & 0 < \xi < 1, \\
\Lambda^{-1}, & \xi = 1, \\
\sqrt{\xi^2 - 1} \cot(\Lambda\sqrt{\xi^2 - 1}), & \xi > 1.
\end{cases}
\end{equation}
As a result, the “$Z = 137$ catastrophe” in Eq. (29) disappears and $\mathcal{E}(1S_{1/2}) = 0$ gives
\begin{equation}
\xi_0 = 1 + \frac{\pi^2}{8\Lambda} + O(\Lambda^{-4});
\end{equation}
the state $1S_{1/2}$ energy continuously goes down to the negative energy continuum since $Z\alpha > 1$, and $\mathcal{E}(1S_{1/2}) = -1$ gives
\begin{equation}
\xi_{\text{cr}} = 1 + \frac{\pi^2}{2\Lambda(\Lambda + 2)} + O(\Lambda^{-4})
\end{equation}
as shown in Fig. 6. In Ref. 40,44 it is found that the critical value $\xi^{(n)} = Z_{\text{cr}}\alpha$ for the energy-levels $nS_{1/2}$ and $nP_{1/2}$ to reach the negative energy continuum is equal to
\begin{equation}
\xi^{(n)} = 1 + \frac{n^2\pi^2}{2\Lambda^2} + O(\Lambda^{-3}).
\end{equation}
The critical value increases rapidly with increasing $n$. As a result, it is found that $Z_{cr} \simeq 173$ is a critical value at which the lowest energy-level of the bound state $1S_{1/2}$ encounters the negative energy continuum, while other bound states encounter the negative energy continuum at $Z_{cr} > 173$ (see also Ref. 38 for a numerical estimation of the same spectrum). We refer the readers to Ref. 40–45 for mathematical and numerical details.

When $Z > Z_{cr} = 173$, the lowest energy-level of the bound state $1S_{1/2}$ enters the negative energy continuum. Its energy-level can be estimated as follows:

$$E(1S_{1/2}) = m_e c^2 - \frac{Z \alpha}{\bar{r}} < -m_e c^2,$$

(49)

where $\bar{r}$ is the average radius of the $1S_{1/2}$ state’s orbit, and the binding energy of this state satisfies $Z \alpha/\bar{r} > 2m_e c^2$. If this bound state is unoccupied, the bare nucleus gains a binding energy $Z \alpha/\bar{r}$ larger than $2m_e c^2$, and becomes unstable against the production of an electron-positron pair. Assuming this pair-production occurs around the radius $\bar{r}$, we have energies for the electron ($\epsilon_-$) and positron ($\epsilon_+$) given by:

$$\epsilon_- = \sqrt{|\mathbf{p}_-|^2 + m_e^2 c^4} - \frac{Z \alpha}{\bar{r}}, \quad \epsilon_+ = \sqrt{|\mathbf{p}_+|^2 + m_e^2 c^4} + \frac{Z \alpha}{\bar{r}},$$

(50)

where $\mathbf{p}_\pm$ are electron and positron momenta, and $\mathbf{p}_- = -\mathbf{p}_+$. The total energy required for the production of a pair is:

$$\epsilon_{-+} = \epsilon_- + \epsilon_+ = 2\sqrt{|\mathbf{p}_-|^2 + m_e^2 c^4},$$

(51)

which is independent of the potential $V(\bar{r})$. The potential energies $\pm eV(\bar{r})$ of the electron and positron cancel each other out and do not contribute to the total energy (51) required for pair production. This energy (51) is acquired from the binding energy ($Z \alpha/\bar{r} > 2m_e c^2$) by the electron filling into the bound state $1S_{1/2}$. A part of the binding energy becomes the kinetic energy of positron that goes out. This is analogous to the familiar case that a proton ($Z = 1$) catches an electron into the ground state $1S_{1/2}$, and a photon is emitted with the energy not less than 13.6 eV. In the same way, more electron-positron pairs are produced, when $Z > Z_{cr} = 173$ the energy-levels of the next bound states $2P_{1/2}, 2S_{3/2}, \ldots$ enter the negative energy continuum, provided these bound states of the bare nucleus are unoccupied.

### 3.4. Positron production

Ref. 46,47 proposed that when $Z > Z_{cr}$, the bare nucleus spontaneously produces pairs of electrons and positrons: the two positrons go off to infinity and the effective charge of the bare nucleus decreases by two electrons, which corresponds exactly to filling the K-shell.\(^a\) A more detailed investigation was made for the solution of the

---

\(^a\)Hyperfine structure of $1S_{1/2}$ state: single and triplet.

\(^b\)The supposition was made in Ref. 46,47 that the electron density of $1S_{1/2}$ state, as well as the vacuum polarization density, is delocalized at $Z \rightarrow Z_{cr}$. Later it was proven to be incorrect.\(^{41,42,44}\)
Dirac equation at $Z \sim Z_{cr}$, when the lowest electron level $1S_{1/2}$ merges with the negative energy continuum, by Ref. 40–43,48. This further clarified the situation, showing that at $Z \gtrsim Z_{cr}$, an imaginary resonance energy of Dirac equation appears

$$
\epsilon = \epsilon_0 - \frac{i \Gamma}{2},
$$

where

$$
\epsilon_0 = -m_e c^2 - a(Z - Z_{cr}),
$$

$$
\Gamma \sim \theta(Z - Z_{cr}) \exp \left( -b \sqrt{Z_{cr} - Z} \right),
$$

and $a, b$ are constants, depending on the cutoff $\Lambda$ (for example, $b = 1.73$ for $Z = Z_{cr} = 173$, see Ref. 41,42,44). The energy and momentum of the emitted positrons are $|\epsilon_0|$ and $|p| = \sqrt{|\epsilon_0| - m_e c^2}$.

The kinetic energy of the two positrons at infinity is given by

$$
\epsilon_p = |\epsilon_0| - m_e c^2 = a(Z - Z_{cr}) + \ldots,
$$

which is proportional to $Z - Z_{cr}$ (as long as $(Z - Z_{cr}) \ll Z_{cr}$) and tends to zero as $Z \sim Z_{cr}$. The pair-production resonance at the energy (52) is extremely narrow and practically all positrons are emitted with almost same kinetic energy for $Z \sim Z_{cr}$, i.e., nearly monoenergetic spectra (sharp line structure). Apart from a pre-exponential factor, $\Gamma$ in Eq. (54) coincides with the probability of positron production, i.e., the penetrability of the Coulomb barrier. The related problems of vacuum charge density due to electrons filling into the K-shell and charge renormalization due to the change of wave function of electron states are discussed by Ref. 49–53. An extensive and detailed review on this theoretical issue can be found in Ref. 35,44,45,54.

On the other hand, some theoretical work has been done studying the possibility that pair production due to bound states encountering the negative energy continuum is prevented from occurring by higher order processes of quantum field theory, such as charge renormalization, electron self-energy and nonlinearities in electrodynamics and even the Dirac field itself. However, these studies show that various effects modify $Z_{cr}$ by a few percent, but have no way to prevent the binding energy from increasing to $2m_e c^2$ as $Z$ increases, without simultaneously contradicting the existing precise experimental data on stable atoms.

Following this, special attention has been given to understand the process of creating a nucleus with $Z > Z_{cr}$ by collision of two nuclei of charge $Z_1$ and $Z_2$ such that $Z = Z_1 + Z_2 \geq Z_{cr}$.

To observe the emission of positrons coming from pair production occurring near an overcritical nucleus temporarily formed by two nuclei, the following necessary conditions have to be fulfilled: (i) the atomic number of an overcritical nucleus must be larger than $Z_{cr} = 173$; (ii) the lifetime (the sticking time of two-nuclear collisions) of the overcritical nucleus must be much longer than the characteristic time $(\hbar/m_e c^2)$ of pair production; (iii) inner shells (K-shell) of the overcritical nucleus should be unoccupied.
When in the course of a heavy-ion collision the two nuclei come into contact, some deep-inelastic reactions have been claimed to exist for a certain time $\Delta t$. The duration $\Delta t$ of this contact (sticking time) is expected to depend on the nuclei involved in the reaction and on beam energy. For very heavy nuclei, the Coulomb interaction is the dominant force between the nuclei, so that the sticking times $\Delta t$ are typically much shorter and on the average probably do not exceed $1 \sim 2 \cdot 10^{-21}$ sec.$^{35}$ Accordingly the calculations (see Fig. 7) also show that the time when the binding energy is overcritical is very short, about $1.2 \cdot 10^{-21}$ sec. Theoretical attempts have been proposed to study the nuclear aspects of heavy-ion collisions at energies very close to the Coulomb barrier and search for conditions, which would serve as a trigger for prolonged nuclear reaction times, (the sticking time $\Delta t$) to enhance the amplitude of pair production.$^{35,62,65-67}$ Up to now no conclusive theoretical or experimental evidence exists for long nuclear delay times in very heavy collision systems.

![Energy expectation values of the 1s$\sigma$ state in a U+U collision at 10 GeV/nucleon.](image)

The unit of time is $\hbar/mc^2$. This figure is reproduced from Fig. 4 in Ref. 35.

It is worth noting that several other dynamical processes contribute to the production of positrons in undercritical as well as in overcritical collision systems.$^{35-38}$ Due to the time-energy uncertainty relation (collision broadening), the energy-spectrum of such positrons has a rather broad and oscillating structure, considerably different from a sharp line structure that we would expect from pair-production positron emission alone.
3.5. Experiments

As already remarked, if the sticking time $\Delta t_s$ could be prolonged, the probability of pair production in vacuum around the super heavy nucleus would be enhanced. As a consequence, the spectrum of emitted positrons is expected to develop a sharp line structure, indicating the spontaneous vacuum decay caused by the overcritical electric field of a forming super heavy nuclear system with $Z \geq Z_{cr}$. If the sticking time $\Delta t_s$ is not long enough and the sharp line of pair production positrons has not yet well-developed, in the observed positron spectrum it is difficult to distinguish the pair production positrons from positrons created through other different mechanisms. Prolonging the sticking time and identifying pair production positrons among all other particles created in the collision process are important experimental tasks.

For nearly 20 years the study of atomic excitation processes and in particular of positron creation in heavy-ion collisions has been a major research topic at GSI (Darmstadt). The Orange and Epos groups at GSI (Darmstadt) discovered narrow line structures (see Fig. 8) of unexplained origin, first in the single positron energy spectra and later in coincident electron-positron pair emission. Studying more collision systems with a wider range of the combined nuclear charge $Z = Z_1 + Z_2$ they found that narrow line structures are essentially independent of $Z$. This rules out the explanation of a pair-production positron, since the line would be expected at the position of the $1s\sigma$ resonance, i.e., at a kinetic energy given by Eq. (55), which is strongly $Z$ dependent. Attempts to link this positron line to spontaneous pair production have failed. Other attempts to explain this positron line in term of atomic physics and new particle scenario were not successful as well.

The anomalous positron line problem has perplexed experimentalists and theorists alike for more than a decade. Moreover, later results obtained by the Apex collaboration at Argonne National Laboratory showed no statistically significant positron line structures. This is in strong contradiction with the former results obtained by the Orange and Epos groups. However, the analysis of Apex data was challenged in the comment by Ref. 85,86 for the Apex measurement would have been less sensitive to extremely narrow positron lines. A new generation of experiments (Apex at Argonne and the new Epos and Orange setups at GSI) with much improved counting statistics has failed to reproduce the earlier results.

To overcome the problem posed by the short time scale of pair production ($10^{-21}$ sec), hopes rest on the idea to select collision systems in which a nuclear reaction with sufficient sticking time occurs. Whether such a situation can be realized still is an open question. In addition, the anomalous positron line problem and its experimental contradiction overshadow on the field of studying the pair production in heavy ion collisions. In summary, clear experimental signals for electron-positron pair production in heavy ion collisions are still missing at the present time.
Fig. 8. Two typical example of coincident electron-positron spectra measured by the Epose group in the system U+Th (left) and by the Orange group in U+Pb collisions (right). When plotted as a function of the total energy of the electron and positron, very narrow line structures were observed. This figure is reproduced from Fig. 7 in Ref. 35.

4. The extraction of blackholic energy from a black hole by vacuum polarization processes

We recall here the basic steps leading to the study of a critical electric field in a Kerr-Newman black hole. We recall the theoretical framework to apply the Schwinger process in general relativity in the field of a Kerr-Newman geometry as well as the process of extraction of the “blackholic” energy. We then recall the basic concepts of dyadosphere and dyadotorus leading to a photon-electron-positron plasma surrounding the black hole.

4.1. The mass-energy formula of black holes

The same effective potential technique (see Landau and Lifshitz) which allowed the analysis of circular orbits around the black hole was crucial in reaching the equally interesting discovery of the reversible and irreversible transformations of black holes by Christodoulou and Ruffini, which in turn led to the mass-energy formula of the black hole

\[ E_{BH} = M^2c^4 = \left( M_\text{ir}c^2 + \frac{Q^2}{2\rho_+} \right)^2 + \frac{T^2c^2}{\rho_+^2}, \quad (56) \]

with

\[ \frac{1}{\rho_+^2} \left( \frac{G}{c^2} \right) (Q^4 + 4L^2c^2) \leq 1, \quad (57) \]

where

\[ S = 4\pi \rho_+^2 = 4\pi(r_+^2 + \frac{L^2}{c^2M^2}) = 16\pi \left( \frac{G}{c^2} \right) M^2, \quad (58) \]
is the horizon surface area, $M$ is the irreducible mass, $r_+$ is the horizon radius and $\rho_+$ is the quasi-spheroidal cylindrical coordinate of the horizon evaluated at the equatorial plane. Extreme black holes satisfy the equality in Eq. (57).

From Eq. (56) follows that the total energy of the black hole $E_{BH}$ can be split into three different parts: rest mass, Coulomb energy and rotational energy. In principle both Coulomb energy and rotational energy can be extracted from the black hole (Christodoulou and Ruffini88). The maximum extractable rotational energy is 29% and the maximum extractable Coulomb energy is 50% of the total energy, as clearly follows from the upper limit for the existence of a black hole, given by Eq. (57). We refer in the following to both these extractable energies as the blackholic energy. We outline how the extraction of the blackholic energy is indeed made possible by electron-positron pair creation. We also introduce the concept of a “dyadosphere” or “dyadotorus” around a black hole and we will the outline how the evolution of the electron-positron plasma will naturally lead to a self-acceleration process creating the very high Lorentz gamma factor observed in GRBs.

4.2. Vacuum polarization in Kerr-Newman geometries

We already discussed the phenomenon of electron-positron pair production in a strong electric field in a flat space-time. We study the same phenomenon occurring around a black hole with electromagnetic structure (EMBH). For simplicity and in order to give the fundamental energetic estimates, we postulate that the collapse has already occurred and has led to the formation of an EMBH. Clearly, this is done only in order to give an estimate of the transient phase occurring during the gravitational collapse. In reality, an EMBH will never be formed because the vacuum polarization process will carry away its electromagnetic energy during the last phase of gravitational collapse. Indeed being interested in this transient phenomenon, we can estimate its energetics by the conceptual analysis of an already formed EMBH, and this is certainly valid for the estimate of the energetics which we will encounter in a realistic phase of gravitational collapse.

The spacetime around the EMBH is described by the Kerr-Newman geometry whose metric we rewrite here for convenience in Boyer-Lindquist coordinates $(t, r, \theta, \phi)$

$$ds^2 = \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\Delta}{\Sigma} \left( dt - a \sin^2 \theta d\phi \right)^2 + \frac{\sin^2 \theta}{\Sigma} \left[ (r^2 + a^2) d\phi - adt \right]^2, \quad (59)$$

where $\Delta = r^2 - 2Mr + a^2 + Q^2$ and $\Sigma = r^2 + a^2 \cos^2 \theta$, as before and as usual $M$ is the mass, $Q$ the charge and $a$ the angular momentum per unit mass of the EMBH. We recall that the Reissner-Nordstrom geometry is the particular case $a = 0$ of a non-rotating black hole. Natural units $G = h = c = 1$ will be adopted in this section.

The electromagnetic vector potential around the Kerr-Newman black hole is given in Boyer-Lindquist coordinates by

$$A = -Q\Sigma^{-1} r(dt - a \sin^2 \theta d\phi). \quad (60)$$
The electromagnetic field tensor is then

\[ F = dA = 2Q \Sigma^{-2} \left[ (r^2 - a^2 \cos^2 \theta) dr \wedge dt - 2a^2 r \cos \theta \sin \theta d\theta \wedge d\phi \right. \]
\[ - a \sin^2 \theta (r^2 - a^2 \cos^2 \theta) dr \wedge d\phi + 2ar(r^2 + a^2) \cos \theta \sin \theta d\theta \wedge d\phi \]. \quad (61) \]

After some preliminary work in Refs. 89–91, the occurrence of pair production in a Kerr-Newman geometry was addressed by Deruelle.\(^9\) In a Reissner-Nordström geometry, QED pair production has been studied by Zaumen\(^9\) and Gibbons.\(^9\) The corresponding problem of QED pair production in the Kerr-Newman geometry was addressed by Damour and Ruffini,\(^9\) who obtained the rate of pair production with particular emphasis on:

- the limitations imposed by pair production on the strength of the electromagnetic field of a black hole;\(^9\)
- the efficiency of extracting rotational and Coulomb energy (the “blackholic” energy) from a black hole by pair production;
- the possibility of having observational consequences of astrophysical interest.

The third point was in fact a far-reaching prevision of possible energy sources for gamma ray bursts that are now one of the most important phenomena under current theoretical and observational study. In the following, we recall the main results of the work by Damour and Ruffini.

In order to study the pair production in the Kerr-Newman geometry, they introduced at each event \((t, r, \theta, \phi)\) a local Lorentz frame associated with a stationary observer \(O\) at the event \((t, r, \theta, \phi)\). A convenient frame is defined by the following orthogonal tetrad\(^9\):

\[ \omega^{(0)} = (\Delta/\Sigma)^{1/2} (dt - a \sin^2 \theta d\phi), \quad (62) \]
\[ \omega^{(1)} = (\Sigma/\Delta)^{1/2} dr, \quad (63) \]
\[ \omega^{(2)} = \Sigma^{1/2} d\theta, \quad (64) \]
\[ \omega^{(3)} = \sin \theta \Sigma^{-1/2} ((r^2 + a^2)d\phi - ar dt). \quad (65) \]

In this Lorentz frame, the electric potential \(A_0\), the electric field \(E\) and the magnetic field \(B\) are given by the following formulas (c.e.g. Ref. 98),

\[ A_0 = \omega_\alpha^{(0)} A^\alpha, \]
\[ E^\alpha = \omega_\beta^{(0)} F^{\alpha\beta}, \]
\[ B^\beta = \frac{1}{2} \omega_\gamma^{(0)} \epsilon^{\gamma\delta\beta} F_{\gamma\delta}. \]

We then obtain

\[ A_0 = -Qr(\Sigma \Delta)^{-1/2}, \quad (66) \]
while the electromagnetic fields \( E \) and \( B \) are parallel to the direction of \( \omega^{(1)} \) and have strengths given by
\[
E_{(1)} = Q \Sigma^{-2} (r^2 - a^2 \cos^2 \theta), \quad (67)
\]
\[
B_{(1)} = Q \Sigma^{-2} 2a r \cos \theta, \quad (68)
\]
respectively. The maximal strength \( E_{\text{max}} \) of the electric field is obtained in the case \( a = 0 \) at the horizon of the EMBH: \( r = r_+ \). We have
\[
E_{\text{max}} = Q^2 / r_+^2. \quad (69)
\]
Equating the maximal electric field strength (69) to the critical value (28), one obtains the maximal black hole mass \( M_{\text{max}} \approx 7.2 \cdot 10^6 M_\odot \) for pair production to occur. For any black hole with mass smaller than \( M_{\text{max}} \), the pair production process can drastically modify its electromagnetic structure.

Both the gravitational and the electromagnetic background fields of the Kerr-Newman black hole are stationary when considering the quantum field of the electron, which has mass \( m_e \) and charge \( e \). If \( m_e M \gg 1 \), then the spatial variation scale \( GM/c^2 \) of the background fields is much larger than the typical wavelength \( \hbar / m_e c \) of the quantum field. As far as purely QED phenomena such as pair production are concerned, it is possible to consider the electric and magnetic fields defined by Eqs. (67,68) as constants in the neighborhood of a few wavelengths around any events \((r, \theta, \phi, t)\). Thus, the analysis and discussion on the Sauter-Euler-Heisenberg-Schwinger process over a flat space-time can be locally applied to the case of the curved Kerr-Newman geometry, based on the equivalence principle.

The rate of pair production around a Kerr-Newman black hole can be obtained from the Schwinger formula (26) for parallel electromagnetic fields \( \varepsilon = E_{(1)} \) and \( \beta = B_{(1)} \) as:
\[
\tilde{\Gamma} = e^2 E_{(1)} B_{(1)} \sum_{n=1}^{\infty} \frac{1}{n} \coth \left( \frac{n \pi B_{(1)}}{E_{(1)}} \right) \exp \left( - \frac{n \pi E_{(1)}}{E_{(1)}} \right). \quad (70)
\]
The total number of pairs produced in a region \( D \) of the space-time is
\[
N = \int_D d^4x \sqrt{-g} \tilde{\Gamma}, \quad (71)
\]
where \( \sqrt{-g} = \Sigma \sin \theta \). In Ref. 95, it was assumed that for each created pair the particle (or antiparticle) with the same sign of charge as the black hole is expelled to infinity with charge \( e \), energy \( \omega \) and angular momentum \( l_\phi \) while the antiparticle is absorbed by the black hole. This implies the decrease of charge, mass and angular momentum of the black hole and a corresponding extraction of all three quantities. The rates of change of the three quantities are then determined by the rate of pair production (70) and by the conservation laws of total charge, energy and angular
momentum
\[
\begin{align*}
\dot{Q} &= -Re,
M &= -R\langle \omega \rangle,
\dot{L} &= -R\langle l_\phi \rangle,
\end{align*}
\]  
(72)

where \( R = \dot{N} \) is the rate of pair production and \( \langle \omega \rangle \) and \( \langle l_\phi \rangle \) represent some suitable mean values for the energy and angular momentum carried by the pairs.

Supposing the maximal variation of black hole charge to be \( \Delta Q = -Q \), one can estimate the maximal number of pairs created and the maximal mass-energy variation. It was concluded in Ref. 95 that the maximal mass-energy variation in the pair production process is larger than \( 10^{41} \) erg and up to \( 10^{58} \) erg, depending on the black hole mass. They concluded at the time “this work naturally leads to a most simple model for the explanation of the recently discovered \( \gamma \)-ray bursts”.

4.3. The “Dyadosphere”

We first recall the three theoretical results which provide the foundation of the EMBH theory.

In 1971 in the article “Introducing the Black Hole”,16 the theorem was advanced that the most general black hole is characterized uniquely by three independent parameters: the mass-energy \( M \), the angular momentum \( L \) and the charge \( Q \) making it an EMBH. Such an ansatz, which came to be known as the “uniqueness theorem” has turned out to be one of the most difficult theorems to be proven in all of physics and mathematics. The progress in the proof has been authoritatively summarized by Ref. 99. The situation can be considered satisfactory from the point of view of the physical and astrophysical considerations. Nevertheless some fundamental mathematical and physical issues concerning the most general perturbation analysis of an EMBH are still the topic of active scientific discussion.100

In 1971 Christodoulou and Ruffini88 obtained the mass-energy formula of a Kerr-Newman black hole, see Eqs.(56–58). As we just recalled, in 1975 there was the Damour and Ruffini work95 on the vacuum polarization of a Kerr-Newman geometry. The key point of this work was the possibility that energy on the order of \( 10^{54} \) ergs could be released almost instantaneously by the vacuum polarization process of a black hole. At the time, however, nothing was known about the distance of GRB sources or their energetics. The number of theories trying to explain GRBs abounded, as mentioned by Ruffini in the Kleinert Festschrift.101

After the discovery in 1997 of the afterglow of GRBs102 and the determination of the cosmological distance of their sources, we noticed the coincidence between their observed energetics and the one theoretically predicted by Damour and Ruffini.95 We therefore returned to these theoretical results with renewed interest developing some additional basic theoretical concepts103–107 such as the dyadosphere and, more recently, the dyadotorus.
As a first simplifying assumption we have developed our considerations in the absence of rotation using spherically symmetric models. The space-time is then described by the Reissner-Nordström geometry, whose spherically symmetric metric is given by

\[ ds^2 = g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2, \]

where

\[ g_{tt}(r) = \alpha^2(r) \]
\[ g_{rr}(r) = \alpha^{-2}(r). \]

The first result we obtained is that the pair creation process does not occur at the horizon of the EMBH: it extends over the entire region outside the horizon in which the electric field exceeds the value \( E^* \) of the order of magnitude of the critical value given by Eq. (28). The pair creation process, as we recalled in the previous sections (see e.g. Fig.5), is a quantum tunneling between the positive and negative energy states, which needs only a level crossing but can occur, of course, also for \( E^* < E_c \) if the extent of the field is large enough, although with decreasing intensity. Such a process occurs also for \( E^* > E_c \). In order to give a scale of the phenomenon, and for definiteness, in Ref. 105 we first consider the case of \( E^* \equiv E_c \). Since the electric field in the Reissner-Nordström geometry has only a radial component given by

\[ E(r) = \frac{Q}{r^2}, \]

this region extends from the horizon radius

\[ r_+ = 1.47 \cdot 10^7 \mu(1 + \sqrt{1 - \xi^2}) \text{ cm}, \]

out to an outer radius

\[ r^* = \left( \frac{\hbar}{mc} \right)^2 \left( \frac{GM}{c^2} \right)^{\frac{1}{2}} \left( \frac{m_p}{m} \right)^{\frac{1}{2}} \left( \frac{e}{q_p} \right)^{\frac{1}{2}} \left( \frac{Q}{\sqrt{GM}} \right)^{\frac{1}{2}} = 1.12 \cdot 10^9 \sqrt{\mu \xi} \text{ cm}, \]

where we have introduced the dimensionless mass and charge parameters \( \mu = \frac{M}{M_\odot} \), \( \xi = \frac{Q}{(4\pi\sqrt{GM})} \leq 1 \), see Fig. 10.

The second result has been to realize that the local number density of electron and positron pairs created in this region as a function of radius is given by

\[ n_{e^+e^-}(r) = \frac{Q}{4\pi r^2} \left( \frac{m}{\hbar} \right) e \left[ 1 - \left( \frac{r}{r^*} \right)^2 \right], \]

and consequently the total number of electron and positron pairs in this region is

\[ N_{e^+e^-} = \frac{Q - Q_e}{e} \left[ 1 + \frac{(r^* - r_+)}{\frac{r_+}{m}} \right], \]

where \( Q_e = E_n r_+^2 \).

The total number of pairs is larger by an enormous factor \( r^*/(\hbar/mc) > 10^{18} \) than the value \( Q/e \) which a naive estimate of the discharge of the EMBH would have predicted. Due to this enormous amplification factor in the number of pairs created, the region between the horizon and \( r^* \) is dominated by an essentially high
density neutral plasma of electron-positron pairs. We have defined this region as the dyadosphere of the EMBH from the Greek duas, duadsos for pairs. Consequently we have called $r^*$ the dyadosphere radius $r^* \equiv r_{ds}$.\textsuperscript{103-105} The vacuum polarization process occurs as if the entire dyadosphere is subdivided into a concentric set of shells of capacitors each of thickness $\hbar/m_e c$ and each producing a number of $e^+e^-$ pairs on the order of $\sim Q/e$ (see Fig. 10). The energy density of the electron-positron pairs is given by

$$\epsilon(r) = \frac{Q^2}{8\pi r^4} \left( 1 - \left( \frac{r}{r_{ds}} \right)^4 \right),$$  

(79)
Fig. 10. The dyadosphere of a Reissner-Nordström black hole can be represented as equivalent to a concentric set of capacitor shells, each one of thickness $\hbar/m_e c$ and producing a number of $e^+e^-$ pairs of the order of $Q/e$ on a time scale of $10^{-21}$ s, where $Q$ is the EMBH charge. The shells extend in a region of thickness $\Delta r$, from the horizon $r_+$ out to the dyadosphere outer radius $r_{ds}$ (see text). The system evolves to a thermalised plasma configuration.

(see Figs. 2–3 of Ref. 104). The total energy of pairs converted from the static electric energy and deposited within the dyadosphere is then

$$E_{\text{dyas}} = \frac{1}{2} \frac{Q^2}{r_+} \left( 1 - \frac{r_+}{r_{ds}} \right)^2 \left[ 1 - \left( \frac{r_+}{r_{ds}} \right)^4 \right].$$

(80)

As we will see in the following this is one of the two fundamental parameters of the EMBH theory (see Fig. 11). In the limit $\frac{r_+}{r_{ds}} \to 0$, Eq. (80) leads to $E_{\text{dyas}} \to \frac{1}{2} \frac{Q^2}{r_+}$, which coincides with the energy extractable from EMBHs by reversible processes.
\(M_{\text{ir}} = \text{const.})\), namely \(E_{BH} - M_{\text{ir}} = \frac{1}{2} Q^2\) \(88\) see Fig. 9. Due to the very large pair density given by Eq. (77) and to the sizes of the cross-sections for the process \(e^+e^- \rightarrow \gamma + \gamma\), the system has been assumed to thermalize to a plasma configuration for which

\[n_{e^+} = n_{e^-} \sim n_\gamma \sim n_{e^+e^-}, \tag{81}\]

where \(n_{e^+e^-}\) is the total number density of \(e^+e^-\)-pairs created in the dyadosphere. \(104,105\) This assumption has been in the meantime rigorously proven by Ak-senov, Ruffini and Vereshchagin. \(110\)

The third result which we have introduced again for simplicity is that for a given \(E_{\text{dyas}}\) we have assumed either a constant average energy density over the entire dyadosphere volume, or a more compact configuration with energy density equal to its peak value. These are the two possible initial conditions for the evolution of the dyadosphere (see Fig. 11).

![Fig. 11. Left) Selected lines corresponding to fixed values of the \(E_{\text{dyas}}\) are given as a function of the two parameters \(\mu, \xi\), only the solutions below the continuous heavy line are physically relevant. The configurations above the continuous heavy lines correspond to unphysical solutions with \(r_{ds} < r_+\). Right) Two different approximations for the energy density profile inside the dyadosphere. The first one (dashed line) fixes the energy density equal to its peak value, and computes an "effective" dyadosphere radius accordingly. The second one (dotted line) fixes the dyadosphere radius to its correct value, and assumes an uniform energy density over the dyadosphere volume. The total energy in the dyadosphere is of course the same in both cases. The solid curve represents the real energy density profile.

The above theoretical results permit a good estimate of the general energetics processes originating in the dyadosphere, assuming an already formed EMBH. In reality, if the GRB data become accurate enough, the full dynamical description of the dyadosphere formation will be needed in order to explain the observational data by the general relativistic effects and characteristic time scales of the approach to the EMBH horizon. \(111-114\)
4.4. The “Dyadotorus”

We turn now to examine how the presence of rotation modifies the geometry of the surface containing the region where electron-positron pairs are created as well as the conditions for the existence of such a surface. Due to the axial symmetry of the problem, we have called this region the “dyadotorus”. Following Damour\textsuperscript{116,117} we introduce at each point of the spacetime the orthogonal Carter tetrad

\[\omega^{(0)} = (\Delta/\Sigma)^{1/2}(dt - a \sin^2 \theta d\varphi),\]
\[\omega^{(1)} = (\Sigma/\Delta)^{1/2}dr,\]
\[\omega^{(2)} = \Sigma^{1/2}d\theta,\]
\[\omega^{(3)} = \sin \theta \Sigma^{-1/2}[(r^2 + a^2) d\varphi - adt],\]

where \(\Sigma = r^2 + a^2 \cos^2 \theta\) and \(\Delta = r^2 - 2Mr + a^2 + Q^2\), \(M\) being the mass, \(Q\) the electric charge and \(a\) the angular momentum per unit mass of the black hole. Thus, in the Lorentz frame defined by the above tetrad the Kerr–Newman metric reads

\[ds^2 = -(\omega^{(0)})^2 + (\omega^{(1)})^2 + (\omega^{(2)})^2 + (\omega^{(3)})^2,\] \hspace{1cm} (82)

and the outer horizon of the black hole is localized at \(r_+ = M + \sqrt{M^2 - (a^2 + Q^2)}\).

In this frame the electric and magnetic field obtained from the potential

\[A = -Qr(\Sigma/\Delta)^{1/2} \omega^{(0)},\] \hspace{1cm} (83)

are parallel, i.e.,

\[E_{(1)} = Q\Sigma^{-2}(r^2 - a^2 \cos^2 \theta),\] \hspace{1cm} (84)
\[B_{(1)} = 2aQ\Sigma^{-2}r \cos \theta.\] \hspace{1cm} (85)

The rate \(R\) of pair creation using the Schwinger approach is\textsuperscript{95}

\[R = \int 2 \text{Im } \mathcal{L} \sqrt{|g|} d^4 x,\] \hspace{1cm} (86)

where

\[2 \text{Im } \mathcal{L} = (4\pi)^{-1}(E_{(1)}\epsilon/\pi)^2 \sum_{n=1}^\infty n^{-2}(n\pi B_{(1)}/E_{(1)}) \coth(n\pi B_{(1)}/E_{(1)}) \exp(-n\pi m_c^2/E_{(1)}),\] \hspace{1cm} (87)

where \(\epsilon = m_e^2/E_c\).

We define the dyadotorus by the condition \(|\mathbf{E}| = \kappa E_c\), where \(10^{-1} \leq \kappa \leq 10\). Solving for \(r\) and introducing the dimensionless quantities \(\xi = Q/M, \alpha = a/M, \mu = M/M_\odot, \mathcal{E} = \kappa E_c M_\odot \approx 1.873 \times 10^{-6}\) and \(\tilde{r} = r/M\) (with \(M_\odot \approx 1.477 \times 10^5\) cm) we get

\[\left(\frac{\tilde{r}_+^2}{M}\right)^2 = \frac{\xi^2}{2\mu^2} - \alpha^2 \cos^2 \theta \pm \sqrt{\frac{\xi^4}{4\mu^4} - \frac{2\xi}{\mu^2} \alpha^2 \cos^2 \theta},\] \hspace{1cm} (88)

where the \(\pm\) signs correspond to the two different parts of the surface.
The two parts of the surface join at the particular values $\theta^*$ and $\pi - \theta^*$ of the polar angle where $\theta^* = \arccos\left(\frac{1}{2\sqrt{\alpha}}\sqrt{\mu E}\right)$. The requirement that $\cos\theta^* \leq 1$ can be solved for instance for the charge parameter $\xi$, giving a range of values of $\xi$ for which the dyadotorus takes one of the shapes (see fig. 12)

$$\text{surface} = \begin{cases} \text{ellipsoid-like} & \text{if } \xi \geq \xi^*, \\ \text{torus-like} & \text{if } \xi < \xi^* \end{cases}$$

(89)

where $\xi^* = 8\mu E \alpha^2$.

In Fig. 12 we show some examples of the dyadotorus geometry for different sets of parameters for an extreme Kerr–Newman black hole ($M^2 = a^2 + Q^2$), we can see the transition from a toroidal geometry to an ellipsoidal one depending on the value of the black hole charge.

Fig. 13 shows instead the projections of the surfaces corresponding to different values of the ratio $|E|/E_c \equiv \kappa$ for the same choice of parameters as in Fig. 12 (b), as an example. We see that the region enclosed by such surfaces shrinks for increasing values of $\kappa$.

5. On the observability of electron-positron pairs created by vacuum polarization in Earth-bound experiments and in astrophysics

In summary, from the considerations we have presented in the previous sections three different earth-bound experiments and one astrophysical observation have been proposed for identifying the polarization of the electronic vacuum due to a supercritical electric field postulated by Sauter-Heisenberg-Euler-Schwinger (see Ref. 11,13,17,118):

(1) In collisions of heavy ions near the Coulomb barrier, as first proposed in Ref. 46, 47 (see also Ref. 44,48,119). Despite some apparently encouraging results (see Ref. 120), such efforts have failed so far due to the small contact time of the colliding ions. Typically the electromagnetic energy involved in the collisions of heavy ions with impact parameter $l_1 \sim 10^{-12}$ cm is $E_1 \sim 10^{-6}$ erg and the lifetime of the diatomic system is $t_1 \sim 10^{-22}$ s.

(2) In collisions of an electron beam with optical laser pulses: a signal of positrons above background has been observed in collisions of a 46.6 GeV electron beam with terawatt pulses of optical laser in an experiment at the Final Focus Test Beam at SLAC, it is not clear if this experimental result is an evidence for the vacuum polarization phenomenon. The energy of the laser pulses was $E_2 \sim 10^7$ erg, concentrated in a space-time region of spacial linear extension (focal length) $l_2 \sim 10^{-3}$ cm and temporal extension (pulse duration) $t_2 \sim 10^{-12}$ s.

(3) At the focus of an X-ray free electron laser (XFEL) (see Ref. 123–125 and references therein). Proposals for this experiment exist at the TESLA collider at DESY and at the LCLS facility at SLAC. Typically the electromagnetic...
energy at the focus of an XFEL can be $E_3 \sim 10^6$ erg, concentrated in a space-time region of spacial linear extension (spot radius) $l_3 \sim 10^{-8}$ cm and temporal extension (coherent spike length) $t_3 \sim 10^{-13}$ s.\textsuperscript{123}

and from astrophysics

(1) Around an electromagnetic black hole (black hole),\textsuperscript{95,104,105} giving rise to the
Fig. 13. The projections of the surfaces corresponding to different values of the ratio $|E|/E_c \equiv k$ are shown for the same choice of parameters as in Fig. 12 (b), as an example. The gray shaded region is part of the “dyadotorus” corresponding to the case $\kappa = 1$ as plotted in Fig. 12 (b). The region delimited by dashed curves corresponds to $\kappa = 0.8$, i.e., to a value of the strength of the electric field smaller than the critical one, and contains the dyadotorus; the latter in turn contains the white region corresponding to $\kappa = 1.4$, i.e., to a value of the strength of the electric field greater than the critical one.

observed phenomenon of GRBs. The electromagnetic energy of a black hole of mass $M \sim 10M_\odot$ and charge $Q \sim 0.1M/\sqrt{G}$ is $E_4 \sim 10^{54}$ erg and it is deposited in a space-time region of spacial linear extension $l_4 \sim 10^5 cm$ and temporal extension (collapse time) $t_4 \sim 10^{-2}$ s.

As we will see in the following, the creation of an electron-positron plasma is indeed essential to explain not only the GRB energetics but also the unprecedentedly large Lorentz gamma factor observed in GRBs.

6. Electrodynamics for nuclear matter in bulk

We have seen how critical fields may be generated in heavy ion collisions, in collisions of electron beams with optical laser pulses and in X-ray free electron lasers. The explanation of GRBs leads to a theoretical framework postulating the existence of critical and overcritical fields in black holes, in order to extract their blackholic energy. It becomes natural, then, to ask if there is any mechanism which can lead to the existence of a critical field not only on the above mentioned microscopic scale,
but also on macroscopic scales, possibly to be encountered at the onset of the process of gravitational collapse. It is clear that such processes should be common to a large variety of initial conditions occurring in the gravitational collapse either of a neutron star or of two binary neutron stars or again of a binary system formed by a neutron star and a white dwarf or, finally, in the general formation of an intermediate mass black hole. It is already clear from the work of Damour and Ruffini that the gravitational collapse giving birth to the black holes in active galactic nuclei with masses much larger that $10^6 M_{\odot}$ cannot give rise to instantaneous vacuum polarization processes leading to GRBs. For this reason we present here an alternative treatment of the electrodynamics for nuclear matter in bulk, presenting for the first time a unified approach to neutron star physics and nuclear physics, covering the range of atomic number from $A \sim 10^2$ to $A \sim 10^{57}$.

6.1. The Thomas-Fermi equations for heavy ions

It is well known that the Thomas-Fermi equation is the exact theory for atoms, molecules and solids as $Z \to \infty$.\textsuperscript{131} We show in this section that the relativistic Thomas-Fermi theory developed to study atoms for heavy nuclei with $Z \simeq 10^6$\textsuperscript{132} gives important basic new information about the state of nuclear matter in bulk in the limit of $N \simeq (m_{\text{Planck}}/m_n)^3$ nucleons of mass $m_n$ and about its electrodynamic properties.

The analysis of bulk nuclear matter in neutron stars composed of a degenerate gas of neutrons, protons and electrons has traditionally been approached by microscopically implementing the charge neutrality condition by requiring the electron density $n_e(x)$ to coincide with the proton density $n_p(x)$

$$n_e(x) = n_p(x). \quad (90)$$

It is clear, however, that especially when conditions close to gravitational collapse occur, there is an ultra-relativistic component of degenerate electrons whose confinement requires the existence of very strong electromagnetic fields, in order to guarantee the overall charge neutrality of the neutron star. Under these conditions Eq. (90) will necessarily be violated. We will show here that they will develop electric fields close to the critical value $E_c$ introduced by Ref. 11,13,17–19:

$$E_c = \frac{m_n c^3}{e\hbar}. \quad (91)$$

Special attention to the existence of critical electric fields and the possible condition for electron-positron ($e^+e^-$) pair creation out of the vacuum in the case of heavy bare nuclei with the atomic number $Z \geq 173$ has been given by Ref. 36,42,44,47,62,133. They analyzed the specific pair creation process of an electron-positron pair around both a point-like and extended bare nucleus by direct integration of Dirac equation. These considerations have been extrapolated to much heavier nuclei $Z \gg 1600$, implying the creation of a large number of $e^+e^-$ pairs by using a statistical approach based on the relativistic Thomas-Fermi equation by
Ref. 134,135. Using substantially the same statistical approach based on the relativistic Thomas-Fermi equation, Ref. 132,136 have analyzed the electron densities around an extended nucleus in a neutral atom all the way up to $Z \approx 6000$. They have shown the effect of the penetration of the electron orbitals well inside the nucleus, leading to a screening of the nuclei positive charge and to the concept of an “effective” nuclear charge distribution.

All of this work assumes for the radius of the extended nucleus the semi-empirical formula,

\[ R_c \approx r_0 A^{1/3}, \quad r_0 = 1.2 \cdot 10^{-13} \text{cm}, \]  

(92)

where the mass number $A = N_n + N_p$, $N_n$ and $N_p$ are the neutron and proton numbers. The approximate relation between $A$ and the atomic number $Z = N_p$

\[ Z \approx \frac{A}{2} \]  

(93)

was adopted in Ref. 134,135, or the empirical formula

\[ Z \approx \left( \frac{2}{A} \right)^{1/3} - 1 \]  

(94)

was adopted in Ref. 132,136.

6.2. Electroweak equilibrium in Nuclear Matter in Bulk

We outline an alternative approach of the description of nuclear matter in bulk: it generalizes the above treatments, already developed and tested for the study of heavy nuclei, to the case of $N \approx (m_{\text{Planck}}/m_n)^3$ nucleons. This more general approach differs in many aspects from the ones in the current literature and reproduces the above treatments in the limiting case of $A$ smaller than $10^6$. We will look for a solution implementing the condition of overall charge neutrality of the star as given by

\[ N_e = N_p, \]  

(95)

which significantly modifies Eq. (90), since now $N_e(N_p)$ is the total number of electrons (protons) of the equilibrium configuration.

We present here only a simplified prototype of this approach, outlining the essential relative role of the four fundamental interactions present in the neutron star physics: the gravitational, weak, strong and electromagnetic interactions. In addition, we also implement the fundamental role of Fermi-Dirac statistics and the phase space blocking due to the Pauli principle in the degenerate configuration. The new results essentially depend from the coordinated action of the above five theoretical components and cannot be obtained if any one of them is neglected.

Let us first recall the role of gravity. In the case of neutron stars, unlike the case of nuclei where its effects can be neglected, gravitation has the fundamental role of defining the basic parameters of the equilibrium configuration. As pointed out by Ref. 23 at a Newtonian level and by Ref. 138 in general relativity, configurations of...
equilibrium exist at approximately one solar mass and at an average density around the nuclear density. This result is obtainable considering only the gravitational interaction of a system of Fermi degenerate self-gravitating neutrons, neglecting all other particles and interactions. This situation can be formulated within a Thomas-Fermi self-gravitating model (see e.g. Ref. 139).

In the present case of our simplified prototype model directed at revealing new electrodynamic properties, the role of gravity is taken into account simply by considering in line with the generalization of the above results a mass-radius relation for the baryonic core

$$R_{NS} = R_{c} \approx \frac{\hbar}{m_{n} \pi c m_{Planck}}.$$  \hspace{1cm} (96)

This formula generalizes the one given by Eq. (92) extending its validity to \(N \approx (m_{Planck}/m_{n})^{3}\), leading to a baryonic core radius \(R_{c} \approx 10\) km. We also recall that a more detailed analysis of nuclear matter in bulk in neutron stars (see e.g. Ref. 140, 141) shows that at mass densities larger than the “melting” density of

$$\rho_{c} = 4.34 \cdot 10^{13} g/cm^{3},$$  \hspace{1cm} (97)

all nuclei disappear. In the description of nuclear matter in bulk we have to consider then the three Fermi degenerate gases of neutrons, protons and electrons. In turn this naturally leads to considering the role of strong and weak interactions among the nucleons. In the nucleus, the role of the strong and weak interaction, with a short range of one Fermi, is to bind the nucleons, with a binding energy of 8 MeV, in order to balance the Coulomb repulsion of the protons. In the neutron star case we have seen that the neutron confinement is due to gravity. We still assume that an essential role of the strong interactions is to balance the effective Coulomb repulsion due to the protons, partly screened by the electron distribution inside the neutron star core. We shall verify, for self-consistency, the validity of this assumption for the final equilibrium solution we will obtain.

We now turn to the essential weak interaction role in establishing the relative balance between neutrons, protons and electrons via the direct and inverse \(\beta\)-decay

$$p + e \rightarrow n + \nu_{e},$$  \hspace{1cm} (98)

$$n \rightarrow p + e + \bar{\nu}_{e}.$$  \hspace{1cm} (99)

Since neutrinos escape from the star and the Fermi energy of the electrons is zero, as we will show below, the only non-vanishing terms in the equilibrium condition given by the weak interactions are

$$[(P_{n}^{F}c)^{2} + M_{n}^{2}c^{4}]^{1/2} - M_{n}c^{2} = [(P_{p}^{F}c)^{2} + M_{p}^{2}c^{4}]^{1/2} - M_{p}c^{2} + |e|V_{coul}^{p},$$  \hspace{1cm} (100)

where \(P_{n}^{F}\) and \(P_{p}^{F}\) are respectively, the neutron and proton Fermi momenta, and \(V_{coul}^{p}\) is the Coulomb potential of the protons. At this point, having fixed all these physical constraints, the main task is to find the electron distributions satisfying not only the Dirac-Fermi statistics but also the electrostatic Maxwell equations.
The condition of equilibrium for the Fermi degenerate electrons implies a zero value for the Fermi energy

\[ \left( (P_F^e)^2 + m^2 c^4 \right)^{1/2} - mc^2 + eV_{\text{coul}}(r) = 0, \]

where \( P_F^e \) is the electron Fermi momentum and \( V_{\text{coul}}(r) \) is the Coulomb potential.

6.3. Relativistic Thomas-Fermi Equation for Nuclear Matter in Bulk

In line with the procedure already followed for heavy atoms\textsuperscript{132,136} we adopt here the relativistic Thomas-Fermi Equation

\[ \frac{1}{x} \frac{d^2 \chi(x)}{dx^2} = -4\pi \alpha \left\{ \theta(x - x_c) - \frac{1}{3\pi^2} \left[ \left( \frac{\chi(x)}{x} + \beta \right)^2 - \beta^2 \right]^{3/2} \right\}, \]

where \( \alpha = e^2 / (\hbar c) \), \( \theta(x - x_c) \) represents the normalized proton density distribution, the variables \( x \) and \( \chi \) are related to the radial coordinate and the electron Coulomb potential \( V_{\text{coul}} \) by

\[ x = \frac{r}{R_c} \left( \frac{3N_p}{4\pi} \right)^{1/3}; \quad eV_{\text{coul}}(r) \equiv \frac{\chi(r)}{r}, \]

and the constants \( x_c, (r = R_c) \) and \( \beta \) are respectively

\[ x_c \equiv \left( \frac{3N_p}{4\pi} \right)^{1/3}; \quad \beta \equiv \frac{mcR_c}{\hbar} \left( \frac{4\pi}{3N_p} \right)^{1/3}. \]

The solution has the boundary conditions

\[ \chi(0) = 0; \quad \chi(\infty) = 0, \]

with the continuity of the function \( \chi \) and its first derivative \( \chi' \) at the boundary of the core \( R_c \). The crucial point is the determination of the eigenvalue of the first derivative at the center

\[ \chi'(0) = \text{const.}, \]

which has to be determined by satisfying the above boundary conditions (105) and constraints given by Eq. (100) and Eq. (95).

The difficulty of the integration of the Thomas-Fermi equations is certainly one of the most celebrated chapters in theoretical physics and mathematical physics, still challenging a proof of the existence and uniqueness of the solution and strenuously avoiding the occurrence of exact analytic solutions. We recall after the original papers of Ref. 142,143, the works of Ref. 144–147 all the way to the many hundred papers reviewed in the classical articles of Ref. 131,148,149. The situation here is more difficult since we are working on the special relativistic generalization of the Thomas-Fermi equation. We must therefore proceed by numerical integration in this case as well. The difficulty of this numerical task is further enhanced by a consistency check in order to satisfy all the various constraints.
We start the computations by assuming a total number of protons and a value of the core radius $R_c$. We integrate the Thomas-Fermi equation and determine the number of neutrons from the Eq. (100). We iterate the procedure until a value of $A$ is reached consistent with our choice of the core radius. The paramount difficulty of the problem is the numerical determination of the eigenvalue in Eq. (106) which already for $A \approx 10^4$ had presented remarkable numerical difficulties. In the present context we have been faced for a few months with an apparently insurmountable numerical task: the determination of the eigenvalue seemed to necessitate a significant number of decimal places in the first derivative (106) comparable to the number of the electrons in the problem! We shall discuss elsewhere the way we overcame this difficulty by splitting the problem on the basis of the physical interpretation of the solution.

The solution is given in Fig. (14) and Fig. (15).

![Fig. 14. The solution $\chi$ of the relativistic Thomas-Fermi equation for $A = 10^{57}$ and core radius $R_c = 10\text{km}$ is plotted as a function of radial coordinate. The left red line corresponds to the internal solution and it is plotted as a function of radial coordinate in units of $R_c$ in logarithmic scale. The right blue line corresponds to the solution external to the core and it is plotted as function of the distance $\Delta r$ from the surface in a logarithmic scale in centimeters.]

A relevant quantity for exploring the physical significance of the solution is given by the number of electrons within a given radius $r$

$$N_e(r) = \int_0^r 4\pi (r')^2 n_e(r') \, dr'.$$  \hspace{1cm} (107)

This allows the determination of the distribution of the electrons inside and outside the core for selected values of the $A$ parameter, and follows the progressive penetration of the electrons in the core as $A$ increases [see Fig. (16)]. Then we can evaluate the net charge inside the core

$$N_{\text{net}} = N_p - N_e(R_c) < N_p,$$  \hspace{1cm} (108)

generalizing the results in Ref. 132,136 and consequently determine the electric field
at the core surface, as well as inside and outside the core [see Fig. (17)] and evaluate as well the Fermi degenerate electron distribution outside the core [see Fig. (18)].

It is interesting to explore the solution of the problem under the same conditions and constraints imposed by the fundamental interactions and the quantum statistics and imposing the corresponding Eq. (95) instead of Eq. (90). Indeed a solution exists and is much simpler

\[ n_n(x) = n_p(x) = n_e(x) = 0, \quad \chi = 0. \]  

(109)

6.4. The energetic stability of the solution

Before drawing our conclusions we should check the theoretical consistency of the solution. We obtain an overall neutral configuration for the nuclear matter in bulk, with a positively charged baryonic core with

\[ N_{\text{net}} = 0.92 \left( \frac{m}{m_\pi} \right)^2 \left( \frac{e}{m_n \sqrt{G}} \right) \left( \frac{1}{\alpha} \right)^2, \]  

(110)

and an electric field at the baryonic core surface (see Fig. (17))

\[ \frac{E}{E_c} = 0.92. \]  

(111)

The corresponding Coulomb repulsive energy per nucleon is given by

\[ U_{\text{max, Coul}} = \frac{1}{24} \left( \frac{m}{m_\pi} \right)^3 mc^2 \approx 1.78 \cdot 10^{-6}(\text{MeV}), \]  

(112)

well below the nucleon binding energy per nucleon. It is also important to verify that this charge core is gravitationally stable. We have in fact

\[ \frac{Q}{\sqrt{GM}} = \alpha^{-1/2} \left( \frac{m}{m_\pi} \right)^2 \approx 1.56 \cdot 10^{-4}. \]  

(113)
Fig. 16. The electron number (10^7) in units of the total proton number \( N_p \) is given as function of radial distance in units of the core radius \( R_c \) for selected values of \( A \), again in a logarithmic scale. It is clear that by increasing the value of \( A \), the penetration of electrons inside the core increases. The detail shown in Fig. (17) and Fig. (18) demonstrates how for \( N \approx (m_{\text{Planck}}/m_n)^3 \) a relatively small tail of electrons outside the core exists and generates on the baryonic core surface an electric field close to the critical value. A significant electron density outside the core is found.

Fig. 17. The electric field in units of the critical field \( E_c \) is plotted around the core radius \( R_c \). The left (right) diagram in the red (blue) refers the region just inside (outside) the core radius plotted logarithmically. By increasing the density of the star the field approaches the critical field.
The electric field of the baryonic core is screened to infinity by an electron distribution given in Fig. (18).

As has been the case previously, any new solution for a Thomas-Fermi system has relevance and finds its justification in the domain of theoretical and mathematical physics. We expect that as in the case of other solutions that have appeared in the literature of the relativistic Thomas-Fermi equations, this new one presented here will find important applications in physics and astrophysics. There are a variety of new effects that such a generalized approach naturally leads to: (1) the energetics of the global neutrality solution is greatly different from the one obtained from the condition of local neutrality; (2) the formation process for a neutron star can also have specific new signatures, due to reaching a more tightly bound system; (3) we expect important consequences on the initial conditions in the physics of gravitational collapse of the baryonic core as soon as the protons and neutrons become relativistic and the critical mass for gravitational collapse to a black hole is reached. The consequent collapse to a black hole will have very different energetics properties, since the initial conditions will imply the existence of a critical electric field. Such a field will naturally lead to very strong processes of pair creation during the following phases of gravitational collapse. This research is ongoing.

We now turn to the interpretation of the GRB data within the above theoretical
framework and recall some basic interpretational paradigms that we have introduced in order to reach a systematic understanding of these sources.

7. The first paradigm: The Relative Space-Time Transformation (RSTT) paradigm

The ongoing dialogue between our work and that of others who model GRBs still rests on some elementary considerations presented by Einstein in his classic article of 1905. These considerations are quite general and even precede Einstein’s derivation of the Lorentz transformations from first principles. We recall here Einstein’s words: “We might, of course, content ourselves with time values determined by an observer stationed together with the watch at the origin of the coordinates, and coordinating the corresponding positions of the hands with light signals, given out by every event to be timed, and reaching him through empty space. But this coordination has the disadvantage that it is not independent of the standpoint of the observer with the watch or clock, as we know from experience.

Einstein’s message is simply illustrated in Fig. 19. If we consider in an inertial frame a source (solid line) moving with high speed and emitting light signals (dashed lines) along the direction of its motion, a far away observer will measure a delay \( \Delta t_a \) between the arrival time of two signals respectively emitted at the origin and after a time interval \( \Delta t \) in the laboratory frame, which in our case is the frame where the black hole is at rest. The real velocity of the source is given by

\[
v = \frac{\Delta r}{\Delta t}
\]

and the apparent velocity is given by:

\[
v_{\text{app}} = \frac{\Delta r}{\Delta t_a},
\]

As pointed out by Einstein, the adoption of coordinating light signals simply by their arrival time as in Eq. (115), without an adequate definition of synchronization, is incorrect and leads to insurmountable difficulties as well as to apparently ‘superluminal velocities as soon as motions close to the speed of light are considered.

The use of \( \Delta t_a \) as a time coordinate, often tacitly adopted by astronomers, should be done, if at all, cautiously. The relation between \( \Delta t_a \) and the correct time parameterization in the laboratory frame has to be taken into account

\[
\Delta t_a = \Delta t - \frac{\Delta r}{c} = \Delta t - \frac{1}{c} \int_{t_1}^{t_1 + \Delta t} v(t') \, dt'.
\]

In other words, the relation between the arrival time and the laboratory time cannot be done without a knowledge of the speed along the entire world line of the source. In the case of GRBs, such a world line starts at the moment of gravitational collapse. It is of course clear that the parameterization in the laboratory frame has to take into account the cosmological redshift \( z \) of the source. We then have at the detector

\[
\Delta t_a^d = (1 + z) \Delta t_a.
\]
Fig. 19. Relation between the arrival time $t_a$ and the laboratory time $t$. Details in Ref. 126,152.

In the current GRB literature, Eq. (116) has been systematically neglected by addressing only the afterglow description and neglecting the previous history of the source. Often the integral equation has been approximated by a clearly incorrect instantaneous value

$$
\Delta t_a \approx \Delta t^2 \gamma^2.
$$

The approach has been adopted to consider the afterglow part of the GRB phenomenon separately without knowledge of the entire equation of motion of the source.

This point of view has reached its most extreme expression in the work reviewed...
by Ref. 153,154, where the so-called “prompt radiation”, lasting on the order of 10^2 s, is considered as a burst emitted by the prolonged activity of an “inner engine. In these models, generally referred to as the “internal shock model, the emission of the afterglow is assumed to follow the “prompt radiation” phase.155–159 As we outline in the following sections, such an extreme point of view originates from the inability to obtain the time scale of the “prompt radiation” from a burst structure. These authors consequently appeal to the existence of an “ad hoc” inner engine in the GRB source to solve this problem.

We show in the following sections how this difficulty has been overcome in our approach by interpreting the “prompt radiation” as an integral part of the afterglow and not as a burst. This explanation can be reached only through a relativistically correct theoretical description of the entire afterglow (see next sections). Within the framework of special relativity we show that it is not possible to describe a GRB phenomenon by disregarding the knowledge of the entire past world line of the source. We show that at 10^2 seconds the emission occurs from a region of dimensions of approximately 10^{16} cm, well within the region of activity of the afterglow. This point was not appreciated in the current literature due to the neglect of the apparent superluminal effects implied by the use of the “pathological” parametrization of the GRB phenomenon by the arrival time of light signals.

We now turn to the first paradigm, the relative space-time transformation (RSTT) paradigm,126 which emphasizes the importance of a global analysis of the GRB phenomenon encompassing both the optically thick and the afterglow phases. Since all the data are received in terms of the detector arrival time, it is essential to know the equations of motion of all relativistic phases of the GRB sources with \( \gamma > 1 \) in order to reconstruct the corresponding time coordinate in the laboratory frame, see Eq. (116). Contrary to other phenomena in nonrelativistic physics or astrophysics, where every phase can be examined separately from the others, in the case of GRBs all the phases are inter-related by their signals received in the arrival time \( t^a_d \). In order to describe the physics of the source, there is the need to derive the laboratory time \( t \) as a function of the arrival time \( t^a_d \) along the entire past world line of the source using Eq. (117).

An additional difference, also linked to special relativity, between our treatment and others in the current literature relates to the assumption of the existence of scaling laws in the afterglow phase: the power law dependence of the Lorentz gamma factor on the radial coordinate is usually systematically assumed. From the proper use of the relativistic transformations and by the direct numerical and analytic integration of the special relativistic equations of motion we demonstrate (see next sections) that no simple power-law relation can be derived for the equations of motion of the system. This situation is not new for workers in relativistic theories: scaling laws exist in the extreme ultrarelativistic regimes and in the Newtonian ones but not in the intermediate fully relativistic regimes (see e.g. Ref. 96).
8. The second paradigm: The Interpretation of the Burst Structure (IBS) paradigm

We turn now to the second paradigm, which is more complex since it deals with all the different phases of the GRB phenomenon. We first address the dynamical phases following the dyadosphere formation.

After the vacuum polarization process around a black hole, one of the topics of the greatest scientific interest is the analysis of the dynamics of the electron-positron plasma formed in the dyadosphere. This issue was addressed by us in a collaboration with Jim Wilson at Livermore. The numerical simulations of this problem were developed at Livermore, while the semi-analytic approach was developed in Rome (see Ruffini et al.\textsuperscript{106,107} and next sections). The corresponding treatment in the framework of the Cavallo, Rees et al. analysis was performed by Piran et al.\textsuperscript{160} also using a numerical approach, by Bisnovatyi-Kogan & Murzina\textsuperscript{161} using an analytic approach and by Mészáros et al.\textsuperscript{162} using a numerical and semi-analytic approach.

Although some similarities exist between these treatments, they are significantly different in the theoretical details and in the final results (see Ref. 163 and next sections). Since the final result of the GRB model is extremely sensitive to any departure from the correct treatment, it is indeed very important to detect at every step the appearance of possible fatal errors.

8.1. The optically thick phase of the fireshell

A conclusion common to all these treatments is that the electron-positron plasma is initially optically thick and expands till transparency reaching very high values of the Lorentz gamma factor. A second point, which is also common, is the discovery of a clearly new feature: the plasma shell expands but the Lorentz contraction is such that its width in the laboratory frame appears to be constant. This self acceleration of the thin shell is the distinguishing factor of GRBs, conceptually very different from the physics of a fireball developed by the inner pressure of an atomic bomb explosion in the Earth’s atmosphere. In the case of GRBs the region interior to the shell is inert and with pressure totally negligible: the entire dynamics occurs on the shell itself. For this reason, we refer in the following to the self accelerating shell as the “fireshell.”

There is a major difference between our approach and those of Piran, Mészáros and Rees in that we assume the dyadosphere to be initially filled only with an electron-positron plasma. Such a plasma expands in substantial agreement with the results presented in the work of Ref. 161. In our model the fireshell of electron-positron pairs and photons (PEM pulse)\textsuperscript{106} evolves and encounters the remnant of the star progenitor of the newly formed black hole. The fireshell is then loaded with baryons. A new fireshell is formed of electron-positron-photons and baryons (PEMB pulse)\textsuperscript{107} which expands all the way until transparency is reached. At transparency the emitted photons give origin to what we define as the Proper-GRB (P-GRB, see Ref. 127 and Fig. 20).
In our approach, the baryon loading is measured by a dimensionless quantity

$$B = \frac{M_B c^2}{E_{\text{dyn}}},$$

(119)
which gives direct information about the mass $M_B = N_B m_p$ of the remnant, where $m_p$ is the proton mass. The corresponding treatment done by Piran and collaborators\cite{160,164} and by Ref. 162 differs in one important respect: the baryonic loading is assumed to occur from the beginning of the electron-positron pair formation and no relation to the mass of the remnant of the collapsed progenitor star is attributed to it.

A further difference also exists between our description of the rate equation for the electron-positron pairs and the ones by those authors. While our results are comparable with the ones obtained by Piran under the same initial conditions, the set of approximations adopted by Ref. 162 appears to be too radical and leads to very different results violating energy and momentum conservation (see next sections and Ref. 163).

From our analysis\cite{107} it also becomes clear that such an expanding dynamical evolution can only occur for values of $B \leq 10^{-2}$ (see Fig. 21). This prediction, as we will show shortly in the many GRB sources considered, is very satisfactorily confirmed by observations and is indeed essential in order to reach the high values of the Lorentz gamma factor observed in GRBs.

From the value of the $B$ parameter, related to the mass of the remnant, it therefore follows that the collapse to a black hole leading to a GRB is drastically different from the collapse to a neutron star. While in the case of a neutron star collapse a very large amount of matter is expelled, in many instances well above the mass of the neutron star itself, in the case of black holes leading to a GRB only a very small fraction of the initial mass ($\sim 10^{-2}$ or less) is expelled. The collapse to a black hole giving rise to a GRB appears to be much smoother than any collapse process considered until today: almost 99.9% of the star has to collapse at once to form the black hole!

We summarize in Fig. 20 the optically thick phase of the fireshell evolution: we start from a given dyadosphere of energy $E_{dy}$; the fireshell self-accelerates outward; an abrupt decrease in the value of the Lorentz gamma factor occurs due to the engulfment of the baryonic loading followed by a further self-acceleration until the fireshell becomes transparent.

The photon emission at this transparency point is the P-GRB. An accelerated beam of baryons with an initial Lorentz gamma factor $\gamma_0$ starts to interact with the interstellar medium at typical distances from the black hole of $r_0 \sim 10^{14}$ cm and at a photon arrival time at the detector on the Earth’s surface of $t_d \sim 0.1$ s. These values determine the initial conditions of the afterglow.

### 8.2. Hydrodynamics of the pair plasma

We give a systematic derivation of the main equations, present a critical review of existing models for isotropic relativistic fireballs, and compare and contrast these models, following Ref. 163. In the next section, following Ref. 165 we derive basic equations and describe the approximations involved. Then we present the
Fig. 21. A sequence of snapshots of the coordinate baryon energy density is shown from the one dimensional hydrodynamic calculations of the Livermore code. The radial coordinate is given in units of dyadoosphere radii ($r_{ds}$). At $r \approx 100r_{ds}$, there is located a baryonic matter shell corresponding to a baryon loading $B = 1.3 \times 10^{-2}$. For this baryon shell mass we see a significant departure from the constant thickness solution for the fireshell dynamics and a clear instability occurs. Details in Ref. 107. As we will see, this result, peculiar to our treatment, will play a major role in the theoretical interpretation of GRBs.

model$^{106,107}$ which differs from other models in the literature in that it describes the dynamics of the fireshell taking into account the rate equations for electron-positron pairs. Then we compare and contrast the above mentioned models.
8.2.1. Local, global and average conservation laws

**Particle number.** The first relevant equation represents continuity of relativistic flux and reads

\[(nU^\mu)_\mu = \frac{1}{\sqrt{-g}} \partial (\sqrt{-g} nU^\mu) = 0, \quad (120)\]

where \(n\) is the number density of relativistic fluid and \(U^\mu\) is its velocity field. Defining the particle number by

\[N = \int_V \sqrt{-g} nU^0 dV, \quad (121)\]

we see that

\[dN dt = - \int_V \sqrt{-g} nU^i dV = - \oint_{\Sigma} \sqrt{-g} nU^i dS, \quad (122)\]

where we have used the Ostrogradsky-Gauss theorem. Thus if particles do not cross the surface \(\Sigma\) bounding the volume \(V\) considered under consideration, the total number of particles is constant during the system evolution.

Now assume spherical symmetry\(^d\), which is usually done for fireball descriptions. With spherical spatial coordinates \(x^i = \{r, \vartheta, \varphi\}\) the interval is

\[ds^2 = -dt^2 + r^2 d\vartheta^2 + r^2 \sin^2 \vartheta d\varphi^2. \quad (123)\]

Assuming the absence of fluxes through the boundary \(\Sigma\) we rewrite (120)

\[\frac{\partial (n\gamma)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 n \sqrt{\gamma^2 - 1} \right) = 0. \quad (124)\]

Integrating this equation over the volume from some initial \(r_i(t)\) to a radius \(r_e(t)\) which we assume to be comoving with the fluid

\[\frac{dr_i(t)}{dt} = \beta(r_i, t), \quad \frac{dr_e(t)}{dt} = \beta(r_e, t), \quad (125)\]

\(^c\)Greek indices denote four-dimensional components and run from 0 to 3 while Latin indices run from 1 to 3. General relativistic effects are neglected, which is a good approximation, but we use the general definition of energy-momentum conservation to take into account the most general coordinate system.

\(^d\)The only nonvanishing components of the energy-momentum tensor are \(T^{00}, T^{01}, T^{10}, T^{11}, T^{22}\) and \(T^{33}\). The factor \(\sqrt{-g} = r^2 \sin \vartheta\) in all expressions above becomes simply a volume measure and the differentials are \(dV = dr dr d\vartheta, dS = d\vartheta d\varphi\), so the differential laboratory volume can be written as \(dV = \sqrt{-g} dV = r^2 \sin \vartheta dr d\vartheta d\varphi\).
and ignoring a factor $4\pi$ we have

$$\int_{r_i}^{r_e} \frac{\partial}{\partial t} (n\gamma) r^2 dr + \int_{r_i}^{r_e} \frac{\partial}{\partial r} \left( r^2 n \sqrt{\gamma^2 - 1} \right) dr =$$

$$\frac{\partial}{\partial t} \int_{r_i}^{r_e} (n\gamma) r^2 dr - \frac{\partial}{\partial r} \left( \frac{dr}{dt} n(r_e,t) \gamma(r_e,t) r_e^2 + \frac{dr}{dt} n(r_i,t) \gamma(r_i,t) r_i^2 + r_e^2 n(r_e,t) \sqrt{\gamma^2(r_e,t) - 1} - r_i^2 n(r_i,t) \sqrt{\gamma^2(r_i,t) - 1} \right) =$$

$$= \frac{d}{dt} \int_{r_i}^{r_e} (n\gamma) r^2 dr = 0,$$

(126)

Since we deal with arbitrary comoving boundaries, this means that the number of particles in each shell between the boundaries is conserved as well as the total number of particles integrated over all shells, in other words,

$$N = 4\pi \int_0^{R(t)} n\gamma r^2 dr = \text{const},$$

(127)

where $R(t)$ is the external radius of the fireshell.

Following Ref. 160 one can transform (124) from the variables $(t,r)$ to the new variables $(s = t - r, r)$ and then show that

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 n \sqrt{\gamma^2 - 1} \right) = -\frac{\partial}{\partial s} \left( \frac{n}{\gamma + \sqrt{\gamma^2 - 1}} \right).$$

(128)

Now assume the expansion velocity is ultrarelativistic,

$$\gamma \gg 1.$$  

(129)

In this approximation, therefore,

$$dN = 4\pi n\gamma r^2 dr \approx \text{const}.$$  

(130)

Relations (130) and (127) then imply

$$4\pi \int_{r_i}^{r_e} (n\gamma r^2) dr = 4\pi \left[ n(r,t) \gamma(r,t) r_e^2 \right] \int_{r_i}^{r_e} dr = 4\pi \left( n\gamma r^2 \right) \Delta \approx \text{const},$$

(131)

where the first argument of the functions $n(r,t)$ and $\gamma(r,t)$ is restricted to the interval $r_i < r < r_e$ and

$$\Delta \equiv r_e - r_i \approx \text{const}.$$  

(132)

This means that the fluid shell does not broaden, but rather has a constant thickness. This fact proves the constant thickness approximation, adopted in 106,107.
The volume element measured by the observer outside the fireshell (to be referred to as the lab frame in what follows), for which it appears to be moving with velocity $\beta$ is just

$$dV = 4\pi r^2 dr,$$

while the volume element comoving with the fireshell, for which the fluid is at rest, is

$$dV = 4\pi \gamma r^2 dr,$$

with the conversion of the volumes

$$dV = \gamma dV.$$

Then the average value of the Lorentz factor is defined as follows

$$\langle \gamma \rangle \equiv \frac{4\pi \int \gamma r^2 dr}{4\pi \int r^2 dr} = \frac{V}{V}.$$

Now we can formulate the conservation law for the average value of the number density in the lab frame

$$\langle n \rangle_{\text{lab}} \equiv \frac{N}{V} = \frac{4\pi \int_{r_i}^{r} n \gamma r^2 dr}{4\pi \int_{r_i}^{r} r^2 dr}.$$

Assuming $r \gg \Delta$ we then obtain

$$\langle n \rangle_{\text{lab}} \approx \frac{4\pi W \gamma^2 \Delta}{4\pi r^2 \Delta} = n(r, t) \gamma(r, t) \propto r^{-2}.$$

Therefore, the average number density in the lab frame scales like $r^{-2}$.

At the same time, recalling the expression for the divergence of the four-velocity $U^\mu$;

$$\mu \in \frac{1}{V} \frac{dV}{d\tau},$$

where $\tau$ is the proper time, and remembering that $U^\mu \frac{d}{d\tau} \frac{\partial}{\partial x^\mu} = \frac{d}{d\tau}$, from (120) we get

$$\langle n U^\mu \rangle_{\mu} = U^\mu n_{,\mu} + n U^\mu_{,\mu} = \frac{dn}{d\tau} + n \frac{dV}{V d\tau} = 0,$$

$$d \ln n + d \ln V = 0.$$

This means that the number of particles is conserved along the flow lines of the fluid. The solution of this equation provides the definition for the comoving average
number density

\[ \langle n \rangle_{\text{com}} \equiv \frac{N}{V} = \frac{4\pi}{4\pi r_e^2} \int_{r_i}^{r_e} \gamma r^2 \, dr = \frac{\langle n \rangle_{\text{lab}}}{\langle \gamma \rangle}. \]  

(141)

Clearly, the condition (141) gives a link between the description of the fireshell evolution in terms of local functions entering (124) on one side of the equation, and global quantities (136) and (138) on the other. The presence of the global conservation (127) in both these cases ensures equivalence of the local (124) and the average (137) descriptions for the fireshell, unless its detailed structure is considered.

**Energy-momentum conservation.** The basis of the description of a relativistic fireshell is the energy-momentum principle. It allows one to obtain the relativistic hydrodynamic equations, or equations of motion for the fireshell, the energy and momentum conservation equations which are used extensively to describe interaction of relativistic baryons of the fireshell with the interstellar matter, and the boundary conditions which are used to understand shock wave propagation in the decelerating baryons and in the outer medium.

Consider energy-momentum conservation in the most general form:

\[ \left( T_{\mu}^{\, \nu} \right)_{; \nu} = \frac{\partial}{\partial x^{\nu}} \left( \sqrt{-g} T_{\mu}^{\, \nu} \right) + \sqrt{-g} \Gamma_{\mu \nu}^{\alpha} T_{\alpha}^{\lambda} = 0, \]  

(142)

where \( \Gamma_{\mu \nu}^{\alpha} \) are the Christoffel symbols and \( g \) is the determinant of the metric tensor. Integrating over the entire three-dimensional volume we obtain

\[ \int_V T_{\mu}^{\, \nu} \, dV = 0. \]  

(143)

Integrating over the entire four-dimensional volume and applying the divergence theorem we get\textsuperscript{166}

\[ \int_V \int_t T_{\mu}^{\, \nu} \, dV \, dt = \oint_V T_{\mu}^{\, \nu} \lambda_\nu dV = 0, \]  

(144)

where \( \lambda_\nu \) are covariant components of the outer normal to the three-dimensional hypersurface (volume \( V \)) enclosing the spacetime region.

Now suppose that there is a discontinuity on the fluid flow. Taking the volume to be a spherical shell and choosing the coordinate system in which the discontinuity is at rest so that in (144) for normal vectors to the discontinuity hypersurface \( \lambda_\alpha \), we have

\[ \lambda_\alpha \lambda^\alpha = 1, \quad \lambda_0 = 0. \]  

(145)
Let the radius of the shell $R_s$ be very large and the shell thickness $\Delta$ very small. With $R_s \to \infty$ and $\Delta \to 0$ from (144) we get

$$\langle T^\alpha \rangle = 0,$$

(146)

where the brackets mean that the quantity inside is the same on both sides of the discontinuity surface. This equation together with continuity condition for particle density flux $\langle n U \rangle = 0$ was used by Ref. 166 to obtain the relativistic Rankine-Hugoniot equations. These equations govern shock wave dynamics which are supposed to appear during the collision of the baryonic material left from the fireshell with the interstellar medium.\(^\text{167}\)

The origin of the afterglow could be connected\(^\text{168–170}\) to the conversion of kinetic energy into radiative energy in these shocks.

Consider now the energy-momentum tensor of the perfect fluid in the lab frame (where the fluid was initially at rest)

$$T^\mu_\nu = p g^\mu_\nu + \omega U^\mu U^\nu,$$

(147)

where $\omega = \rho + p$ is the proper enthalpy, $p$ is the proper pressure and $\rho$ is the proper internal energy density.

Rewrite (142) in the spherically symmetric case

$$\frac{\partial T^0_0}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 T^0_1 \right) = 0,$$

(148)

$$\frac{\partial T^1_0}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 T^1_1 \right) - \frac{1}{r} \left( T^2_2 + T^3_3 \right) = 0,$$

(149)

arriving at the equations of motion of a relativistic fireshell\(^\text{106,160,167}\)

$$\frac{\partial (\gamma^2 \omega)}{\partial t} - \frac{\partial p}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \gamma^2 \beta \omega \right) = 0,$$

(150)

$$\frac{\partial (\gamma^2 \beta \omega)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 (\gamma^2 - 1) \omega \right] + \frac{\partial p}{\partial r} = 0,$$

(151)

where the four-velocity and the relativistic Lorentz factor are defined as follows\(^a\)

$$U^\mu = (\gamma, \gamma \beta, 0, 0), \quad \gamma \equiv (1 - \beta^2)^{-1/2},$$

(152)

and $\beta$ is the radial velocity.

The total momentum of the spherically symmetric expanding shell vanishes. However, from the local conservation equations (150) one finds that the radial component of the four-momentum vector does not vanish. In analogy with the continuity equation (124) we integrate the first equation in (150) over the volume starting from some internal radius $r_i(t)$ up to some external radius $r_e(t)$, and ignoring a factor throughout this chapter we set the speed of light equal to 1.

\(^a\)Throughout this chapter we set the speed of light equal to 1.
If the boundaries \( r_i(t) \) and \( r_e(t) \) are comoving with the fluid we have

\[
\frac{d}{dt} \int_{r_i}^{r_e} (\gamma^2 \omega - p) r^2 \, dr = r_e^2 p(r_e) - r_i^2 p(r_i).
\]

Further, if one assumes (129), one gets the following result

\[
E = 4\pi \int_0^{R(t)} \gamma^2 \omega r^2 \, dr = \text{const.}
\]

The differential conservation law follows from the same arguments which lead to (130), so we also have

\[
dE = 4\pi \gamma^2 \omega r^2 \, dr \approx \text{const.}
\]

Analogously to (137) we introduce the average energy density in the lab frame

\[
\langle \rho \rangle_{\text{lab}} \equiv \frac{E}{V} = \frac{4\pi \int_{r_i}^{r_e} (\gamma^2 \omega) r^2 \, dr}{4\pi \int_{r_i}^{r_e} r^2 \, dr},
\]

Taking the polytropic equation of state with the thermal index

\[
\Gamma \equiv 1 + \frac{p}{\rho},
\]

and requiring also \( r \gg \Delta \) and (129) we find from (158)

\[
\langle \rho \rangle_{\text{lab}} \approx \rho(r) \gamma^2(r) \propto r^{-2}.
\]

The radial momentum equation follows from (151)
\[
\int_{r_1}^{r_0} \frac{\partial (\gamma^2 \beta \omega)}{\partial t} r^2 dr + \int_{r_1}^{r_0} \frac{1}{r^2} \frac{\partial}{\partial r} [r^2 (\gamma^2 - 1) \omega] r^2 dr + \int_{r_1}^{r_0} \frac{\partial p}{\partial r} r^2 dr \\
= \frac{\partial}{\partial t} \int_{r_1}^{r_0} \gamma^2 \beta \omega r^2 dr + r^2 \gamma^2 (r_0) \omega(r_0) \beta^2 (r_0) - r^2 \gamma^2 (r_0) \omega(r_0) \beta^2 (r_0) + \\
+ r^2 \left( \gamma^2 (r_1) - 1 \right) \omega(r_1) - r^2 \gamma^2 (r_1) - 1 \omega(r_1) + \int_{r_1}^{r_0} \frac{\partial p}{\partial r} r^2 dr = 0. 
\]

This leads to
\[
\frac{d}{dt} \int_{r_1}^{r_0} (\gamma^2 \beta \omega) r^2 dr = 2 \int_{r_1}^{r_0} p r dr + r^2 p(r_1) - r^2 p(r_0). 
\]

For the radial momentum we have
\[
\frac{dP_{\text{tot}}}{dt} = \frac{d}{dt} \int_0^{\mathcal{R}(t)} 4\pi (\gamma^2 \beta \omega) r^2 dr = 8\pi \int_0^{\mathcal{R}(t)} p r dr. 
\]

The left hand side of this equation is the time derivative of the radial momentum, i.e., the radial “force”. The right hand side is the integral of the pressure over all the shells, so it is clear that unless the pressure in the fireshell is zero, it experiences self-acceleration due to internal pressure.

**Entropy conservation.** Yet another relevant equation is entropy conservation which may be obtained from (142) by projection along the flow line
\[
U^\mu (T^\nu_{\mu})_{\nu} = (U^\nu T^\mu_{\nu})_{\nu} - T^\mu_{\nu} (U^\nu)_{\nu} = 0. 
\]

The second term on the last line vanishes since \(U^\nu U^\mu = -1\), so we have another conservation equation
\[
- U^\mu (T^\nu_{\mu})_{\nu} = (\rho U^\mu)_{\mu} + p U^\mu_{\mu} = 0. 
\]

This conservation law corresponds to another conserved quantity, the entropy. In fact, Eq. (165) can be rewritten as
\[
(\rho U^\mu)_{\mu} + p U^\mu_{\mu} = (\omega U^\mu)_{\mu} - U^\mu p_{\mu} = 0. 
\]

Now using the continuity equation (120) and the identity \(\omega U^\mu = n U^\mu \left( \frac{\varpi}{n} \right) \) we find
\[
(\omega U^\mu)_{\mu} - U^\mu p_{\mu} = n U^\mu \left( \frac{\varpi}{n} \right) - \frac{1}{n} p_{\mu} = 0. 
\]

But the functions inside the brackets are scalars, and therefore covariant derivatives can be replaced by ordinary derivatives. Then we recall the second law of thermodynamics\(^{171}\)
\[
d \left( \frac{\varpi}{n} \right) = T d \left( \frac{\varpi}{n} \right) + \frac{1}{n} dp, 
\]
and finally obtain
\[ nTU^\mu \left( \gamma \frac{\partial}{\partial n} \right)_\mu = 0, \quad (169) \]
which can be rewritten using (120) as
\[ (\sigma U^\mu)_\mu = 0. \quad (170) \]
This is the continuity equation for the entropy. Since it has exactly the same form as (120), all the conservation equations such as (130) and (127) hold for the entropy as well,
\[ d\sigma = 4\pi (\sigma \gamma) r^2 dr \approx \text{const}, \quad (171) \]
\[ S = \frac{R(t)}{\gamma} \int_0^d d\sigma = \text{const}. \quad (172) \]
Assuming (159) we find from (165) and (139) the following result
\[ U^\mu \rho_\mu + \Gamma pU^\mu_\mu = d\ln \rho + \Gamma d\ln V = 0, \quad (173) \]
\[ \langle \rho \rangle_{\text{com}} V^T = \text{const}. \]
Finally, due to the similarity of Eqs. (130) and (171), the average entropy can be defined in the same manner as (137).

**Analogy with a Friedmann Universe.** It is easy to show that the conservation equations (130),(157) and (171) imply an analogy between the fireball and the Friedmann Universe, noticed first by Shemi and Piran.\[160,164\] In fact, this analogy is valid for a polytropic equation of state (159) and the ultrarelativistic expansion condition (129). First, using (159) and the integral form of (168) we obtain
\[ \sigma = \frac{\omega}{T}, \]
which leads to
\[ \rho \propto \sigma^\Gamma, \quad (174) \]
we rewrite the above mentioned conservation equations using (159)
\[ n\gamma r^2 = \text{const}, \]
\[ \rho\gamma r^2 = \text{const}, \]
\[ \rho\gamma^2 r^2 = \text{const}. \quad (175) \]
From these equations we then easily find
\[ \gamma \propto r^{\frac{2(\Gamma-1)}{2\Gamma-1}}, \]
\[ n \propto r^{-\frac{\Gamma-1}{2\Gamma}}, \]
\[ \rho \propto r^{-\frac{\Gamma}{2\Gamma}}. \quad (176) \]
Taking the ultrarelativistic equation of state with $\Gamma = 4/3$ we immediately obtain
\begin{align}
\gamma &\propto r, \\
n &\propto r^{-3}, \\
\rho &\propto r^{-4},
\end{align}
(177)
as opposed to the nonrelativistic equation of state with $\Gamma = 1$ with different scaling
\begin{align}
\gamma &\propto \text{const}, \\
n &\propto r^{-2}, \\
\rho &\propto r^{-2}.
\end{align}
(178)

Actually, scaling laws (177) take place for the homogeneous isotropic radiation-dominated Universe.$^{160,164}$ This fact allowed the authors of Ref. 160 to speak about the frozen-pulse profile for $\gamma \gg 1$ where number, energy and entropy density is conserved within each differential shell with thickness $dr$, although the radial distribution of matter and energy can be inhomogeneous.

Although for observer inside the radiation-dominated fireshell it looks indistinguishable from a portion of radiation-dominated Universe, for the observer outside it looks drastically different. In fact, the validity of differential conservation laws (130),(157) and (171) together with integral ones (127),(156) and (172) implies constant thickness approximation assumed in 106,107.

Clearly, if the condition (129) is satisfied, also
\begin{equation}
\Delta \ll R(t)
\end{equation}
(179)
is valid. Given the scalings (177) we then find
\begin{equation}
V = 4\pi \int_{R(t)-\Delta}^{R(t)} \gamma r^2 dr \simeq 4\pi \int_{R(t)-\Delta}^{R(t)} \delta r^3 dr \simeq 4\pi R^3,
\end{equation}
(180)where we put $\gamma = \delta r$, $\delta$ is a constant. At the same time,
\begin{equation}
V = 4\pi \int_{0}^{R(t)} r^2 dr = \frac{4\pi}{3} R^3.
\end{equation}
(181)

Equality of (180) and (181) up to a numerical factor suggests that the initially homogeneous energy and particle number distribution looks highly compressed in the lab frame expanding with ultrarelativistic velocity and compression factor $\gamma$.

### 8.2.2. Self-acceleration of the fireshell

For a fireshell which is initially optically thick the total energy is conserved. Assume that the fireshell consists of relativistic electrons, positrons and photons, and also that some admixture of a plasma in the form of photons and electrons is present such that the total charge is zero. While electrons are relativistic, the protons are not. The equation of state for pairs and electrons in such a case is given to a good approximation by that of an ultrarelativistic fluid: $p_{e^\pm, \gamma} = \rho_{e^\pm, \gamma}/3$. At the same
time for protons we have \( p_p \simeq 0 \). Therefore positrons and electrons together with photons can be considered to be one fluid with \( p_e = \rho_e / 3 \) since they are strongly coupled since the medium is optically thick. Instead protons have low pressure and little internal energy compared to their rest mass energy.

According to (156) we find

\[
\int_0^{R(t)} (\gamma^2 \omega - p)r^2 dr = \int_0^{R(t)} \gamma^2 \rho_r r^2 dr + \frac{4}{3} \int_0^{R(t)} \gamma^2 \rho_r r^2 dr. \tag{182}
\]

These two terms are the rest mass energy of protons \( M_B \) and the energy of the ultrarelativistic fluid \( E \) correspondingly, so we arrive at a simple result, expressing the total energy of the expanding relativistic shell in the lab frame

\[
\gamma (E + M) = \text{const}, \tag{183}
\]

which reads simply as \( E + M = \text{const} \) in the comoving frame taking into account the conversion of volumes (135).

For homogeneous distributions of matter, energy density and pressure the integrals (126), (155) and (162) reduce to

\[
n\gamma V = \text{const}, \tag{184}
\]

\[
[\gamma^2 (\rho + p)] V = \text{const},
\]

while in the comoving frame instead we would have

\[
nV = \text{const}, \tag{185}
\]

\[
\rho V = \text{const}, \tag{186}
\]

which means the energy and number of particles do not change.

From the above we have

\[
nu_{\text{com}}^0 V = nV = \text{const} = nu_{\text{lab}}^0 V = n\gamma V, \tag{187}
\]

\[
\mathcal{T}_{\text{com}}^0 V = \rho V = \text{const} = T_{\text{lab}}^0 V = [\gamma^2 (\rho + p)] V, \tag{188}
\]

remembering that all quantities \( n, \rho, p \) are always defined as comoving ones.

Energy conservation (182) for (129) implies

\[
\gamma = \gamma_0 \sqrt{\frac{\rho_0^0 + \Gamma \rho_0 V_0}{\rho_p + \Gamma \rho V}}. \tag{189}
\]

Clearly all the equations given above can be written for the average values of the number and energy densities.
8.2.3. Quasi-analytic model of GRBs

The first detailed models for the expansion of a relativistic fireball were suggested in the beginning of nineties.\cite{160,162,164} Independent calculations were performed in Ref. 106 and 107. The main difference of these last two articles from the other models in the literature is that initially not photons but pairs are created by an overcritical electric field, and these pairs produce photons later. This plasma referred to as the pair-electro-magnetic (PEM) pulse expands initially into the vacuum surrounding the black hole reaching relativistic velocities very quickly. Then the collision with the baryonic remnant of the collapsed star takes place and the PEM pulse becomes a pair-electro-magnetic-baryonic (PEMB) pulse, see Ref. 152 for details. This difference is not large, since it was shown that the final gamma factor does not depend on the distance from the baryonic remnant or on the parameters of the black hole. The only crucial parameters are again the initial energy $E_0$ and baryonic admixture $B$.

The model is based on the numerical integration of the relativistic energy-momentum conservation equations (150,151) together with the baryonic number conservation equation (120). However, the most important point distinct from all previous models is that the rate equation for electron-positron pairs is added to the model and integrated simultaneously in order to give a more detailed description of the transparency. This latter fact leads to quantitative differences in predictions of the model with respect to the simplified models in the literature.

Here we concentrate on the simple quasi-analytical treatment presented in 106,107, see also 152. The PEMB pulse is supposed to contain a finite number of shells each with a flat density profile. Their dynamics is governed by the set of equations derived in the previous subsections. We collect (141),(173) and (189) together (omitting brackets for brevity)

$$\frac{n_B^0}{n_B} = \frac{V}{V_0} = \frac{\gamma}{\gamma_0},$$

$$\frac{\rho_0}{\rho} = \left(\frac{V}{V_0}\right)^\Gamma = \left(\frac{\gamma}{\gamma_0}\right)^\Gamma,$$

$$\frac{\gamma}{\gamma_0} = \sqrt{\frac{\rho_0}{\rho} + \Gamma \rho_0 V_0},$$

where subscript “0” denotes initial values, and all quantities are assumed to be averaged over a finite distribution of shells with constant width and density profiles. All components such as photons, electrons, positrons and plasma ions give a contribution to the energy density and pressure. This set of equations is equivalent to (200) and (202) (see below). The next step is to take into account the rate equation for positrons and elections, accounting for non-instant transparency:

$$(n_{e\pm} U^\mu)_{\mu} = \bar{\sigma} v (n_{e\pm}^2 (T) - n_{e\pm}^2),$$
or, integrating over the volume

\[ \frac{\partial}{\partial t} N_{e^{\pm}} = -N_{e^{\pm}} \frac{1}{V} \frac{\partial V}{\partial t} + \frac{1}{\gamma^2} (N_{e^{\pm}}^2(T) - N_{e^{\pm}}^2), \]

(194)

where \( \sigma \) is the mean pair annihilation-creation cross section and \( v \) is the thermal velocity of \( e^{\pm} \)-pairs. The coordinate number density of \( e^{\pm} \)-pairs in equilibrium is \( N_{e^{\pm}}(T) = \gamma n_{e^{\pm}}(T) \) and the coordinate number density of \( e^{\pm} \)-pairs is \( N_{e^{\pm}} = \gamma n_{e^{\pm}} \).

For \( T > m_e c^2 \) we have \( n_{e^{\pm}}(T) \approx n_{\gamma}(T) \), i.e., the number densities of pairs and photons are nearly equal. The pair number densities are given by appropriate Fermi integrals with zero chemical potential at the equilibrium temperature \( T \).

In a previous subsection we proved the applicability of the constant thickness approximation for the fireshell. The second conclusion appears to be crucial, since it shows that there is a critical loading of baryons at which their presence produces a turbulence in the outflow from the fireshell, and its motion becomes very complicated and the fireshell evolution does not lead in general to a GRB.

Exactly because of this reason, the optically thick fireshell never reaches a radius as large as \( r_b = r_0 \eta^2 \) which is discussed in 162, see section (8.2.4), since to do this the baryonic fraction should exceed the critical value \( B_c = 10^{-2} \). For larger values of \( B_c \) the theory reviewed here does not apply. This means in particular that all the conclusions of Ref. 162 obtained for \( r > r_b \) are invalid. In fact, for \( B < B_c \) the gamma factor even does not reach saturation.

Notice that another way to obtain the constraint \( B < B_c \) is to require the optical depth of the emitting region to be smaller than 1, leading to the requirement that the Lorentz factor be greater than \( \gamma \geq 10^2 \), see the introduction. At the same time, there is a simple relation between the Lorentz factor and the baryonic loading parameter \( B = \gamma^{-1} \) in the region \( 10^{-2} < B < 10^{-4} \), see Fig. 23, which leads to \( B \leq 10^{-2} \).

The fundamental result coming from this model are the diagrams shown in Fig. 22 and Fig. 23. The first one shows basically which portion of the initial energy is emitted in the form of gamma rays \( E_\gamma \) when the fireshell reaches the transparency condition \( \tau \approx 1 \) and how much energy gets converted into the kinetic energy \( E_k \) of the baryons left after pair annihilation and the photons escape.
Fig. 22. Relative energy release in the form of photons emitted at the transparency point (solid line) and the kinetic energy of the plasma (dashed line) of the baryons in terms of the initial energy of the fireball depending on parameter $B$ obtained on the basis of quasi-analytic model. The thick line denotes the total energy of the system in terms of the initial energy.

The second diagram gives the value of gamma factor at the moment when the system reaches transparency.

Energy conservation holds, namely

$$E_0 = E_\gamma + E_k.$$  \hspace{1cm} (195)

Clearly when the baryon abundance is low, most energy is emitted when the fireshell becomes transparent. It is remarkable that almost all initial energy is converted into the kinetic energy of the baryons already in the region of validity of the constant thickness approximation $B < 10^{-2}$, so the region $10^{-8} < B < 10^{-2}$ is the most interesting from this point of view.

8.2.4. Alternative models

Shemi and Piran model. In this section we discuss the model proposed by Ref. 164. This quantitative model gives a rather good general picture of relativistic fireballs. Shemi and Piran found that the temperature at which the fireball becomes optically thin is determined to be

$$T_{esc} = \min(T_g, T_p),$$  \hspace{1cm} (196)
Fig. 23. Relativistic gamma factor of the fireball when it reaches transparency depending on the value of parameter $B$. The dashed line gives the asymptotic value.

where $T_g$ and $T_p$ is the temperature when it reaches transparency with respect to the gas (plasma) or the pairs:

$$T_g^2 \simeq \frac{45}{8\pi^3} \frac{m_p}{m_e} \frac{1}{\alpha^2 g_0^2} \frac{1}{T_0^2 R_0^2} \eta,$$

$$T_p \simeq 0.032,$$

where $m_p$ is the proton mass, $g_0 = \frac{11}{4}$, $\alpha = \frac{1}{137}$, and the dimensionless temperature $T$ and radius $R$ of the fireball are measured in units of $\frac{m_e c^2}{\hbar}$ and $\lambda_e \equiv \frac{\hbar}{m_e c}$ correspondingly, and the subscript “0” denotes initial values. The temperature at the transparency point in the case when the plasma admixture is unimportant is nearly a constant for a range of parameters of interest and it almost equals

$$T_p = 15 \text{ keV}.$$  

Adiabatic expansion of the fireball implies

$$\frac{E}{E_0} = \frac{T}{T_0} = \frac{R_0}{R}.$$  

where

- $m_p$ is the mass of the proton,
- $g_0$ is the statistical weight factor
- $\alpha$ is the fine-structure constant
- $\eta$ is a dimensionless parameter
- $T_0$ is the initial temperature
- $R_0$ is the initial radius
- $E_0$ is the initial energy
- $E$, $T$, and $R$ are the current temperature, energy, and radius
- $\lambda_e$ is the wave length of the electron
- $\hbar$ is the reduced Planck constant
- $m_e$ is the mass of the electron
- $c$ is the speed of light
where \( E = \frac{E_m}{m c^2} \) is a radiative energy. From the energy conservation (142), assuming the fluid to be pressureless and its energy density profile to be constant, we have in the coordinate frame

\[
\int T_0^0 dV = \gamma E_{\text{tot}} = \text{const.} \quad (201)
\]

Assuming at initial moment \( \gamma_0 = 1 \) and remembering that \( E_{\text{tot}} = E + M c^2 \) we arrive at the following fundamental expression for the relativistic gamma factor \( \gamma \) at the transparency point:

\[
\gamma = \frac{E_0 + M c^2}{E + M c^2} = \frac{\eta + 1}{(\frac{E_0}{E})\eta + 1}, \quad (202)
\]

where \( M = \frac{M}{m} \).

One can use this relation to get such important characteristics of the GRB as the observed temperature and observed energy. In fact, they can be expressed as follows:

\[
T_{\text{obs}} = \gamma T_{\text{esc}}, \quad (203)
\]

\[
E_{\text{obs}} = E_0 \frac{T_{\text{obs}}}{T_{\text{esc}}}. \quad (204)
\]

These results are shown in Fig. 24. In the limit of small \( \eta \) we have \( \gamma = (1 + \eta) \), while for very large \( \eta \) the value of the gamma factor at the transparency point is \( \gamma = T_0/T_{\text{esc}} \), and it has a maximum at intermediate values of \( \eta \). We denote by a dashed thick line the limiting value of the \( \eta \) parameter \( \eta_0 \equiv B^{1,1} \). For \( \eta < \eta_0 \), the approximations used to construct the model do not hold. It is clear that because of the presence of bound \( \eta_0 \), the value \( \gamma = \eta \) can be reached only asymptotically. In effect, the value \( \eta_0 \) cuts the region where saturation of the gamma factor happens before the moment when the fireball becomes transparent.

It was found that for relatively large \( \eta \geq 10^5 \) the photons emitted when the fireball becomes transparent carry most of the initial energy. However, since the observed temperature in GRBs is smaller than the initial temperature of the fireball, one may assume that a large part of the initial energy is converted to kinetic energy of the plasma.

**Shemi, Piran and Narayan model.** 160 presents a generalization of this model to arbitrary initial density profile of the fireball. These authors performed numerical integrations of energy-momentum relativistic conservation equations (150),(151) and baryon number conservation equation (122). They were mainly interested in the evolution of the observed temperature, gamma factor and other quantities as the the radius increases. Their study results in a number of important conclusions, namely:
Fig. 24. The relativistic gamma factor (upper dashed line), the observed temperature (solid line), and the ratio of observed energy to the initial energy of the fireball (lower dashed line) as a function of baryonic loading parameter, see Ref. 164. The values of parameters are the same as in the cited paper. The thick dashed line denotes the limiting value of the baryonic loading. Its values when the gamma factor reaches a maximum and becomes constant are also shown.

- The expanding fireball has two basic phases: a radiation dominated phase and a matter-dominated phase. In the former, the gamma factor grows linearly with the radius of the fireball: $\gamma \propto r$, while in the latter the gamma factor reaches the asymptotic value $\gamma \simeq \eta + 1$.
- The numerical solutions are reproduced with good accuracy by the frozen-pulse approximation when the pulse width is given by initial radius of the fireball.

The last conclusion is important, since the fireball becomes a fireshell, and the volume $V$ can be calculated as

$$V = 4\pi R^2 \Delta,$$

where $\Delta \simeq R_0$ is the width of the leading shell with constant energy density profile and $R$ is the radius of the fireball.

They also present the following scaling solution:

$$R = R_0 \left( \frac{\gamma_0}{D^3} \right)^{1/2},$$

$$\frac{1}{D} \equiv \frac{\gamma_0}{\gamma} + \frac{3\gamma_0}{4\gamma\eta} - \frac{3}{4\eta},$$

$$\Delta \simeq R_0.$$
where the subscript “0” denotes some initial time when \( \gamma \gtrsim 2 \), which can be inverted to give \( \gamma(R) \).

**Mészáros, Laguna and Rees model.** The next step in developing this model was made by Ref. 162. In order to reconcile the model with observations, these authors proposed a generalization to the anisotropic (jet) case. Nevertheless, their analytic results apply to the case of homogeneous isotropic fireballs and we will follow their analytical isotropic model in this section.

Starting from the same point as Shemi and Piran, consider (200) and (202). The analytic part of the paper describes the geometry of the fireball, the gamma factor behavior and the final energy balance between radiation and kinetic energy. Magnetic field effects are also considered, but we are not interested in this part here.

Three basic regimes are found in Ref. 162 for the evolution of the fireball. In two first regimes there is a correspondence between the analysis in the paper and results of Ref. 160, so the constant thickness approximation holds. It is claimed in 162 that when the radius of the fireball reaches very large values comparable to \( R_b = R_0 \eta^2 \), a noticeable departure from the constant width of the fireball occurs. However, it is important to note that the fireball becomes transparent much earlier and this effect never becomes important (see section 8.2.3).

The crucial quantity presented in the paper is \( \Gamma_m \)—the maximum possible bulk Lorentz factor achievable for a given initial radiation energy \( E_0 \) deposited within a given initial radius \( R_0 \):

\[
\Gamma_m \equiv \eta_m = (\tau_0 \eta)^{1/3} = (\Sigma_0 \eta)^{1/3},
\]

(208)

\[
\Sigma_0 = \frac{M}{4\pi R_0^2}, \quad \kappa = \frac{\sigma_T}{m_p},
\]

(209)

where \( \Sigma_0 \) is initial baryon (plasma) mass surface density.

All subsequent calculations in the paper Ref. 162 involve this quantity. It is evident from (208) that a linear dependence between the gamma factor \( \Gamma \) and parameter \( \eta \) is assumed. However, this is certainly not true as can be seen from Fig. 23. We will come back to this point in the following section.

Another important quantity is given in this paper, namely

\[
\Gamma_p = \frac{T_0}{T_p}
\]

(210)

This is just the asymptotic behavior of the gamma factor at Fig. 24 for very large \( \eta \). Using it, the authors calculate the value of the \( \eta \) parameter above which the pair dominated regime occurs:

\[
\eta_p = \frac{\Gamma_m^3}{\Gamma_p^2}
\]

(211)
This means that above $\eta_p$ the presence of baryons in the fireball is insufficient to keep the fireball opaque after pairs are annihilated and almost all initial energy deposited in the fireball is emitted immediately.

The estimate of the final radiation to kinetic energy ratio\textsuperscript{162} is incorrect, because kinetic and radiation energies do not sum up to the initial energy of the fireball thus violating energy conservation. This is illustrated in Fig. 25. The correct analytic diagram is instead presented in Fig. 22.

![Fig. 25. The ratios of radiation and kinetic energy to the initial energy of the fireball predicted by the Mészáros, Laguna and Rees model. The thick line denotes the total energy of the system in terms of the initial energy. Energy conservation does not hold.](image)

**Approximate results.** All models for isotropic fireballs are based on the following points:

1. flat space-time,
2. relativistic energy-momentum principle,
3. baryonic number conservation.

Although the model\textsuperscript{106,107} starts with Reissner-Nordström geometry, the numerical code is written for the case of flat space-time simply because curved space-time effects become insignificant soon after the fireshell reaches relativistic expansion velocities. The presence of the rate equation in the model\textsuperscript{106,107} has a strong physical basis and its absence in the other treatments means incompleteness of their mod-
els. Indeed, the number density of pairs influences the speed of expansion of the fireshell. However, in this section we neglect the rate equation and discuss those points common among all the models being considered.

First of all, let us return to Fig. 23. For almost all values of the $\eta$ parameter the gamma factor is determined by the gas (i.e., plasma or baryons) admixture according to (197); consider this case in what follows. For given initial energy and radius this temperature depends only on $\eta$, so one can write:

$$\gamma = \frac{\eta + 1}{(\frac{4\pi}{3})\eta + 1} = \frac{\eta + 1}{a\eta^2 + 1},$$

where

$$a = 2.1 \cdot 10^5 T_0^{-2} \rho_0^{-0.5}. \quad (213)$$

From this formula one has immediately the two asymptotic regimes, namely:

$$\gamma = \begin{cases} \eta + 1, & \eta < \eta_{\text{max}}, \\ \frac{1}{a\eta^2}, & \eta > \eta_{\text{max}}. \end{cases} \quad (214)$$

Notice, that the constant $a$ is an extremely small number, so that after obtaining a precise value of $\eta_{\text{max}}$ by equating to zero the derivative of function (212), one can expand the result in a Taylor series and get to the lowest order in $a$ the result:

$$\eta_{\text{max}} \simeq \left(2 \cdot \frac{a}{2}\right)^{\frac{1}{2}} - 2, \quad (215)$$

$$\gamma_{\text{max}} \equiv \gamma(\eta_{\text{max}}) \simeq \frac{1}{3} \left[1 + \left(2 \cdot \frac{a}{2}\right)^{\frac{1}{2}}\right]. \quad (216)$$

In particular, in the case shown in Fig. 24 one has $\eta_{\text{max}} = 2.8 \cdot 10^6$, $\gamma_{\text{max}} = 9.3 \cdot 10^4$ while according to (208), $\Gamma_m = \eta_{\text{max}} = 1.75 \cdot 10^5$. Clearly, our result is much more accurate. Actually, the value $\Gamma_m$ in (208) is obtained from equating asymptotes in (214) and there exists the following relation:

$$\Gamma_m = (\tau_0 \eta)^{-1/2}. \quad (217)$$

Now we are ready to explain why the observed temperature (and consequently the observed energy) does not depend on $\eta$ in the region $\eta_{\text{max}} < \eta < \eta_p$. From the second line in (214) it follows that the gamma factor in this region behaves like $\gamma \propto \eta^{-1/2}$, while $T_{\text{esc}} \propto \eta^{1/2}$. These two exactly compensate each other leading to the independence of the observed quantities on $\eta$ in this region. This remains the same for $\eta > \eta_p$ also, since here $T_{\text{esc}} = T_p = \text{const}$ and from (212) $\gamma = \text{const.}$
Nakar, Piran and Sari revision. Recently a revision of the fireball model was made in Ref. 172. These authors presented a new diagram for the final Lorentz gamma factor and for the energy budget of the fireball. Their work was motivated by the observation of giant flares with the subsequent afterglow spreading up to the radio region with a thermal spectrum. They concluded that the fireball has to be loaded by either baryons or magnetic field, and cannot be only a pure $e^\pm, \gamma$ plasma in order to have $10^{-3}$ of the total energy radiated in the giant flare.

In analogy with cosmology the authors define the number density of pairs which survive because the expansion rate becomes larger than the annihilation rate $f$ which gives the condition
\[ n_{\pm} \approx \frac{1}{\sigma_T R_0}. \] (218)

Then, recalling (200), if we want to estimate the number of pairs it turn out to be
\[ N_{\pm} = \frac{4\pi R_0 c t}{\sigma_T} \left( \frac{T_0}{T_{\pm}} \right)^2, \] (219)
where we identify $\Delta = ct$ in (205). In Ref. 172 the authors obtained the third power of the ratio of temperatures which influences all of their subsequent results.

Having reached the conclusion that the afterglow cannot be obtained as the result of interaction of the $e^\pm, \gamma$ plasma with the CircumBurst Medium (CBM), the authors turn to baryonic loading considerations. They attempt to define critical values of the loading parameter $\eta$ finding in general 4 such values, in particular:
\[ \eta_1 = \frac{E_0 \sigma_T}{4\pi R_0 c t m_e c^2} \left( \frac{T_{\pm}}{T_0} \right)^3, \] (220)
\[ \eta_2 = \frac{E_0 \sigma_T}{4\pi R_0 c t m_p c^2} \left( \frac{T_{\pm}}{T_0} \right)^3, \] (221)
\[ \eta_3 = \left( \frac{E_0 \sigma_T}{4\pi R_0 c t m_p c^2} \right)^{1/4}. \] (222)

We recall that the first two quantities are based on the formula for $N_{\pm}$ and should contain factors of $\left( \frac{T_{\pm}}{T_0} \right)^2$ instead.

The first ‘critical’ value, $\eta_1$, comes from the condition $N_p m_p = N_{\pm} m_e$, where $N_p \equiv \frac{E_0}{m_e c^2}$ is just the number of protons in the plasma admixture. It does not correspond to any critical change in the physics of the phenomena; for instance it cannot be interpreted as the equality of masses (equal inertia) of pairs and baryons since the former is mainly due to their total energy $E_{\pm}$, while the latter to their rest mass $N_p m_p$. This value is, however, close to the one defined above $\eta_1 \approx \eta_p$.

\[ \text{This effect is accounted for automatically in our approach where rate equations for pairs include an expansion term.} \]

\[ \text{The last value } \eta_4 \text{ corresponds to the case of heavy loading where spreading of the expanding shell is observed, and is not considered here.} \]
The second 'critical' value, $\eta_2$, corresponds to the condition $N_p = N_\pm$, namely equality of the numbers of protons and pairs. It is also incorrectly interpreted as equal contributions to the Thompson scattering. In fact, the cross-section for Thompson scattering of protons contains an additional factor $\left(\frac{me}{mp}\right)^2$ with respect to the usual formula for electrons.

The definition of the third 'critical' value, $\eta_3$, is not clear, but an important feature is its closeness to the critical value $\eta_c$ quoted above.

Assuming adiabatic conditions (200) the authors present the new diagram for the final gamma factor and energy budget of the pair-baryonic plasma at transparency. In fact, this diagram, shown in our Fig. 27 by the dashed curve for the parameter $B$, is very similar to the one obtained in Ref. 173, which considered the hydrodynamics of relativistic $e^\pm, \gamma$ winds. That problem is very different from ours, because of different boundary conditions. In particular, in the wind energy conservation, (142) does not hold; the reason is that constant energy (mass) supply takes place parametrized in Ref. 173 by $\dot{E}(\dot{M})$. In that paper in fact authors present the diagram for the asymptotic value of the Lorentz gamma factor depending on the ratio $\frac{\dot{E}}{\dot{M}}$ which is very different from the quantity $\eta$.

Surprisingly, the fundamental result about the presence of a maximum in the diagram for the gamma factor on Fig. 27 which was found by the same authors previously in Ref. 164 (see Fig. 24) that comes from the energy conservation (202) is ignored in 172. It can be understood in the following way. For small loading (small $B$) the more baryons are present in the plasma the larger the number density of corresponding electrons becomes, and the larger the optical depth is. Therefore, transparency is reached later, which gives a larger gamma factor at transparency. On the other hand, for heavy baryon loading (relatively large $B$) the more baryons are present, the more inertia the plasma has, and by energy conservation, the less the final gamma factor has to be.

8.2.5. Significance of the rate equation

The rate equation describes the evolution of the number densities for electrons and positrons. In analytic models it is assumed that pairs are annihilated instantly when the transparency condition is fulfilled. Moreover, the dynamics of expansion is influenced by the electron-positron energy density as can be seen from (190)–(194). Therefore, it is important to clarify whether neglecting the rate equation is a crude approximation or not.

Using Eq. (202) one can obtain the analytic dependence of the energy emitted at the transparency point on parameter $B$ and we compare them in Fig. 26.

We also show the difference between numerical results based on integration of Eqs. (190–194) and analytic results from the Shemi and Piran model. The values of

\footnote{Note that the authors of Ref. 173 also use rate equations describing decoupling plasma from photons.}
Fig. 26. Relative energy release in the form of photons emitted at the transparency point of the GRB in terms of the initial energy of the fireball depending on parameter $B$. Thick line represents numerical results and it is the same as in Fig. 22. The normal line shows the results for the analytic model of Ref. 164. The dashed line shows the difference between the exact numerical and approximate analytical results.

The parameters are: $\mu = 10^3$ and $\xi = 0.1$ (which correspond to $E_0 = 2.87 \cdot 10^{54}$ ergs and $R_0 = 1.08 \cdot 10^9$ cm). One can see that the difference peaks at intermediate values of $B$. The crucial deviations, however, appear for large $B$, where analytical predictions for the observed energy are about two orders of magnitude smaller than the numerical ones. This is due to the difference in predictions of the radius of the fireshell at the transparency moment. In fact, the analytical model overestimates this value by about two orders of magnitude for $B = 10^{-2}$. So for large $B$ with correct treatment of pair dynamics the fireshell becomes transparent earlier comparing to the analytical treatment.

At the same time, the difference between numerical and analytical results for gamma factor is significant for small $B$ as illustrated at Fig. 27. While both results coincide for $B > 10^{-4}$ there is a constant difference for the range of values $10^{-8} < B < 10^{-4}$ and asymptotic constant values for the gamma factor are also different. Besides, this asymptotic behavior takes place for larger values of $B$ in disagreement with analytical expectations. Thus the acceleration of the fireshell for small $B$ is larger if one accounts for pair dynamics.

It is clear that the error coming from neglecting the rate equation is significant. This implies that the simple analytic model of Shemi and Piran gives only a qual-
Fig. 27. Relativistic gamma factor when transparency is reached. The thick line denotes exact numerical results, the normal line corresponds to the analytical estimate from the Shemi and Piran model, while the dotted line denotes the asymptotic value of the baryonic loading parameter. The dashed line shows the results of Nakar, Piran and Sari.

Iterative picture of the fireshell evolution and in order to get the correct description of the fireshell one cannot neglect the rate equation.

Moreover, the difference between the exact numerical model\textsuperscript{106,107} and the approximate analytical models\textsuperscript{164} becomes apparent in various physical aspects, namely in predictions of the radius of the shell when it reaches transparency, the gamma factor at transparency and the ratio between the energy released in the form of photons and that converted into kinetic form. The last point is crucial. It is assumed in the literature that the entire initial energy of the fireshell gets converted into kinetic energy of the shell during adiabatic expansion. Indeed, taking a typical value of the parameter $B$ like $10^{-3}$ we find that according to the Shemi and Piran model we have only 0.2\% of the initial energy left in the form of photons. However, exact numerical computations\textsuperscript{106,107} give 3.7\% for the energy of photons radiated when the fireshell reaches transparency, which is a significant value and it cannot be neglected.

To summarize the above discussion, we present the results of this survey in the Table 1. It is important to notice again that comparing to the simplified analytic treatment, accounting for the rate of change of electron-positron pairs densities gives quantitatively different results for the ratio of kinetic versus photon energies produced in the GRB and the gamma factor at the transparency moment, which in
Table 1. Comparison of different models for fireballs.

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<td>no, no, no</td>
<td>yes</td>
<td>no</td>
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<td>yes</td>
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The turn leads to different afterglow properties. Therefore, although analytical models presented in sections 8.2.4 and 8.2.4 agree and give the correct qualitative description of the fireshell, one should use the numerical approach described in section 8.2.3 in order to compare the theory and observations.

Fig. 28. The Lorentz factor as a function of the radius for selected values of the baryonic loading parameter. Filled circles show the radius at the moment of transparency. Squares show the moment of departure from thermal distributions of electron-positron pairs. The star denotes the moment when the temperature of the fireshell equals 511 keV.

Reaching transparency. To demonstrate the richness of physical phenomena associated with gamma-ray bursts we provide below several illustrations. We calculate the temperature in the comoving and laboratory frames of the plasma as well as
Fig. 29. The temperature of the fireshell as a function of the radius for selected values of the baryonic loading parameter. Filled circles show the radius at the moment of transparency.

the Lorentz factor with the code described in Ref. 106,107 for different values of the parameter $B$. We show our results in Fig. 28 and 29. From Fig. 29 it is clear that during early phases of the expansion the temperature decreases down to 0.511 MeV, as shown by the star in Fig. 28, and the ratio of electron-positron pairs to photons becomes exponentially suppressed. However, because of the accelerated expansion the apparent temperature in the observer’s frame remains almost constant, see Fig. 29. Then, after collision with a baryonic remnant after which the Lorentz factor decreases, the plasma continues to expand. At a certain moment, shown by squares in Fig. 28, a departure from a thermal distribution of electron-positron pairs occurs, due to the fact that the rate of the reaction $\gamma\gamma \rightarrow e^+e^-$ becomes smaller than the expansion rate. From that moment electron-positron pairs freeze out, analogous to what happens in the early Universe. Finally, transparency is reached, as denoted by circles in Fig. 28. For high values of the baryonic loading $B > 10^{-4}$ the comoving temperature decreases in the late stages of expansion in the same way as the observed temperature.

Total opacity due to pair production and due to Compton scattering is shown in Fig. 30, while the ratio of separate contributions, i.e., opacity due to pairs and baryons is shown in Fig. 31. The expansion starts with $\tau \gg 1$ and the optical depth starts to decrease. Then, after collision with the baryonic remnant containing also associated electrons, the opacity may increase, but only for large baryonic loading.
The afterglow

After reaching transparency and the emission of the P-GRB, the accelerated baryonic matter (the ABM pulse) interacts with the CBM and gives rise to the afterglow (see Fig. 32). Also in the descriptions of this last phase many differences exist between our treatment and the other ones in the current literature (see next sections).

We first look to the initial value problem. The initial conditions of the afterglow era are determined at the end of the optically thick era when the P-GRB is emitted. As recalled in the last section, the transparency condition is determined by a time of arrival $t_d$, a value of the gamma Lorentz factor $\gamma_0$, a value of the radial coordinate $r_0$, and the amount of baryonic matter $M_B$, all of which are only functions of the two parameters $E_{dyn}$ and $B$ (see Eq. (119)).

This connection to the optically thick era is missing in the current approach in the literature which attributes the origin of the “prompt radiation” to an unspecified inner engine activity (see Ref. 153 and references therein). The initial conditions at

$B > 10^{-4}$. The change of exponential decrease into a power law decrease seen in Fig. 30 corresponds to the departure of distributions of electrons and positrons from thermal ones. Finally, as one can see from Fig. 31 at the late stage of expansion of the fireshell the opacity is dominated by pairs for $B < 10^{-7}$ and by Compton scattering for $B > 10^{-7}$.

8.3. The afterglow

After reaching transparency and the emission of the P-GRB, the accelerated baryonic matter (the ABM pulse) interacts with the CBM and gives rise to the afterglow (see Fig. 32). Also in the descriptions of this last phase many differences exist between our treatment and the other ones in the current literature (see next sections).

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This connection to the optically thick era is missing in the current approach in the literature which attributes the origin of the “prompt radiation” to an unspecified inner engine activity (see Ref. 153 and references therein). The initial conditions at
Fig. 31. The ratio of the opacity due to electron-positron pairs to the opacity due to electrons associated with baryons in the fireshell.

Fig. 32. The GRB afterglow phase is represented here together with the optically thick phase (see Fig. 20). The value of the Lorentz gamma factor is given here from the transparency point all the way to the ultrarelativistic, relativistic and nonrelativistic regimes. Details in Ref. 152.

the beginning of the afterglow era are obtained by a best fit of the later parts of the afterglow. This approach is quite unsatisfactory since the theoretical treatments currently adopted in the description of the afterglow are not appropriate,174–176 The fit which uses an inappropriate theoretical treatment leads necessarily to the wrong
conclusions as well as, in turn, to the determination of incorrect initial conditions.

The order of magnitude estimate usually quoted for the characteristic time scale to be expected for a burst emitted by a GRB at the moment of transparency at the end of the optically thick expansion phase is given by \( \tau \sim GM/c^3 \). For a 10\( M_\odot \) black hole this will give \( \sim 10^{-3} \) s. There are reasons today not to take seriously such an order of magnitude estimate.\(^{177}\) In any case this time is much shorter than the ones typically observed in “prompt radiation” of the long bursts, from a few seconds all the way to 10\(^2\) s. In the current literature (see e.g. Ref. 153 and references therein), in order to explain the “prompt radiation” and overcome the above difficulty it has been generally assumed that its origin should be related to a prolonged “inner engine” activity preceding the afterglow which is not well identified.

To us this explanation has always appeared logically inconsistent since there remain to be explained not one but two very different mechanisms, independent of each other, of similar and extremely large energetics. This approach has generated an additional very negative result: it has distracted everybody working in the field from the earlier very interesting work on the optically thick phase of GRBs.

The way out of this dichotomy in our model is different: 1) indeed the optically thick phase exists and is crucial to the GRB phenomenon and terminates with a burst: the P-GRB; 2) the “prompt radiation” follows the P-GRB; 3) the “prompt radiation” is not a burst: it is actually the temporally extended peak emission of the afterglow (E-APE). The observed structures of the prompt radiation can all be traced back to inhomogeneities in the interstellar medium (see Fig. 33 and Ref. 129).

This approach was first tested on GRB 991216. Both the relative intensity and time separation of the P-GRB and the afterglow were duly explained (see Fig. 33) choosing a total energy of the plasma \( E_{\text{tot}} \) = \( 4.83 \times 10^{54} \) erg and a baryon loading \( B = 3.0 \times 10^{-3} \) (see Ref. Ref. 127,129,152,178). Similarly, the temporal substructure in the prompt emission was explicitly shown to be related to the CBM inhomogeneities (see the following sections).

Following this early analysis and the subsequent ones on additional sources, it became clear that the CBM structure revealed by our analysis is quite different from the traditional description in the current literature. Far from considering analogies with shock wave processes developed within a fluidodynamic approach, it appears to us that the correct CBM description is a discrete one, composed of overdense “blobs” of typical size \( \Delta R \sim 10^{14} \) cm widely spaced in underdense and inert regions.

We can then formulate the second paradigm, the interpretation of the burst structure (IBS) paradigm,\(^{127}\) which covers three fundamental issues leading to the unequivocal identification of the canonical GRB structure:

a) the existence of two different components: the P-GRB and the afterglow related by precise equations determining their relative amplitude and temporal sequence (see Fig. 34, Ref. 152 and next section);

b) what in the literature has been addressed as the “prompt emission” and considered as a burst, in our model is not a burst at all—instead it is just the emission from the peak of the afterglow (see the clear confirmation of this result by the Swift
Fig. 33. The detailed features of GRB 991216 shown by our theoretical models are reproduced here: the P-GRB, the “prompt radiation” and what is generally called the afterglow. It is clear that the prompt emission observed by BATSE coincides with the extended afterglow peak emission (E-APE) and has been considered as a burst only as a consequence of the high noise threshold in the observations. The small precursor is identified with the P-GRB. Details in Ref. 127,129,152,178.

data of e.g. GRB 050315 in the next sections and in Ref. 180,181; c) the crucial role of the parameter $B$ in determining the relative amplitude of the P-GRB to the afterglow and discriminating between the short and the long bursts (see Fig. 35). Both short and long bursts arise from the same physical phenomena:
the gravitational collapse to a black hole endowed with electromagnetic structure
and the formation of its dyadosphere.

The fundamental diagram determining the relative intensity of the P-GRB and
the afterglow as a function of the dimensionless parameter $B$ is shown in Fig. 35. The
main difference relates to the amount of baryonic matter engulfed by the electron-
positron plasma in their optically thick phase prior to transparency. For $B < 10^{-5}$
the intensity of the P-GRB is larger and dominates the afterglow. This corresponds
to the “genuine” short bursts.\cite{182} For $10^{-5} < B \leq 10^{-2}$ the afterglow dominates
the GRB. For $B > 10^{-2}$ we may observe a third class of “bursts”, eventually related
to a turbulent process occurring prior to transparency.\cite{107} This third family should
be characterized by smaller values of the Lorentz gamma factors than in the case
of the short or long bursts.

Particularly enlightening for the gradual transition to the short bursts as a
function of the $B$ parameter is the diagram showing how the GRB 991216 bolometric
light curve would scale changing the only the value of $B$ (see Fig. 36).

Moving from these two paradigms, and the prototypical case of GRB 991216,
we have extended our analysis to a larger number of sources, such as GRB
970228,\cite{182} GRB 980425,\cite{183,184} GRB 030329,\cite{185} GRB 031203,\cite{186} GRB 050315,\cite{180}
GRB 060218\cite{ruffini2007} which have led to a confirmation of the validity of our canonical GRB structure (see Fig. 37). In addition, progress has been made in our theoretical comprehension, which will be presented in the following sections.
Fig. 36. The bolometric luminosity of a source with the same total energy and CBM distribution of GRB 991216 is represented here for selected values of the $B$ parameter, ranging from $B = 10^{-2}$ to $B = 10^{-4}$. The actual value for the GRB 991216 is $B = 3.0 \times 10^{-3}$. As expected, for smaller values of the $B$ parameter the intensity of the P-GRB increases and the total energy of the afterglow decreases. What is most remarkable is that the luminosity in the early part of the afterglow becomes very spiky and the peak luminosity actually increases.

8.3.1. The simple model for the afterglow

When electron-positron pairs are annihilated and the transparency condition $\tau \approx 1$ is satisfied, photons escape and the only ingredient of the fireshell which is left is a relativistically expanding shell of protons and electrons. The latter are not important kinematically and one can say that the shell consists of baryons. This shell propagates in the interstellar medium sweeping up the cold gas. The constant width approximation is good enough to describe this process, however, we do not restrict ourselves to this case. The only requirement is that the collision is inelastic and the energy released in the collision is shared within the whole shell for a short time.

Consider the collision process in the lab reference frame where the CBM is initially at rest. Assuming that the expanding baryons as well as CBM are cold, the total energy of the shell is $E = M_B c^2 \gamma = M \gamma$ with $\gamma$ being the Lorentz factor of the shell and $M$ is the rest mass of the shell (together with its thermal energy) in units of energy. The mass-energy of the CBM swept up within an infinitesimal time interval is $dm$. The energy released during this process is $dE$. In our definitions
In order to determine the value of the $B$ parameter and the total energy we have performed the complete fit of each source. In particular, for each source we have fit the observed luminosities in selected energy bands of the entire afterglow including the prompt emission. We have verified that in each source the hard-to-soft spectral evolution is correctly fit and we have compared the theoretically computed spectral lag with the observations. Where applicable, we have also computed the relative intensity and temporal separation between the P-GRB and the peak of the afterglow and compared these values with the observed ones. The absence of spectral lag in the P-GRB is automatically verified by our model.

$$\gamma \beta = \sqrt{\gamma^2 - 1}.$$ Finally, the gamma factor after the collision becomes $\gamma + \gamma'$. Rewrite energy and momentum conservation:

$$M \gamma + dm = (M + dm + dE)(\gamma + d\gamma), \quad (223)$$

$$M \sqrt{\gamma^2 - 1} = (M + dm + dE)\sqrt{(\gamma + d\gamma)^2 - 1}. \quad (224)$$

This set of equations is equivalent to the one used in 152. From (224) we get:

$$dE = -dm - M \sqrt{\frac{\gamma^2 - 1}{(\gamma + d\gamma)^2 - 1}}. \quad (225)$$

Substituting the last equality into (223) we arrive at

$$d\gamma = \frac{dm}{M} + \gamma(1 - b), \quad (226)$$

where

$$b = \sqrt{1 + 2\gamma \left(\frac{dm}{M}\right) + \left(\frac{dm}{M}\right)^2}. \quad (227)$$
Substituting these expressions into (225) we find

\[ dE = -dm - M(1 - b). \]  
\[ (228) \]

Since \( dm \) is infinitesimally small we can expand \( b \) in a series in \( dm/M \)

\[ b \approx 1 - \frac{dm}{M\gamma}, \]
\[ (229) \]

so we finally obtain:

\[ dE = (\gamma - 1)dm, \]
\[ (230) \]

\[ d\gamma = -\left(\gamma^2 - 1\right)\frac{dm}{M}. \]
\[ (231) \]

These equations are used to describe collision of the baryonic remnant of the fireshell with the CBM.\textsuperscript{107,152,153} Note that it is a mistake to assume that \( dE = 0 \) and use only (224) to obtain a differential equation for \( \gamma \) as was done in Ref. 170. The total mass-energy swept up during this process is given by

\[ dM = M + dm + (1 - \delta)dE - M = dm + (1 - \delta)dE, \]
\[ (232) \]

where \( \delta \) denotes the portion of the energy that is radiated. There are basically two approximations which follow from this expression, namely the fully radiative condition \( \delta = 1 \) and the adiabatic condition \( \delta = 0 \). With the adiabatic condition no energy is radiated and one must assume that the kinetic energy of the decelerating baryons is converted into radiation in dissipation processes in shocks which are supposed to form during the collision,\textsuperscript{168,169} see also 153. With the radiative condition no additional mechanisms are required to describe the afterglow since it results from the emission of part of the energy released during inelastic collision of accelerated baryonic pulse with the CBM.\textsuperscript{152}

\subsection*{8.3.2. Blandford and McKee radiative solution}

Eqs. (230),(231) can be found already in Ref. 167. However, the meaning of the quantities used there as well as the derivation are incorrect.

Following Ref. 167 consider the relativistic shock (blast) wave resulting from an impulsive energy release, propagating into the homogeneous external medium with the Lorentz factor \( \gamma \). Authors assume that the shock wave sweeps up external matter. They treat this swept-up matter as being in a thin, cold shell.

Consider the reference frame where the shock is at rest. For an observer in this frame the external medium has relativistic Lorentz factor \( \gamma \). The shock front is a spherical massless surface. For our observer there is a flow of matter, energy and momentum through this surface. We can calculate these quantities using (142) and
(122). Recall that the external medium is cold so $p \ll \rho$. Thus we get

$$\frac{dE_{\text{tot}}}{dt} = -\oint_S T^{01} dS = -\int_S \rho \gamma^2 \beta dS = -4\pi R^2 \rho \beta \gamma^2,$$

(233)

$$\frac{dP}{dt} = -\oint_S T^{11} dS = -\int_S \rho (\gamma^2 - 1) dS = -4\pi R^2 \rho (\gamma^2 - 1),$$

(234)

$$\frac{dm}{dt} = -\oint_S \rho U^1 dS = -4\pi R^2 \rho \beta \gamma,$$

(235)

where $P$ is a radial momentum of external medium, $E_{\text{tot}}$ is its total energy. The kinetic energy $E_k$ change is simply connected to its flux:

$$\frac{dE_k}{dt} = \frac{dE_{\text{tot}}}{dt} - \frac{dm}{dt} = -4\pi R^2 \rho \beta \gamma (\gamma - 1).$$

(236)

This formula coincides with Eq. (84) in Ref. 167. The next equation (85) is of the form of our (231). This implies that in order to derive this result these authors considered the full derivative of kinetic energy of the external medium with respect to the expanding shock wave, namely:

$$\frac{dE_k}{dt} = (\gamma - 1) \frac{dm}{dt} + m \frac{d\gamma}{dt},$$

(237)

and equate these two expressions. However, if one proceeds in the same manner with the radial momentum equation, one arrives at a strange result: they become inconsistent. The reason is the following.

Eqs. (233),(234),(235) and (236) are of course correct. The next step is in doubt. To write down the total derivative means to assume that the Lorentz factor changes with time. This means that the properties of the external medium change. However, the problem considered by Blandford and McKee is an idealized one, namely the hydrodynamics of relativistic shocks without consideration of any microphysics\(^1\). In this context the damping and dissipation effect cannot be incorporated directly and the shock wave generally speaking will propagate with constant velocity. Apart from difficulties in definition of the quantity $m$, the mass of external medium which can be thought of as infinite, the last term in (237) cannot exist and therefore there is no way to get (85) from (84) in 167.

From the physical point of view, instead of a massless shock (even if it is present) a massive shell must be considered\(^2\). The only natural way to deal with this problem is to consider interaction of this shell with a small shell of external medium. The problem is consequently equivalent to the interaction of two massive particles which is obvious to treat with the help of conservation equations instead of shock equations.

Thus, equations (230),(231) can be derived only from energy and radial momentum conservation equations as was done in 188 and independently in Ref. 107.

\(^1\)Microphysical interactions responsible for changes in the blast wave can be Coulomb interactions within the plasma, ionization of external medium losses, interactions with magnetic field and various radiative processes.

\(^2\)In some approximation it can be considered as the relativistic piston problem.
9. The “canonical GRB” bolometric light curve

We assume that the internal energy due to kinetic collision is instantly radiated away and that the corresponding emission is isotropic. Let $\Delta \varepsilon$ be the internal energy density developed in the collision. In the comoving frame the energy per unit of volume and per solid angle is simply

$$\left( \frac{dE}{dVd\Omega} \right)_o = \frac{\Delta \varepsilon}{4\pi}$$

(238)
due to the fact that the emission is isotropic in this frame. The total number of photons emitted is an invariant quantity independent of the frame used. Thus we can compute this quantity as seen by an observer in the comoving frame (which we denote with the subscript “$o$”) and by an observer in the laboratory frame (which we denote with no subscripts). Doing this we find:

$$\frac{dN_\gamma}{dt d\Omega d\Sigma} = \left( \frac{dN_\gamma}{dt d\Omega d\Sigma} \right)_o \Lambda^{-3} \cos \vartheta,$$

(239)

where $\vartheta$ is the angle between the radial expansion velocity of a point on the fireshell surface and the line of sight, $\cos \vartheta$ comes from the projection of the elementary surface of the shell on the direction of propagation and $\Lambda = \gamma(1 - \beta \cos \vartheta)$ is the Doppler factor introduced in the two following differential transformation

$$d\Omega_o = d\Omega \times \Lambda^{-2}$$

(240)

for the solid angle transformation and

$$dt_o = dt \times \Lambda^{-1}$$

(241)

for the time transformation. An extra $\Lambda$ factor comes from the energy transformation:

$$E_o = E \times \Lambda$$

(242)

(see also Ref. 189). Thus finally we obtain:

$$\frac{dE}{dt d\Omega d\Sigma} = \left( \frac{dE}{dt d\Omega d\Sigma} \right)_o \Lambda^{-4} \cos \vartheta.$$

(243)

Doing this we clearly identify $\left( \frac{dE}{dt d\Omega d\Sigma} \right)_o$ as the energy density in the comoving frame up to a factor $\frac{1}{4\pi}$ (see Eq. (238)). Then we have:

$$\frac{dE}{dt d\Omega} = \int_{\text{shell}} \frac{\Delta \varepsilon}{4\pi} \nu \cos \vartheta \Lambda^{-4} d\Sigma,$$

(244)

where the integration in $d\Sigma$ is performed over the visible area of the ABM pulse at laboratory time $t$, namely with $0 \leq \vartheta \leq \vartheta_{\max}$ and $\vartheta_{\max}$ is the boundary of the visible region defined by:

$$\cos \vartheta_{\max} = \frac{v}{c}.$$  

(245)
Eq. (244) gives us the energy emitted toward the observer per unit solid angle and per unit laboratory time $t$ in the laboratory frame.

What we really need is the energy emitted per unit solid angle and per unit detector arrival time $t_d^a$, so we must use the complete relation between $t_d^a$ and $t$ given by:

$$t_d^a = (1 + z) \left[ t - \frac{r}{c}(t) \cos \vartheta + \frac{r^*}{c} \right],$$

where $r^*$ is the initial size of the fireshell. First we have to multiply the integrand in Eq. (244) by the factor $(dt/dt_d^a)$ to transform the energy density generated per unit of laboratory time $t$ into the energy density generated per unit arrival time $t_d^a$. Then we have to integrate with respect to $d\Sigma$ over the EquiTemporal Surfaces (EQTS) corresponding to arrival time $t_d^a$ instead of the ABM pulse visible area at laboratory time $t$. The analog of Eq. (244) for the source luminosity in detector arrival time is then:

$$\frac{dE}{dt_d^a d\Omega} = \int_{\text{EQTS}} \frac{\Delta \varepsilon}{4\pi} v \cos \vartheta \Lambda^{-4} \frac{dt}{dt_d^a} d\Sigma.$$  \hspace{1cm} (247)

It is important to note that, in the present case of GRB 991216, the Doppler factor $\Lambda^{-4}$ in Eq. (247) enhances the apparent luminosity of the burst compared to the intrinsic luminosity by a factor which at the peak of the afterglow is in the range between $10^{10}$ and $10^{12}$.

We are now able to reproduce in Fig. 34 the general behavior of the luminosity starting from the P-GRB to the latest phases of the afterglow as a function of the arrival time. It is generally agreed that the GRB afterglow originates from an ultrarelativistic shell of baryons with an initial Lorentz factor $\gamma_0 \sim 200$–300 with respect to the CBM (see e.g. Ref. 152,174 and references therein). Using GRB 991216 as a prototype, in Ref. 126,127 we have shown how from the time varying bolometric intensity of the afterglow it is possible to infer the average density $\langle n_{\text{cbm}} \rangle = 1$ particle/cm$^3$ of the CBM in a region of approximately $10^{17}$ cm surrounding the black hole giving rise to the GRB phenomenon.

The summary of these general results are shown in Fig. 33, where the P-GRB, the emission at the peak of the afterglow in relation to the “prompt emission” and the latest part of the afterglow are clearly identified for the source GRB 991216. Details in Ref. 152.

Summarizing, unlike treatments in the current literature (see e.g. Ref. 190,191 and references therein), we define a “canonical GRB” light curve with two sharply different components (see Fig. 33): 1,127,182

1. **The P-GRB**: it has the imprint of the black hole formation, a harder spectrum and no spectral lag.\textsuperscript{177,192}

2. **The afterglow**: it presents a clear hard-to-soft behavior,\textsuperscript{180,186,193} the peak of the afterglow contributes to what is usually called the “prompt emission”.\textsuperscript{127,180,187}
The ratio between the total time-integrated luminosity of the P-GRB (namely, its total energy) and the corresponding one for the afterglow is the crucial quantity for the identification of a GRBs’ nature. Such a ratio, as well as the temporal separation between the corresponding peaks, is a function of the $B$ parameter.$^{127}$

When the P-GRB is the leading contribution to the emission and the afterglow is negligible we have a “genuine” short GRB.$^{127}$ This is the case where $B \lesssim 10^{-5}$ (see Fig. 37); in the limit $B \to 0$ the afterglow vanishes (see Fig. 37). In the other GRBs, with $10^{-4} \lesssim B \lesssim 10^{-2}$, the afterglow contribution is generally predominant (see Fig. 37; for the existence of the upper limit $B \lesssim 10^{-2}$ see Ref. 107,187). Still, this case presents two distinct possibilities:

- The afterglow peak luminosity is larger than the P-GRB peak luminosity. A clear example of this situation is GRB 991216, represented in Fig. 33.

- The afterglow peak luminosity is smaller than the P-GRB one. A clear example of this situation is GRB 970228, represented in Fig. 53.

The simultaneous occurrence of an afterglow with total time-integrated luminosity larger than the P-GRB one, but with a smaller peak luminosity, is indeed explainable in terms of a peculiarly small average value of the CBM density, compatible with a galactic halo environment, and not due to the intrinsic nature of the source (see Fig. 53).$^{182}$ Such a small average CBM density deflates the afterglow peak luminosity. Of course, such a deflated afterglow lasts much longer, since the total time-integrated luminosity in the afterglow is fixed by the value of the $B$ parameter (see above and Fig. 55). In this sense, GRBs belonging to this class are only “fake” short GRBs. This is GRB class identified by Ref. 194, which also GRB 060614 belongs to, and which has GRB 970228 as a prototype.$^{182}$

Our “canonical GRB” scenario, therefore, especially points out the need to distinguish between “genuine” and “fake” short GRBs:

- The “genuine” short GRBs inherit their features from an intrinsic property of their sources. The very small fireshell baryon loading, in fact, implies that the afterglow time-integrated luminosity is negligible with respect to the P-GRB one.

- The “fake” short GRBs instead inherit their features from the environment. The very small CBM density in fact implies that the afterglow peak luminosity is lower than the P-GRB one, even if the afterglow total time-integrated luminosity is higher. This deflated afterglow peak can be observed as a “soft bump” following the P-GRB spike, as in GRB 970228,$^{182}$ GRB 060614 (Caito et al., in preparation), and the sources analyzed by Ref. 194.

A sketch of the different possibilities depending on the fireshell baryon loading $B$ and the average CBM density $\langle n_{\text{cbm}} \rangle$ is given in Fig. 38.
Fig. 38. A sketch summarizing the different possibilities predicted by the “canonical GRB” scenario depending on the fireshell baryon loading $B$ and the average CBM density $\langle n_{cbm} \rangle$.

10. The spectra of the afterglow

In our approach we focus uniquely on the X and gamma ray radiation, which appears to be conceptually more predictable in terms of fundamental processes than the optical and radio emission. It is perfectly predictable by a set of constitutive equations, which leads to directly verifiable and very stable features in the spectral distribution of the observed GRB afterglows. In line with the observations of GRB 991216 and other GRB sources, we assume in the following that the X and gamma ray luminosity represents approximately 90% of the energy of the afterglow, while the optical and radio emission represents only the remaining 10%.

This approach differs significantly from the other ones in the current literature, where attempts are made to explain at once all the multi-wavelength emission in the radio, optical, X and gamma ray as coming from a common origin which is linked to boosted synchrotron emission. Such an approach has been shown to have a variety of difficulties and cannot anyway have the instantaneous variability needed to explain the structure in the “prompt radiation” in an external shock scenario, which is indeed confirmed by our model.

Here the fundamental new assumption is adopted (see also Ref. 193) that the X and gamma ray radiation during the entire afterglow phase has a thermal spectrum in the comoving frame. The temperature is then given by:

$$T_s = \left[ \frac{\Delta E_{\text{int}}}{4\pi r^2 \Delta \tau \sigma R} \right]^{1/4},$$

(248)

where $\Delta E_{\text{int}}$ is the internal energy developed in the collision with the CBM in a time interval $\Delta \tau$ in the co-moving frame, $\sigma$ is the Stefan-Boltzmann constant and

$$R = A_{\text{eff}}/A_{\text{vis}},$$

(249)

is the ratio between the “effective emitting area” of the ABM pulse of radius $r$ and its total visible area, which accounts for the CBM filamentary structure. Due to the CBM inhomogeneities the ABM emitting region is in fact far from being

Attachment 7
homogeneous. In GRB 991216 such a factor is observed to be decreasing during the afterglow between: \(3.01 \times 10^{-8} \geq R \geq 5.01 \times 10^{-12}\). The temperature in the comoving frame corresponding to the density distribution described in Ref. 129 is shown in Fig. 39.

![Fig. 39. The temperature in the comoving frame of the shock front corresponding to the density distribution with the six spikes A, B, C, D, E, F presented in Ref. 197. The green line corresponds to an homogeneous distribution with \(n_{chm} = 1\). Details in Ref. 197.](image)

We are now ready to evaluate the source luminosity in a given energy band. The source luminosity at a detector arrival time \(t_{da}\) per unit solid angle \(d\Omega\) and in the energy band \([\nu_1, \nu_2]\) is given by: 

\[
\frac{dE^{[\nu_1, \nu_2]}_\Omega}{dE T_{arr}} = \int_{EQTS} \frac{\Delta \varepsilon}{4\pi} v \cos \vartheta \Lambda^{-4} \frac{dt}{d\Sigma} W(\nu_1, \nu_2, T_{arr}) d\Sigma, \tag{250}
\]

where \(\Delta \varepsilon = \Delta E_{int}/V\) is the energy density released in the interaction of the ABM pulse with the CBM inhomogeneities measured in the comoving frame, \(\Lambda = \gamma(1 - (v/c) \cos \vartheta)\) is the Doppler factor, \(W(\nu_1, \nu_2, T_{arr})\) is an “effective weight” required to evaluate only the contributions in the energy band \([\nu_1, \nu_2]\), \(d\Sigma\) is the surface element of the EQTS at detector arrival time \(t_{da}\) on which the integration is performed (see also Ref. 197) and \(T_{arr}\) is the observed temperature of the radiation emitted from...
The “effective weight” $W(\nu_1, \nu_2, T_{arr})$ is given by the ratio of the integral over the given energy band of a Planckian distribution at a temperature $T_{arr}$ to the total integral $aT_{arr}^4$:

$$W(\nu_1, \nu_2, T_{arr}) = \frac{1}{aT_{arr}^4} \int_{\nu_1}^{\nu_2} \rho(T_{arr}, \nu) \frac{h\nu}{c} \frac{d}{d\nu} \left( e^{h\nu/(kT_{arr})} - 1 \right)$$  \hspace{1cm} (252)

where $\rho(T_{arr}, \nu)$ is the Planckian distribution at temperature $T_{arr}$:

$$\rho(T_{arr}, \nu) = \frac{2}{\pi^2} \frac{h\nu}{c} \frac{1}{e^{h\nu/(kT_{arr})} - 1}$$  \hspace{1cm} (253)

11. Application to GRB 031203: the first spectral analysis

The first verification of the above theoretical framework came from the analysis of GRB 031203. GRB 031203 was observed by IBIS, on board the INTEGRAL satellite, as well as by XMM and Chandra in the $2-10$ keV band, and by VLT in the radio band. It appears as a typical long burst, with a simple profile and a duration of $\approx 40$ s. The burst fluence in the $20-200$ keV band is $(2.0 \pm 0.4) \times 10^{-6}$ erg/cm$^2$, and the measured redshift is $z = 0.106$. We analyze in the following the gamma ray signal received by INTEGRAL. The observations in other wavelengths, in analogy with the case of GRB 980425, could be related to the supernova event, as also suggested by Ref. 200, and they will be examined elsewhere.

The INTEGRAL observations find a direct explanation in our theoretical model. We reproduce correctly the observed time variability of the prompt emission (see Fig. 40). The radiation produced by the interaction with the CBM of the baryonic matter shell, accelerated by the fireshell, agrees with observations both for intensity and time structure.

The progress in reproducing the X and $\gamma$–ray emission as originating from a thermal spectrum in the comoving frame of the burst leads to the characterization of the instantaneous spectral properties which are shown to drift from hard to soft during the evolution of the system. The convolution of these instantaneous spectra over the observational time scale is in very good agreement with the observed power-law spectral shape.

11.1. The initial conditions

The best fit of the observational data leads to a total energy of the electron-positron plasma $E_{tot}^{em} = 1.85 \times 10^{50}$ erg. Assuming a black hole mass $M = 10 M_\odot$, we then have a black hole charge to mass ratio $\xi = 6.8 \times 10^{-3}$; the plasma is created between the radii $r_1 = 2.95 \times 10^{8}$ cm and $r_2 = 2.81 \times 10^{7}$ cm with an initial temperature $T = 1.52$ MeV and a total number of pairs $N_{e\pm} = 2.98 \times 10^{55}$. The amount of baryonic matter in the remnant is $B = 7.4 \times 10^{-3}$. 

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attachment 7
After the transparency point and the P-GRB emission, the initial Lorentz gamma factor of the accelerated baryons is $\gamma = 132.8$ at an arrival time at the detector $t_{da} = 8.14 \times 10^{-3}$ s and a distance from the Black Hole $r_s = 6.02 \times 10^{12}$ cm. The CBM parameters are: $< n_{cbm} > = 0.3$ particle/cm$^3$ and $< R > = 7.81 \times 10^{-9}$.

11.2. The GRB luminosity in fixed energy bands

The aim of our model is to derive from first principles both the luminosity in selected energy bands and the time resolved/integrated spectra. We recall that the luminosity in selected energy bands is evaluated integrating over the EQTS the energy density released in the interaction of the accelerated baryons with the CBM measured in the co-moving frame, duly boosted in the observer frame. The radiation viewed in the comoving frame of the accelerated baryonic matter is assumed to have a thermal spectrum and to be produced by the interaction of the CBM with the front of the expanding baryonic shell.

In order to evaluate the contributions in the band $[\nu_1, \nu_2]$ we have to multiply the bolometric luminosity by an “effective weight” $W(\nu_1, \nu_2, T_{arr})$, where $T_{arr}$ is the observed temperature. $W(\nu_1, \nu_2, T_{arr})$ is given by the ratio of the integral over the given energy band of a Planckian distribution at temperature $T_{arr}$ to the total integral $aT_{arr}^4$. The resulting expression for the emitted luminosity is Eq. (250).

11.3. The “prompt emission”

In order to compare our theoretical prediction with the observations, it is important to notice that there is a shift between the initial time of the GRB event and the moment in which the satellite instrument has been triggered. In fact, in our model the GRB emission starts at the transparency point when the P-GRB is emitted. If the P-GRB is under the threshold of the instrument, the trigger starts a few seconds later with respect to the real beginning of the event. Therefore it is crucial, in the theoretical analysis, to estimate and take this time delay into proper account. In the present case it results in $\Delta t_{da} = 3.5$ s (see the bold red line in Fig. 40). In what follows, the detector arrival time is referred to the onset of the instrument.

The structure of the prompt emission of GRB 031203, which is a single peak with a slow decay, is reproduced assuming an CBM which does not have a constant density but instead several density spikes with $< n_{cbm} > = 0.16$ particle/cm$^3$. Such density spikes corresponding to the main peak are modeled as three spherical shells with width $\Delta$ and density contrast $\Delta n/n$: we adopted for the first peak $\Delta = 3.0 \times 10^{15}$ cm and $\Delta n/n = 8$, for the second peak $\Delta = 1.0 \times 10^{15}$ cm and $\Delta n/n = 1.5$ and for the third one $\Delta = 7.0 \times 10^{14}$ cm and $\Delta n/n = 1$. To describe the details of the CBM filamentary structure we would require intensity vs. time information with an arbitrarily high resolving power. With the finite resolution of the INTEGRAL instrument, we can only describe the average density distribution compatible with the given accuracy. Only structures at scales of $10^{15}$ cm can be identified. Smaller structures would need a stronger signal and/or a smaller time resolution of the
detector. The three clouds here considered are necessary and sufficient to reproduce the observed light curve: a smaller number would not fit the data, while a larger number is unnecessary and would be indeterminable.

The result (see Fig. 40) shows a good agreement with the light curve reported by Ref. 201, and it provides further evidence for the possibility of reproducing light curves with a complex time variability through CBM inhomogeneities.\textsuperscript{129,152,178}

11.4. The instantaneous spectrum

In addition to the the luminosity in fixed energy bands we can also derive the instantaneous photon number spectrum $N(E)$. In Fig. 41 are shown samples of time-resolved spectra for five different values of the arrival time which cover the whole duration of the event.

It is clear from this picture that, although the spectrum in the comoving frame of the expanding pulse is thermal, the shape of the final spectrum in the laboratory frame is clearly not thermal. In fact, as explained in Ref. 193, each single instantaneous spectrum is the result of an integration of hundreds of thermal spectra over the corresponding EQTS. This calculation produces a nonthermal instantaneous spectrum in the observer frame (see Fig. 41).

Another distinguishing feature of the GRBs spectra which is also present in these
instantaneous spectra, as shown in Fig. 41, is the hard to soft transition during the evolution of the event.\textsuperscript{153,195,205,206} In fact the peak of the energy distributions $E_p$ drift monotonically to softer frequencies with time (see Fig. 42). This feature explains the change in the power-law low energy spectral index $\alpha$\textsuperscript{207} which at the beginning of the prompt emission of the burst ($\tau_{da} = 2$ s) is $\alpha = 0.75$, and progressively decreases for later times (see Fig. 41). In this way the link between $E_p$ and $\alpha$ identified by Ref. 205 is explicitly shown. This theoretically predicted evolution of the spectral index during the event unfortunately cannot be detected in this particular burst by INTEGRAL because of the insufficient quality of the data (poor photon statistics, see Ref. 201).

11.5. The time-integrated spectrum: comparison with the observed data

The time-integrated observed GRB spectra show a clear power-law behavior. Within a different framework Shakura, Sunyaev and Zel’dovich (see e.g. Ref. 208 and references therein) argued that it is possible to obtain such power-law spectra from a convolution of many non-power-law instantaneous spectra evolving in time. This
Fig. 42. The energy of the peak of the instantaneous photon number spectrum $N(E)$ is here represented as a function of the arrival time during the “prompt emission” phase. The clear hard to soft behavior is shown.

result was recalled and applied to GRBs by Blinnikov et al.\textsuperscript{209} assuming for the instantaneous spectra a thermal shape with a temperature changing with time. They showed that the integration of such energy distributions over the observation time gives a typical power-law shape possibly consistent with GRB spectra.

Our specific quantitative model is more complicated than the one considered by Blinnikov et al.:\textsuperscript{209} the instantaneous spectrum here is not a black body. Each instantaneous spectrum is obtained by an integration over the corresponding EQTS: it is itself a convolution, weighted by appropriate Lorentz and Doppler factors, of $\sim 10^6$ thermal spectra with variable temperature. Therefore, the time-integrated spectra are not plain convolutions of thermal spectra: they are convolutions of convolutions of thermal spectra (see Fig. 41).

The simple power-law shape of the integrated spectrum is more evident if we sum tens of instantaneous spectra, as in Fig. 43. In this case we divided the prompt emission in three different time interval, and for each one we integrated on time the energy distribution. The resulting three time-integrated spectra have a clear nonthermal behavior, and still show the characteristic hard to soft transition.

Finally, we integrated the photon number spectrum $N(E)$ over the entire duration of the prompt event (see again Fig. 43): in this way we obtain a typical
Fig. 43. Three theoretically predicted time-integrated photon number spectra $N(E)$ are here represented for $0 \leq t_{da} \leq 5$ s, $5 \leq t_{da} \leq 10$ s and $10 \leq t_{da} \leq 20$ s (colored curves). The hard to soft behavior presented in Fig. 42 is confirmed. Moreover, the theoretically predicted time-integrated photon number spectrum $N(E)$ corresponding to the first 20 s of the “prompt emission” (black bold curve) is compared with the data observed by INTEGRAL (green points). This curve is obtained as a convolution of 108 instantaneous spectra, which are enough to get a good agreement with the observed data.

The precise knowledge we have here acquired on GRB 031203 helps in clarifying the overall astrophysical system GRB 031203 – SN 2003lw – the 2$–$10 keV XMM and Chandra data (see sections 17.3 and 17.5, where the late 2$–$10 keV XMM and Chandra data are also discussed).

12. Application to GRB 050315: the first complete light curve fitting

The fit of GRB 991216 was specially important in showing a good agreement between the bolometric luminosity predicted by our theory and the observations, evidencing the clear separation between the P-GRB and the afterglow. The INTEGRAL observations of GRB 031203 have been crucial in leading to the confirmation of our theoretical approach for the spectral shape in the prompt emission of GRBs.
It has been the welcome consequence of the Swift satellite to have the possibility of receiving high quality data continuously spanning from the early part of the afterglow (the prompt emission) to the late phases of the afterglow, leading to the emergence of a standard GRB structure. Our first analysis of the Swift data came with GRB 050315.

GRB 050315 had been triggered and located by the BAT instrument on board the *Swift* satellite at 2005-March-15 20:59:42 UT. The narrow field instrument XRT began observations ~80 s after the BAT trigger, one of the earliest XRT observations yet made, and continued to detect the source for ~10 days. The spectroscopic redshift has been found to be $z = 1.949$.

We present here the results of the fit of the *Swift* data of this source in 5 energy bands in the framework of our theoretical model, pointing out a new step toward the uniqueness of the explanation of the overall GRB structure. We first recall the essential features of our theoretical model; then we fit the GRB 050315 observations by both the BAT and XRT instruments; we also present the instantaneous spectra for selected values of the detector arrival time ranging from 60 s (i.e., during the so called “prompt emission”) all the way to $3.0 \times 10^4$ s (i.e., the latest afterglow phases).

### 12.1. The fit of the observations

The best fit of the observational data leads to a total energy of the black hole dyadosphere, generating the $e^\pm$ plasma, $E_{\text{tot}}^{e^\pm} = 1.46 \times 10^{54}$ erg (the observational *Swift* $E_{\text{iso}}$ is $> 2.62 \times 10^{52}$ erg), so that the plasma is created between the radii $r_1 = 5.88 \times 10^8$ cm and $r_2 = 1.74 \times 10^9$ cm with an initial temperature $T = 2.05 MeV$ and a total number of pairs $N_{e^+e^-} = 7.93 \times 10^{57}$. The second parameter of the theory, the amount $M_B$ of baryonic matter in the plasma, is found to be such that $B = M_B c^2 / E_{\text{dyad}} = 4.55 \times 10^{-3}$. The transparency point and the P-GRB emission occurs then with an initial Lorentz gamma factor of the accelerated baryons $\gamma = 217.81$ at a distance $r = 1.32 \times 10^{14}$ cm from the black hole.

#### 12.1.1. The BAT data

In Fig. 44 we represent our theoretical fit of the BAT observations in the three energy channels 15–25 keV, 25–50 keV and 50–100 keV and in the whole 15–350 keV energy band.

We have already recalled how in our model the GRB emission starts at the transparency point when the P-GRB is emitted; this instant of time is often different from the moment in which the satellite instrument triggers, due to the fact that sometimes the P-GRB is under the instrumental noise threshold or comparable with it. In order to compare our theoretical predictions with the observations, it is important to estimate and take into account this time shift. In the present case of GRB 050315 there has been observed a possible precursor before the trigger. Such a precursor is indeed in agreement with our theoretically predicted P-GRB,
both in its isotropic energy emitted (which we theoretically predict to be $E_{\text{P-GRB}} = 1.98 \times 10^{51}$ erg) and its temporal separation from the peak of the afterglow (which we theoretically predicted to be $\Delta t^d = 51$ s). In Fig. 44a the blue line shows our theoretical prediction for the intensity and temporal position of the P-GRB.

After the P-GRB emission, all the observed radiation is produced by the interaction of the expanding baryonic shell with the interstellar medium. In order to reproduce the complex time variability of the light curve of the prompt emission as well as of the afterglow, we describe the CBM filamentary structure, for simplicity, as a sequence of overdense spherical regions separated by much less dense regions. Such overdense regions are nonhomogeneously filled, leading to an effective emitting area $A_{\text{eff}}$ determined by the dimensionless parameter $\mathcal{R}$.\textsuperscript{193,197} Clearly, in order to describe any detailed structure of the time variability an authentic three-dimensional representation of the CBM structure would be needed. However, this finer description would not change the substantial agreement of the model with the observational data. Anyway, in the “prompt emission” phase, the small angular size of the source visible area due to the relativistic beaming makes such a spherical approximation an excellent one (see also for details Ref. 129).

The structure of the “prompt emission” has been reproduced assuming three
overdense spherical CBM regions with width $\Delta$ and density contrast $\Delta n/\langle n \rangle$: we chose for the first region, at $r = 4.15 \times 10^{16}$ cm, $\Delta = 1.5 \times 10^{15}$ cm and $\Delta n/\langle n \rangle = 5.17$; for the second region, at $r = 4.53 \times 10^{16}$ cm, $\Delta = 7.0 \times 10^{14}$ cm and $\Delta n/\langle n \rangle = 36.0$ and for the third region, at $r = 5.62 \times 10^{16}$ cm, $\Delta = 5.0 \times 10^{14}$ cm and $\Delta n/\langle n \rangle = 85.4$. The CBM mean density during this phase is $\langle n_{cbm} \rangle = 0.81$ particles/cm$^3$ and $\langle R \rangle = 1.4 \times 10^{-7}$. With this choice of the density mask we obtain agreement with the observed light curve, as shown in Fig. 44. A small discrepancy occurs in coincidence with the last peak: this is due to the fact that at this stage the source visible area due to the relativistic beaming is comparable with the size of the clouds, therefore the spherical shell approximation should be properly modified by a detailed analysis of a full three-dimensional treatment of the CBM filamentary structure. Such a topic is currently under investigation (see also for details Ref. 129). Fig. 44 also shows the theoretical fit of the light curves in the three BAT energy channels in which the GRB has been detected (15–25 keV in Fig. 44b, 25–50 keV in Fig. 44c, 50–100 keV in Fig. 44d).

12.1.2. The XRT data

![Graph of XRT data](image)

Fig. 45. Our theoretical fit (blue line) of the XRT observations (green points) of GRB 050315 in the 0.2–10 keV energy band. Theoretical fit of the BAT observations (see Fig. 44a) in the 15–350 keV energy band is also represented (red line).
The same analysis can be applied to explain the features of the XRT light curve in the afterglow phase. It has been recently pointed out\textsuperscript{219} that almost all the GRBs observed by \textit{Swift} show a “canonical behavior”: an initial very steep decay followed by a shallow decay and finally a steeper decay. In order to explain these features many different approaches have been proposed\textsuperscript{191,219–221}. In our treatment these behaviors are automatically described by the same mechanism responsible for the prompt emission described above: the baryonic shell expands in a CBM region, between $r = 9.00 \times 10^{16}$ cm and $r = 5.50 \times 10^{18}$ cm, which is at significantly lower density ($\langle n_{cbm} \rangle = 4.76 \times 10^{-4}$ particles/cm$^3$, $\langle R \rangle = 7.0 \times 10^{16}$) then the one corresponding to the prompt emission, and this produces a slower decrease of the velocity of the baryons with a consequently longer duration of the afterglow emission. The initial steep decay of the observed flux is due to the smaller number of collisions with the CBM. In Fig. 45 is represented our theoretical fit of the XRT data, together with the theoretically computed 15–350 keV light curve of Fig. 44a (without the BAT observational data in order not to overwhelm the picture too much).

What is impressive is that no different scenarios need to be advocated in order to explain the features of the light curves: both the prompt and the afterglow emission are just due to the thermal radiation in the comoving frame produced by inelastic collisions with the CBM properly boosted by the relativistic transformations over the EQTSs.

### 12.2. The instantaneous spectrum

In addition to the luminosity in fixed energy bands we can derive also the instantaneous photon number spectrum $N(E)$ starting from the same assumptions. In Fig. 46 are shown samples of time-resolved spectra for eight different values of the arrival time which cover the entire duration of the event. It is clear from this picture that although the spectrum in the co-moving frame of the expanding pulse is thermal, the shape of the final spectrum in the laboratory frame is clearly nonthermal. In fact, as we have recalled and explained in Ref. 193, each single instantaneous spectrum is the result of an integration of thousands of thermal spectra over the corresponding EQTS. This calculation produces a nonthermal instantaneous spectrum in the observer frame (see Fig. 46).

A distinguishing feature of the GRB spectra which is also present in these instantaneous spectra is the hard to soft transition during the evolution of the event\textsuperscript{153,195,205,206}. In fact the peak of the energy distribution $E_p$ drifts monotonically to softer frequencies with time. This feature is linked to the change in the power-law low energy spectral index $\alpha$\textsuperscript{207} so the correlation between $\alpha$ and $E_p$\textsuperscript{205} is explicitly shown.

It is important to stress that there is no difference in the nature of the spectrum during the prompt and the afterglow phases: the observed energy distribution changes from hard to soft, with continuity, from the “prompt emission” all the way
Fig. 46. Eight theoretically predicted instantaneous photon number spectra $N(E)$ are represented here for different values of the arrival time (colored curves). The hard to soft behavior is confirmed.

to the latest phases of the afterglow.

13. Problems with the definition of “long” GRBs evidenced by GRB 05015

The confirmation by Swift of our prediction of the overall afterglow structure, and especially the coincidence of the “prompt emission” with the peak of the afterglow, opens a new problematic in the definition of the long GRBs. It is clear in fact that the identification of the “prompt emission” in the current GRB literature is not at all intrinsic to the phenomenon but is merely due to the threshold of the instruments used in the observations (e.g. BATSE in the 50–300 keV energy range, or BeppoSAX GRBM in 40–700 keV, or Swift BAT in 15–350 keV). As it is clear from Fig. 47, there is no natural way to identify in the source a special extension of the peak of the afterglow that is not the one purely defined by the experimental threshold. It is clear, therefore, that long GRBs, as defined until today, are just the peak of the afterglow and there is no way, as explained above, to define their “prompt emission” duration as a characteristic signature of the source. As the Swift observations show, the duration of the long GRBs has to coincide with the duration of the entire afterglow. A Kouveliotou-Tavani plot of the long GRBs, done following our interpretation which is clearly supported by the recent Swift data (see Fig. 47),
will present enormous dispersion on the temporal axis.

We recall that in our theory both “short” and “long” GRBs originate from the same process of black hole formation. The major difference between the two is the value of the baryon loading parameter \( B \) (see Fig. 35). In the limit of small baryon loading, all the plasma energy is emitted at the transparency in the P-GRB, with negligible afterglow observed flux. For higher values of the baryon loading, the relative energy content of the P-GRB with respect to the afterglow diminishes (see Ref. 178 and references therein).

14. Application to GRB 060218: the first evidence of the critical value of the baryon loading \( B \sim 10^{-22} \)

GRB 060218 triggered the BAT instrument of Swift on 18 February 2006 at 03:36:02 UT and has a \( T_{90} = (2100 \pm 100) \) s.\(^{222}\) The XRT instrument\(^{222,223}\) began observations \( \sim 153 \) s after the BAT trigger and continued for \( \sim 12.3 \) days.\(^{224}\) The source is characterized by a flat gamma ray light curve and a soft spectrum.\(^{225}\) It has an X-ray light curve with a long, slow rise and gradual decline and it is considered an X-Ray Flash (XRF) since its peak energy occurs at \( E_p = 4.9^{+0.4}_{-0.3} \) keV.\(^{226}\) It has been observed by the Chandra satellite on February 26.78 and March 7.55 UT.
(t ∼ 8.8 and 17.4 days) for 20 and 30 ks respectively.\textsuperscript{227} The spectroscopic redshift has been found to be $z = 0.033$.\textsuperscript{228,229} The corresponding isotropic equivalent energy is $E_{\text{iso}} = (1.9 \pm 0.1) \times 10^{49}$ erg which sets this GRB as a low luminous one, consistent with most of the GRBs associated with supernovas.\textsuperscript{230–232}

GRB 060218 is associated with SN2006aj whose expansion velocity is $v \sim 0.1c$.\textsuperscript{231,233–235} The host galaxy of SN2006aj is a low luminosity, metal poor star-forming dwarf galaxy\textsuperscript{236} with an irregular morphology\textsuperscript{237} similar to that of other GRBs associated with supernovas.\textsuperscript{228,238}

14.1. The fit of the observed data

Fig. 48. GRB 060218 prompt emission: a) our theoretical fit (blue line) of the BAT observations in the 15–150 keV energy band (pink points); b) our theoretical fit (red line) of the XRT observations in the 0.3–10 keV energy band (green points).\textsuperscript{226}

In this section we present the fit of our fireshell model to the observed data (see Figs. 48, 51). The fit leads to a total energy of the $e^\pm$ plasma $E_{\text{tot}}^{e^\pm} = 2.32 \times$
10^{50} \text{ erg}, with an initial temperature $T = 1.86 \text{ MeV}$ and a total number of pairs $N_{e\pm} = 1.79 \times 10^{55}$. The second parameter of the theory, $B = 1.0 \times 10^{-2}$, is the highest value ever observed and is close to the limit for the stability of the adiabatic optically thick acceleration phase of the fireshell (for further details see Ref. 107).

The Lorentz gamma factor obtained solving the fireshell equations of motion\cite{175,176} is $\gamma_0 = 99.2$ at the beginning of the afterglow phase at a distance from the progenitor $r_0 = 7.82 \times 10^{12} \text{ cm}$. It is much larger than $\gamma \sim 5$ estimated by Ref. 239 and Ref. 240.

In Fig. 48 we show the afterglow light curves fitting the prompt emission both in the BAT (15–150 keV) and in the XRT (0.3–10 keV) energy ranges, as expected in our “canonical GRB” scenario.\cite{187} Initially the two luminosities are comparable to each other, but for a detector arrival time $t_d > 1000 \text{ s}$ the XRT curves becomes dominant. The displacement between the peaks of these two light curves leads to a theoretically estimated spectral lag greater than 500 s in perfect agreement with the observations.\cite{241} We obtain that the bolometric luminosity in this early part coincides with the sum of the BAT and XRT light curves (see Fig. 51) and the luminosity in the other energy ranges is negligible.

We recall that at $t_d \sim 10^4 \text{ s}$ there is a sudden enhancement in the radio luminosity and there is an optical luminosity dominated by the SN2006aj emission,\cite{226,227,242} Although our analysis addresses only the BAT and XRT observations, for $r > 10^{18} \text{ cm}$ corresponding to $t_d > 10^4 \text{ s}$ the fit of the XRT data implies two new features:

1) a sudden increase of the $R$ factor from $R = 1.0 \times 10^{-11}$ to $R = 1.6 \times 10^{-6}$, corresponding to a significantly more homogeneous effective CBM distribution (see Fig. 52b);

2) an XRT luminosity much smaller than the bolometric one (see Fig. 51).

These theoretical predictions may account for the energetics of the enhancement of the radio and possibly optical and UV luminosities. Therefore, we identify two different regimes in the afterglow, one for $t_d < 10^4 \text{ s}$ and the other for $t_d > 10^4 \text{ s}$. Nevertheless, there is a unifying feature: the determined effective CBM density decreases with the distance $r$ monotonically and continuously through both these two regimes from $n_{\text{cm}} = 1 \text{ particle/cm}^3$ at $r = r_0$ to $n_{\text{cbm}} = 10^{-6} \text{ particle/cm}^3$ at $r = 6.0 \times 10^{18} \text{ cm}$: $n_{\text{cbm}} \propto r^{-\alpha}$, with $1.0 \lesssim \alpha \lesssim 1.7$ (see Fig. 52a).

Our assumption of spherical symmetry is supported by the observations which set for GRB 060218 an opening beaming angle larger than $\sim 37^\circ$.\cite{226,227,230,232}

### 14.2. The procedure of the fit

The arrival time of each photon at the detector depends on the entire previous history of the fireshell.\cite{126} Moreover, all the observables depend on the EQTS\cite{174,175} which in turn depend crucially on the equations of motion of the fireshell. The CBM engulfment has to be computed self-consistently through the entire dynamical evolution of the fireshell and not separately at each point. Any change in the CBM distribution strongly influences the entire dynamical evolution of the fireshell and, due to the EQTS structure, produces observable effects up to a much later time. For example if we change the density mask at a certain distance from the black
hole we modify the shape of the lightcurve and consequently the evolution changes at larger radii corresponding to later times. Anyway the change of the density is not the only problem to face in the fitting of the source, in fact first of all we have to choose the energy in order to have a Lorentz gamma factor sufficiently high to fit the entire GRB. In order to show the sensitivity of the fitting procedure I also present two examples of fits with the same value of $B$ and a different value of $E_{\text{tot}}^\pm$.

The first example has an $E_{\text{tot}}^\pm = 1.36 \times 10^{50}$ erg. This fit was unsuccessful as we see from the Fig. 49, because the bolometric lightcurve is under the XRT peak of the afterglow. This means that the value of the energy chosen is too small to fit any data points after the peak of the afterglow. So we have to increase the value of the energy to have a better fit. In fact the parameter values have been found with various attempts in order to obtain the best fit.

The second example is characterized by $E_{\text{tot}}^\pm = 1.61 \times 10^{50}$ erg and the all the data are fit except for the last point from $2.0 \times 10^{2}$ s to the end (see Fig. 50). We attempt to fit these last points trying to diminish the $R$ values in order to enhance the energy emission, but again the low value of the Lorentz gamma factor that in this case is 3 prevents the fireshell from expanding. So again in this case the value of the energy chosen is too small, but it is better than the previous attempt. In this case we increased the energy value by 24%, but it is not enough so we decide to increase 16%.

So the final fit is characterized by the $B = 1.0 \times 10^{-2}$ and by the $E_{\text{tot}}^\pm = 2.32 \times 10^{50}$ erg. With this value of the energy we are able to fit all the experimental points.

14.3. The fireshell fragmentation

GRB 060218 presents different peculiarities: the extremely long $T_{90}$, the very low effective CBM density decreasing with the distance and the largest possible value of $B = 10^{-2}$. These peculiarities appear to be correlated. Following Ref. 184, we propose that in the present case the fireshell is fragmented. This implies that the surface of the fireshell does not increase any longer like $r^2$ but like $r^\beta$ with $\beta < 2$. Consequently, the effective CBM density $n_{\text{cbm}}$ is linked to the actual one $n_{\text{act}}^{\text{cbm}}$ by:

$$n_{\text{cbm}} = R_{\text{shell}} n_{\text{act}}^{\text{cbm}}, \quad \text{with} \quad R_{\text{shell}} \equiv (r^*/r)^\alpha,$$

where $r^*$ is the starting radius at which the fragmentation occurs and $\alpha = 2 - \beta$ (see Fig. 52a). For $r^* = r_0$ we have $n_{\text{act}}^{\text{cbm}} = 1$ particles/cm$^3$, as expected for a “canonical GRB" and in agreement with the apparent absence of a massive stellar wind in the CBM.227,242,243

The $R$ parameter defined in Eq. (255) has to take into account both the effect of the fireshell fragmentation ($R_{\text{shell}}$) and the effective CBM porosity ($R_{\text{cbm}}$):

$$R \equiv R_{\text{shell}} \times R_{\text{cbm}}.$$

The phenomenon of the clumpiness of the ejecta, whose measure is the filling factor, is an aspect well known in astrophysics. For example, in the case of Novae
Fig. 49. GRB 060218 light curves with $E_{\text{tot}}^{[1]} = 1.36 \times 10^{50}$ erg: our theoretical fit (blue line) of the 15–150 keV BAT observations (pink points), our theoretical fit (red line) of the 0.3–10 keV XRT observations (green points) and the 0.3–10 keV Chandra observations (black points) are represented together with our theoretically computed bolometric luminosity (black line) (Data from: Ref. 226,227).

the filling factor has been measured to be in the range $10^{-2}–10^{-5}$. Such a filling factor coincides, in our case, with $R_{\text{shel}}$.

14.4. Binaries as progenitors of GRB-supernova systems

The majority of the existing models in the literature appeal to a single astrophysical phenomenon to explain both the GRB and the SN (“collapsar”, see e.g. Ref. 245). On the contrary, a distinguishing feature of our theoretical approach is to distinguish between the supernova and the GRB process. The GRB is assumed to occur during the formation process of a black hole. The supernova is assumed to lead to the formation of a neutron star (NS) or to a complete disruptive explosion without remnants and in no way to the formation of a black hole. In the case of SN2006aj the formation of such a NS has been actually inferred by Ref. 246 because of the large amount of $^{56}\text{Ni}$ ($0.05M_{\odot}$). Moreover the significantly small initial mass of the supernova progenitor star $M \approx 20M_{\odot}$ is expected to form a NS rather than a black hole when its core collapses. In order to fulfill both the above requirement, we assume that the progenitor of the GRB and the supernova consists of a binary system formed by a NS close to its critical mass collapsing to a black hole, and a
Fig. 50. GRB 060218 light curves with $E_{\text{tot}} = 1.61 \times 10^{50}$ erg: our theoretical fit (blue line) of the 15–150 keV BAT observations (pink points), our theoretical fit (red line) of the 0.3–10 keV XRT observations (green points) and the 0.3–10 keV Chandra observations (black points) are represented together with our theoretically computed bolometric luminosity (black line). Data from: Ref. 226,227.

A companion star evolved out of the main sequence originating the supernova. The temporal coincidence between the GRB and the supernova phenomenon is explained in terms of the concept of “induced” gravitational collapse.128,184 There is also the distinct possibility of observing the young born NS out of the supernova (see e.g., Ref. 184 and references therein).

It has been often proposed that GRBs associated with Ib/c supernovas, at a smaller redshift $0.0085 < z < 0.168$ (see e.g. Ref. 249 and references therein), form a different class, less luminous and possibly much more numerous than the high luminosity GRBs at higher redshift.200,233,246,249 Therefore they have been proposed to originate from a separate class of progenitors.200,233 In our model this is explained by the nature of the progenitor system leading to the formation of the black hole with the smallest possible mass: the one formed by the collapse of a just overcritical NS.184

The recent observation of GRB 060614 at $z = 0.125$ without an associated supernova250,251 gives strong support to our scenario, alternative to the collapsar model. Also in this case the progenitor of the GRB appears to be a binary system composed of two NSs or a NS and a white dwarf.
Fig. 51. GRB 060218 complete light curves: our theoretical fit (blue line) of the 15–150 keV BAT observations (pink points), our theoretical fit (red line) of the 0.3–10 keV XRT observations (green points) and the 0.3–10 keV Chandra observations (black points) are represented together with our theoretically computed bolometric luminosity (black line). In this case we have $E_{\text{tot}} = 2.32 \times 10^{50}$ ergs.

14.5. Conclusions on GRB 060218

GRB 060218 presents a variety of peculiarities, including its extremely large $T_{90}$ and its classification as an XRF. Nevertheless, a crucial point of our analysis is that we have successfully applied to this source our “canonical GRB” scenario.

Within our model there is no need for inserting GRB 060218 in a new class of GRBs, such as the XRFs, alternative to the “canonical” ones. This same point recently received strong observational support in the case of GRB 060218 and a consensus by other models in the literature.

The anomalously long $T_{90}$ led us to infer a monotonic decrease in the CBM effective density giving the first clear evidence for the fragmentation of the fireshell, which indeed was predicted for values of the baryon loading $B > 10^{-2}$. For GRB 060218 there is no need within our model for a new or unidentified source such as a magnetar or a collapsar.

GRB 060218 is the first GRB associated with a supernova with complete coverage of data from the onset all the way up to $\sim 10^6$ s. This fact offers an unprecedented opportunity to verify theoretical models on such a GRB class. For example, GRB 060218 fulfills the Ref. 252 relation unlike other sources in its same class. This is particularly significant, since GRB 060218 is the only source in such a class to have an excellent data coverage without gaps. We are currently examining if the missing data in the other sources of such a class may have a prominent role in their
Fig. 52. The CBM distribution parameters: a) the effective CBM number density (red line) monotonically decreases with the distance $r$ following Eq. (254) (green line); b) the $R$ parameter vs. distance.

non-fulfillment of the Ref. 252 relation.

15. Application to GRB 970228: the appearance of “fake” short GRBs

GRB 970228 was detected by the Gamma-Ray Burst Monitor (GRBM, 40–700 keV) and Wide Field Cameras (WFC, 2–26 keV) on board BeppoSAX on February 28, 12:36 UT. The burst prompt emission is characterized by an initial 5 s strong pulse followed, after 30 s, by a set of three additional pulses of decreasing intensity. Eight hours after the initial detection, the NFIs on board BeppoSAX were pointed at the burst location for a first target of opportunity observation and a new X-ray source was detected in the GRB error box: this is the first “afterglow” ever detected. A fading optical transient has been identified in a position consistent with the X-ray transient, coincident with a faint galaxy with redshift
Further observations by the Hubble Space Telescope clearly showed that the optical counterpart was located in the outskirts of a late-type galaxy with an irregular morphology.\textsuperscript{257}

The BeppoSAX observations of GRB 970228 prompt emission revealed a discontinuity in the spectral index between the end of the first pulse and the beginning of the three additional ones.\textsuperscript{102,206,254} The spectrum during the first 3 s of the second pulse is significantly harder than during the last part of the first pulse,\textsuperscript{206,254} while the spectrum of the last three pulses appear to be consistent with the late X-ray afterglow.\textsuperscript{206,254} This was soon recognized by Ref. 206,254 as pointing to an emission mechanism producing the X-ray afterglow already taking place after the first pulse.

The simultaneous occurrence of an afterglow with total time-integrated luminosity larger than the P-GRB one, but with a smaller peak luminosity, is indeed explainable in terms of a peculiarly small average value of the CBM density and not due to the intrinsic nature of the source. In this sense, GRBs belonging to this class are only “fake” short GRBs. We show that GRB 970228 is a very clear example of this situation. We identify the initial spikelike emission with the P-GRB, and the late soft bump with the peak of the afterglow. GRB 970228 shares the same morphology and observational features with the sources analyzed by Ref. 194 as well as with e.g. GRB 050709,\textsuperscript{258} GRB 050724\textsuperscript{259} and GRB 060614.\textsuperscript{260} Therefore, we propose GRB 970228 as a prototype for this new GRB class.

15.1. The analysis of GRB 970228 prompt emission

In Fig. 53 we present the theoretical fit of BeppoSAX GRBM (40–700 keV) and WFC (2–26 keV) light curves of GRB 970228 prompt emission.\textsuperscript{254} Within our “canonical GRB” scenario we identify the first main pulse with the P-GRB and the three additional pulses with the afterglow peak emission, consistent with the above mentioned observations by Ref. 102 and Ref. 254. The last three such pulses have been reproduced assuming three overdense spherical CBM regions (see Fig. 54) with very good agreement (see Fig. 53).

We therefore obtain for the two parameters characterizing the source in our model \( E_{\text{GRB}}^{\text{tot}} = 1.45 \times 10^{54} \) erg and \( B = 5.0 \times 10^{-3} \). This implies an initial \( e^\pm \) plasma created between the radii \( r_1 = 3.52 \times 10^7 \) cm and \( r_2 = 4.87 \times 10^8 \) cm with a total number of \( e^\pm \) pairs \( N_{e^\pm} = 1.6 \times 10^{59} \) and an initial temperature \( T = 1.7 \) MeV. The theoretically estimated total isotropic energy emitted in the P-GRB is \( E_{\text{P-GRB}} = 1.1% E_{\text{GRB}}^{\text{tot}} = 1.54 \times 10^{52} \) erg, in excellent agreement with the one observed in the first main pulse (\( E_{\text{P-GRB}}^{\text{tot}} \sim 1.5 \times 10^{52} \) erg in 2 – 700 keV energy band, see Fig. 53), as expected due to their identification. After the transparency point at \( r_0 = 4.37 \times 10^{14} \) cm from the progenitor, the initial Lorentz gamma factor of the fireshell is \( \gamma_0 = 199. \) On average, during the afterglow peak emission phase we have for the CBM \( \langle \mathcal{R} \rangle = 1.5 \times 10^{-7} \) and \( \langle n_{\text{cbm}} \rangle = 9.5 \times 10^{-4} \) particles/cm\(^3\). This very low average value for the CBM density is compatible with the observed occurrence.
Fig. 53. The “canonical GRB” light curve theoretically computed for the prompt emission of GRB 970228. BeppoSAX GRBM (40–700 keV, above) and WFC (2–26 keV, below) light curves (data points) are compared with the afterglow peak theoretical ones (solid lines). The onset of the afterglow coincides with the end of the P-GRB (represented qualitatively by the dotted lines). For this source we have $B \approx 5.0 \times 10^{-3}$ and $\langle n_{\text{cm}} \rangle \sim 10^{-3}$ particles/cm$^3$.

of GRB 970228 in its host galaxy’s halo$^{255,257,261}$ and it is crucial in explaining the light curve behavior.

The values of $E_{\text{tot}}^\text{grb}$ and $B$ we determined are univocally fixed by two tight constraints. The first one is the total energy emitted by the source all the way up to the latest afterglow phases (i.e., up to $\sim 10^6$ s). The second one is the ratio between the total time-integrated luminosity of the P-GRB and the corresponding one of the entire afterglow (i.e., up to $\sim 10^6$ s). In particular, in GRB 970228 such
a ratio turns out to be $\sim 1.1\%$ (see Fig. 37). However, the P-GRB peak luminosity actually turns out to be much more intense than the afterglow one (see Fig. 53). This is due to the very low average value of the CBM density $\langle n_{cbm} \rangle = 9.5 \times 10^{-4}$ particles/cm$^3$, which produces a less intense afterglow emission. Since the afterglow total time-integrated luminosity is fixed, such a less intense emission lasts longer than what we would expect for an average density $\langle n_{cbm} \rangle \sim 1$ particles/cm$^3$.

15.2. Rescaling the CBM density

We present now an explicit example in order to probe the crucial role of the average CBM density in explaining the relative intensities of the P-GRB and of the afterglow peak in GRB 970228. We keep fixed the basic parameters of the source, namely the total energy $E_{tot}^{\gamma}$ and the baryon loading $B$, therefore keeping fixed the P-GRB and the afterglow total time-integrated luminosities. Then we rescale the CBM density profile given in Fig. 54 by a constant numerical factor in order to raise its average value to the standard one $\langle n_{cbm} \rangle = 1$ particle/cm$^3$. We then compute the corresponding light curve, shown in Fig. 55.

We notice a clear enhancement of the afterglow peak luminosity with respect to the P-GRB one in comparison with the fit of the observational data presented in Fig. 53. The two light curves actually cross at $t_a^d \approx 1.8 \times 10^4$ s since their total time-integrated luminosities must be the same. The GRB “rescaled” to $\langle n_{cbm} \rangle = 1$ particle/cm$^3$ appears to be totally similar to, e.g., GRB 050315$^{180}$ and GRB 991216.$^{152,178,193}$
Fig. 55. The theoretical fit of the BeppoSAX GRBM observations (red line, see Fig. 53) is compared with the afterglow light curve in the 40–700 keV energy band obtained rescaling the CBM density to $\langle n_{cbm} \rangle = 1$ particle/cm$^3$ keeping constant its shape and the values of the fundamental parameters of the theory $E_{tot}$ and $B$ (black line). The P-GRB duration and luminosity (blue line), depending only on $E_{tot}$ and $B$, are not affected by this process of rescaling the CBM density.

It is appropriate to emphasize that, although the two underlying CBM density profiles differ by a constant numerical factor, the two afterglow light curves in Fig. 55 do not. This is because the absolute value of the CBM density at each point affects in a nonlinear way all the following evolution of the fireshell due to the feedback on its dynamics. Moreover, the shape of the surfaces of equal arrival time of the photons at the detector (EQTS) is strongly elongated along the line of sight. Therefore photons coming from the same CBM density region are observed over a very long arrival time interval.

15.3. GRB 970228 and the Amati relation

We turn now to the “Amati relation” between the isotropic equivalent energy emitted in the prompt emission $E_{iso}$ and the peak energy of the corresponding time-integrated spectrum $E_{p,i}$ in the source rest frame. It has been shown by Ref. 252,262 that this correlation holds for almost all the “long” GRBs which have a redshift and an $E_{p,i}$ measured, but not for the ones classified as “short”. If we focus on the “fake” short GRBs, namely the GRBs belonging to this new class, at least in one case (GRB 050724) it has been shown that the correlation is recovered if also the extended emission is considered.

It clearly follows from our treatment that for the “canonical GRBs” with large values of the baryon loading and high $\langle n_{cbm} \rangle$, which presumably are most of the GRBs for which the correlation holds, the leading contribution to the prompt emis-
sion is the afterglow peak emission. The case of the “fake” short GRBs is completely different: it is crucial to consider separately the two components since the P-GRB contribution to the prompt emission in this case is significant.

To test this scenario, we evaluated from our fit of GRB 970228 $E_{\text{iso}}$ and $E_{p,i}$ only for the afterglow peak emission component, i.e., from $t_d = 37$ s to $t_d = 81.6$ s. We found an isotropic energy emitted in the 2–400 keV energy band $E_{\text{iso}} = 1.5 \times 10^{52}$ erg, and $E_{p,i} = 90.3$ keV. As it is clearly shown in Fig. 56, the sole afterglow component of GRB 970228 prompt emission is in perfect agreement with the Amati relation. If this behavior is confirmed for other GRBs belonging to this new class, this will reinforce our identification of the “fake” short GRBs. This result will also provide a theoretical explanation for the apparent absence of such a correlation for the initial spikelike component in the different nature of the P-GRB.

![Fig. 56](image_url)

**Fig. 56.** The estimated values for $E_{p,i}$ and $E_{\text{iso}}$ obtained by our analysis (black dot) compared with the “Amati relation.” The solid line is the best fitting power law and the dashed lines delimit the region corresponding to a vertical logarithmic deviation of 0.4. The uncertainty in the theoretical estimated value for $E_{p,i}$ has been assumed conservatively as 20%.

### 15.4. Conclusions on GRB 970228

We conclude that GRB 970228 is a “canonical GRB” with a large value of the baryon loading quite near to the maximum $B \sim 10^{-2}$ (see Fig. 37). The difference with e.g. GRB 050315 or GRB 991216 is the low average value of the...
CBM density \( \langle n_{cbm} \rangle \sim 10^{-3} \) particles/cm\(^3\) which deflates the afterglow peak luminosity. Hence, the predominance of the P-GRB, coincident with the initial spikelike emission, over the afterglow is just apparent: 98.9% of the total time-integrated luminosity is indeed in the afterglow component. Such a low average CBM density is consistent with the occurrence of GRB 970228 in the galactic halo of its host galaxy.\(^{255,257}\) where lower CBM densities have to be expected.\(^{261}\)

We propose GRB 970228 as the prototype for the new class of GRBs comprising GRB 060614 and the GRBs analyzed by Norris & Bonnell.\(^{194}\) We naturally explain the hardness and the absence of spectral lag in the initial spikelike emission with the physics of the P-GRB originating from the gravitational collapse leading to the black hole formation. The hard-to-soft behavior in the afterglow is also naturally explained by the physics of the relativistic fireshell interacting with the CBM, clearly evidenced in GRB 031203\(^{186}\) and in GRB 050315.\(^{180}\) Also justified is the applicability of the Amati relation to the sole afterglow component.\(^{262,263}\)

This class of GRBs with \( z \sim 0.4 \) appears to be nearer than the other GRBs detected by Swift (\( z \sim 2.3 \)).\(^{264}\) This may be explained by the afterglow peak luminosity deflation. The absence of a jet break in those afterglows has been pointed out,\(^{259,265}\) consistently with our spherically symmetric approach. Their association with non-star-forming host galaxies appears to be consistent with the merging of a compact object binary.\(^{266,267}\) It is here appropriate, however, to caution on this conclusion, since the association of GRB 060614 and GRB 970228 with the explosion of massive stars is not excluded.\(^{250,268}\)

Most of the sources of this class appear indeed not to be related to bright “Hypernovae”, to be in the outskirts of their host galaxies\(^{267}\) and a consistent fraction of them are in galaxy clusters with CBM densities \( \langle n_{cbm} \rangle \sim 10^{-3} \) particles/cm\(^3\).\(^{269,270}\) This suggests a spiraling out binary nature of their progenitor systems\(^{271}\) made of neutron stars and/or white dwarfs leading to a black hole formation.

Moreover, we verified the applicability of the Amati relation to the sole afterglow component in GRB 970228 prompt emission, in analogy with what happens for some of the GRBs belonging to this new class. In fact it has been shown by Ref. 262,263 that the “fake” short GRBs do not fulfill the \( E_{p,i} - E_{iso} \) correlation when the sole spikelike emission is considered, while they do if the long soft bump is included. Since the spikelike emission and the soft bump contributions are comparable, it is natural to expect that the soft bump alone will fulfill the correlation as well.

Within our “canonical GRB” scenario the sharp distinction between the P-GRB and the afterglow provide a natural explanation for the observational features of the two contributions. We naturally explain the hardness and the absence of spectral lag in the initial spikelike emission with the physics of the P-GRB originating from the gravitational collapse leading to the black hole formation. The hard-to-soft behavior in the afterglow is also naturally explained by the physics of the relativistic fireshell interacting with the CBM, clearly evidenced in GRB 031203\(^{186}\) and in GRB 050315.\(^{180}\) Therefore, we expect naturally that the \( E_{p,i} - E_{iso} \) correlation holds only for the afterglow component and not for the P-GRB. Actually we find that the
correlation is recovered for the afterglow peak emission of GRB 970228.

In the original work of Ref. 252,262 only the prompt emission is considered and not the late afterglow one. In our theoretical approach the afterglow peak emission contributes to the prompt emission and continues up to the latest GRB emission. Hence, the meaningful procedure within our model to recover the Amati relation is to look at a correlation between the total isotropic energy and the peak of the time-integrated spectrum of the whole afterglow. A first attempt to obtain such a correlation has already been performed using GRB 050315 as a template, giving very satisfactory results (…).

16. The GRB-Supernova Time Sequence (GSTS) paradigm: the concept of induced gravitational collapse

Following the result of Ref. 273 who discovered the temporal coincidence of GRB 980425 and SN 1998bw, the association of other nearby GRBs with Type Ib/c SNe has been spectroscopically confirmed (see Tab. 2). The approaches in the current literature have attempted to explain both the supernova and the GRB as two aspects of the same astrophysical phenomenon. Hence, GRBs have been assumed to originate from a specially strong supernova process, a hypernova or a collapsar (see e.g. Ref. 245,277–279 and references therein). Both these possibilities imply very dense and strongly wind-like CBM structure.

In our model we assumed that the GRB consistently originates from the gravitational collapse to a black hole. The supernova follows instead the complex pattern of the final evolution of a massive star, possibly leading to a neutron star or to a complete explosion but never to a black hole. The temporal coincidence of the two phenomena, the supernova explosion and the GRB, have then to be explained by the novel concept of “induced gravitational collapse”, introduced in Ref. 128. We have to recognize that still today we do not have a precise description of how this process of “induced gravitational collapse” occurs. At this stage, it is more a
framework to be implemented by additional theoretical work and observations. Two different possible scenarios have been outlined. In the first version we have considered the possibility that the GRBs may have caused the trigger of the supernova event. For the occurrence of this scenario, the companion star had to be in a very special phase of its thermonuclear evolution and three different possibilities were considered:

1. A white dwarf, close to its critical mass. In this case, the GRB may implode the star enough to ignite thermonuclear burning.
2. The GRB enhances in an iron-silicon core the capture of the electrons on the iron nuclei and consequently decreases the Fermi energy of the core, leading to the onset of gravitational instability.
3. The pressure waves of the GRB may trigger a massive and instantaneous nuclear burning process leading to the collapse.

More recently, a quite different possibility has been envisaged: the supernova, originating from a very evolved core, undergoes explosion in presence of a companion neutron star with a mass close to its critical one. The supernova blast wave may then trigger the collapse of the companion neutron star to a black hole and the emission of the GRB (see Fig. 57). It is clear that, in both scenarios, the GRB and the supernova occur in a binary system.

There are many reasons to propose this concept of “induced gravitational collapse”:

1. The fact that GRBs occur from the gravitational collapse to a black hole.
2. The fact that CBM density for the occurrence of GRBs is inferred from the analysis of the afterglow to be on the order of 1 particle/cm$^3$ (see Tab. 2) except for few cases (see e.g. sections 17 and 15). This implies that the process of collapse has occurred in a region of space filled with a very little amount of baryonic matter. The only significant contribution to the baryonic matter component in this process is the one represented by the fireshell baryon loading, which is anyway constrained by the inequality $B \leq 10^{-2}$.
3. The fact that the energetics of the GRBs associated with supernovas appears to be particularly weak is consistent with the energy originating from the gravitational collapse to the smallest possible black hole: the one with mass $M$ just over the neutron star critical mass.

There are also at work very clearly selection effects among the association between supernovas and GRBs:

1. Many type Ib/c supernovas exist without an associated GRB.
2. Some GRBs do not show the presence of an associated supernova, although they are close enough for the supernova to be observed.
3. The presence in all observed GRB-supernova systems of an URCA source, a peculiar late time X-ray emission. These URCA sources have been identified.
Fig. 57. A possible process of gravitational collapse to a black hole “induced” by the Ib/c supernova on a companion neutron star in a close binary system.

and presented for the first time at the Tenth Marcel Grossmann meeting held in Rio de Janeiro (Brazil) in the Village of Urca, and named consequently. They appear to be one of the most novel issues still to be understood about GRBs. We will return on these aspects in the section 17.5.

The issue of triggering the gravitational collapse instability induced by the GRB on the progenitor star of the supernova or, vice versa, by the supernova on the progenitor star of the GRB needs accurate timing. The occurrence of new nuclear physics and/or relativistic phenomena is very likely. The general relativistic instability induced on a nearby star by the formation of a black hole needs some very basic new developments.

Only a very preliminary work exists on this subject, by Jim Wilson and his collaborators. The reason for the complexity in answering such a question is simply stated; unlike the majority of theoretical work on black holes and binary X-ray sources, which deals mainly with one-body black hole solutions in the Newtonian field of a companion star, we now have to address a many-body problem in general relativity. We are starting in these days to reconsider, in this framework, some classic work by Ref. 281–287 which may lead to a new understanding of general relativistic effects in these many-body systems. This is a welcome effect of GRBs.
on the conceptual development of general relativity.

17. Some unexplained features in the GRB-supernova association: the URCA sources

Models of GRBs based on a single source (the “collapsar”) generating both the supernova and the GRB abounds in the literature. Since the two phenomena are qualitatively very different, in our approach we have emphasized the concept of induced gravitational collapse, which occurs strictly in a binary system. The supernova originates from a star evolved out of the main sequence and the GRB from the collapse to a black hole. The concept of induced collapse implies at least two alternative scenarios. In the first, the GRB triggers a supernova explosion in the very last phase of the thermonuclear evolution of a companion star. In the second, the early phases of the SN induce gravitational collapse of a companion neutron star to a black hole. Of course, in absence of a supernova, there is also the possibility that the collapse to a black hole, generating the GRB, occurs in a single star system or in the final collapse of a binary neutron star system. Still, in such a case there is also the possibility that the black hole progenitor is represented by a binary system composed by a white dwarf and/or a neutron star and/or a black hole in various combinations. What is most remarkable is that, following the “uniqueness of the black hole”, all these collapses lead to a common GRB independently of the nature of their progenitors.

Having obtained success in the fit of GRB 991216, as well as of GRB 031203 and GRB 050315 (see sections 11 and 12), we turn to the application of our theoretical analysis to the GRBs associated with supernovas. We start with GRB 980425 / SN 1998bw. We must, however, be cautious about the validity of this fit. From the available data of BeppoSAX, BATSE, XMM and Chandra, only the data of the prompt emission ($t_{\text{d}a} < 10^{7}$ s) and of the latest afterglow phases ($t_{\text{d}a} > 10^{5}$ s all the way to more than $10^{8}$ s!) were available. Our fit refers only to the prompt emission, as usually interpreted as the peak of the afterglow. The fit, therefore, represents an underestimate of the GRB 980425 total energy and in this sense it is not surprising that it does not fit the Ref. 252 relation. The latest afterglow emission, the URCA-1 emission, presents a different problematic which we will shortly address (see below).

17.1. GRB 980425 / SN 1998bw / URCA-1

The best fit of the observational data of GRB 980425 leads to $E_{\text{tot}} = 1.2 \times 10^{48}$ erg and $B = 7.7 \times 10^{-3}$. This implies an initial $e^\pm$ plasma with $N_{e^+e^-} = 3.6 \times 10^{53}$ and with an initial temperature $T = 1.2$ MeV. After the transparency point, the initial Lorentz gamma factor of the accelerated baryons is $\gamma_0 = 124$. The variability of the luminosity, due to the inhomogeneities of the CBM, is characterized by a density contrast $\delta n/n \sim 10^{-1}$ on a length scale of $\Delta \sim 10^{14}$ cm. We determine the effective CBM parameters to be: $\langle n_{\text{cbm}} \rangle = 2.5 \times 10^{-2}$ particle/cm$^3$ and $\langle R \rangle = 1.2 \times 10^{-8}$. 


In Fig. 58 we test our specific theoretical assumptions comparing our theoretically computed light curves in the 40–700 keV and 2–26 keV energy bands with the observations by the BeppoSAX GRBM and WFC during the first 60 s of data. The agreement between observations and theoretical predictions in Fig. 58 is very satisfactory.

In Fig. 59 we summarize some of the problematics implicit in the old pre-Swift era: data are missing in the crucial time interval between 60 s and $10^5$ s, when the BeppoSAX NFI starts to point the GRB 980425 location. In this region we have assumed, for the effective CBM parameters, constant values inferred by the last...
Fig. 59. Theoretical light curves of GRB 980425 in the 40–700 keV (red line), 2–26 keV (green line), 2–10 keV (blue line) energy bands, represented together with URCA-1 observational data. All observations are by BeppoSAX, with the exception of the last two URCA-1 points, which is observed by XMM and Chandra. The follow-up of GRB 980425 with BeppoSAX NFI 10 hours, one week and 6 months after the event revealed the presence of an X-ray source consistent with SN1998bw, confirmed also by observations by XMM and Chandra. The S1 X-ray light curve shows a decay much slower than usual X-ray GRB afterglows. We then address to this peculiar X-ray emission as “URCA-1” (see section 16 and the following sections). In Fig. 60A we represent the URCA-1 observations. The separation between the light curves of GRB 980425 in the 2–700 keV energy band, of SN 1998bw in the optical band, and of the above mentioned URCA-1 observations is evident.

17.2. GRB 030329 / SN 2003dh / URCA-2

For GRB 030329 we have obtained a total energy $E_{\text{tot}}^{\text{tot}} = 2.12 \times 10^{52}$ erg and a baryon loading $B = 4.8 \times 10^{-3}$. This implies an initial $e^\pm$ plasma with $N_{e^+ e^-} = 1.1 \times 10^{17}$ and with an initial temperature $T = 2.1$ MeV. After the transparency point, the initial Lorentz gamma factor of the accelerated baryons is $\gamma_0 = 206$. The effective CBM parameters are $\langle n_{\text{chem}} \rangle = 2.0$ particle/cm$^3$ and
⟨R⟩ = 2.8 × 10^{-9}, with a density contrast δn/n ∼ 10 on a length scale of Δ ∼ 10^{14} cm. The resulting fit of the observations, both of the prompt phase and of the afterglow have been presented in Ref. 185,272. We compare in Fig. 60B the light curves of GRB 030329 in the 2–400 keV energy band, of SN 2003dh in the optical band^{233,248} and of the possible URCA-2 emission observed by XMM-EPIC in 2–10 keV energy band^{291,292}

17.3. GRB 031203 / SN 2003lw / URCA-3

Fig. 60. Theoretically computed light curves of GRB 980425 in the 2–700 keV band (A), of GRB 030329 in the 2–400 keV band (B) and of GRB 031203 in the 2–200 keV band (C) are represented, together with the URCA observational data and qualitative representative curves for their emission, fit with a power law followed by an exponentially decaying part. The luminosity of the supernovas in the 3000 − 24000 Å are also represented.^{233,248}

We will show in section 11 the detailed analysis of GRB 031203 which leads to a total energy E_{\text{tot}} = 1.85 \times 10^{50} \text{ erg} and to a baryon loading B = 7.4 \times 10^{-3}.
This implies an initial $e^\pm$ plasma with $N_{e^\pm} = 3.0 \times 10^{55}$ and with an initial temperature $T = 1.5$ MeV. After the transparency point, the initial Lorentz gamma factor of the accelerated baryons is $\gamma_0 = 132$. The effective CBM parameters are $\langle n_{cbm}\rangle = 1.6 \times 10^{-1}$ particle/cm$^3$ and $\langle R \rangle = 3.7 \times 10^{-9}$, with a density contrast $\delta n/n \sim 10$ on a length scale of $\Delta \sim 10^{15}$ cm. In Fig. 60C we compare the light curves of GRB 031203 in the 2–200 keV energy band, of SN 2003lw in the optical band$^{233,248}$ and of the possible URCA-3 emission observed by XMM-EPIC in the 0.2–10 keV energy band$^{189}$ and by Chandra in the 2–10 keV energy band.$^{200}$

17.4. The GRB / SN / URCA connection

In Tab. 3 we summarize the representative parameters of the above three GRB-supernova systems together with GRB 060218-SN 2006aj, including the very large kinetic energy observed in all supernovas.$^{203}$ Some general conclusions on these weak GRBs at low redshift, associated to SN Ib/c, can be established on the grounds of our analysis:

1) From the detailed fit of their light curves, as well as their accurate spectral analysis, it follows that all the above GRB sources originate consistently from the formation of a black hole. This result extends to this low-energy GRB class at small cosmological redshift the applicability of our model, which now spans over a range of energy of six orders of magnitude from $10^{48}$ to $10^{54}$ ergs.$^{152,180,184,186,204,272}$ Distinctive of this class is the very high value of the baryon loading which in one case (GRB 060218)$^{187}$ is very close to the maximum limit compatible with the dynamical stability of the adiabatic optically thick acceleration phase of the GRBs.$^{107}$ Corresopndingly, the maximum Lorentz gamma factors are systematically smaller than the ones of the more energetic GRBs at large cosmological distances. This in turn implies the smoothness of the observed light curves in the so-called “prompt phase”. The only exception to this is the case of GRB 030329.

2) The accurate fits of the GRBs allow us to also infer some general properties of the CBM. While the size of the clumps of the inhomogeneities is $\Delta \approx 10^{14}$ cm, the effective CBM average density is consistently smaller than in the case of more energetic GRBs: we have in fact $\langle n_{cbm}\rangle$ in the range between $\sim 10^{-6}$ particle/cm$^3$...
(GRB 060218) and $\sim 10^{-1}$ particle/cm$^3$ (GRB 031203), while only in the case of GRB 030329 it is $\sim 2$ particle/cm$^3$.

3) Still within their weakness these four GRB sources present a large variability in their total energy: a factor $10^4$ between GRB 980425 and GRB 030329. Remarkably, the supernova emissions both in their very high kinetic energy and in their bolometric energy appear to be almost constant respectively $10^{52}$ erg and $10^{49}$ erg. The URCA sources present also a remarkably steady behavior around a “standard luminosity” and a typical temporal evolution. The weakness in the energetics of GRB 980425 and GRB 031203, and the sizes of their dyadospheres, suggest that they originate from the formation of the smallest possible black hole, just over the critical mass of the neutron star (see Fig. 57).

17.5. **URCA-1, URCA-2 and URCA-3**

We turn to the search for the nature of URCA-1, URCA-2 and URCA-3. These systems are not yet understood and may have an important role in the comprehension of the astrophysical scenario of GRB sources. It is important to perform additional observations in order to verify if the URCA sources are related to the black hole originating the GRB phenomenon or to the supernova. Even a single observation of an URCA source with a GRB in absence of a would prove their relation with the black hole formation. Such a result is today theoretically unexpected and would open new problematics in relativistic astrophysics and in the physics of black holes. Alternatively, even a single observation of an URCA source during the early expansion phase of a Type Ib/c supernova in absence of a GRB would prove the early expansion phases of the supernova remnants. In the case that none of such two conditions are fulfilled, then the URCA sources must be related to the GRBs occurring in presence of a supernova. In such a case, one of the possibilities would be that for the first time we are observing a newly born neutron star out of the supernova phenomenon unveiled by the GRB. This last possibility would offer new fundamental information about the outcome of the gravitational collapse, and especially about the equations of state at supra-nuclear densities and about a variety of fundamental issues of relativistic astrophysics of neutron stars.

The names of “URCA-1” and “URCA-2” for the peculiar late X-ray emission of GRB 980425 and GRB 030329 were given in the occasion of the Tenth Marcel Grossmann meeting held in Rio de Janeiro (Brazil) in the Village of Urca. Their identification was made at that time and presented at that meeting. However, there are additional reasons for the choice of these names. Another important physical phenomenon was indeed introduced in 1941 in the same Village of Urca by George Gamow and Mario Schoenberg. The need for a rapid cooling process due to neutrino anti-neutrino emission in the process of gravitational collapse leading to the formation of a neutron star was there considered for the first time. It was Gamow who named this cooling as “Urca process”. Since then, a systematic analysis of the theory of neutron star cooling was advanced by Ref. 297–301. The
coming of age of X-ray observatories such as Einstein (1978-1981), EXOSAT (1983-1986), ROSAT (1990-1998), and the contemporary missions of Chandra and XMM-Newton since 1999 dramatically presented an observational situation establishing very embarrassing and stringent upper limits to the surface temperature of neutron stars in well known historical supernova remnants. For some remnants, notably SN 1006 and the Tycho supernova, the upper limits to the surface temperatures were significantly lower than the temperatures given by standard cooling times. Much of the theoretical work has been mainly directed, therefore, to find theoretical arguments in order to explain such low surface temperature $T_s \sim 0.5-1.0 \times 10^6$ K — embarrassingly low, when compared to the initial hot ($\sim 10^{11}$ K) birth of a neutron star in a supernova explosion. Some important contributions in this research have been presented by Ref. 303–307. The youngest neutron star to be searched for thermal emission has been the pulsar PSR J0205+6449 in 3C 58, which is 820 years old! Ref. 308 reported evidence for the detection of thermal emission from the crab nebula pulsar which is, again, 951 years old.

URCA-1, URCA-2 and URCA-3 may explore a totally different regime: the X-ray emission possibly from a recently born neutron star in the first days – months of its existence. The thermal emission from the young neutron star surface would in principle give information on the equations of state in the core at supranuclear densities and on the detailed mechanism of the formation of the neutron star itself with the related neutrino emission. It is also possible that the neutron star is initially fast rotating and its early emission could be dominated by the magnetospheric emission or by accretion processes from the remnant which would overshadow the thermal emission. A periodic signal related to the neutron star rotational period should in principle be observable in a close enough GRB-supernova system. In order to attract attention to this problematic, we have given in Tab. 3 an estimate of the corresponding neutron star radius for URCA-1, URCA-2 and URCA-3. It has been pointed out the different spectral properties between the GRBs and the URCAs. It would be also interesting to compare and contrast the spectra of all URCAs in order to evidence any analogy among them. Observations of a powerful URCA source on time scales of 0.1–10 seconds would be highly desirable.

18. Conclusions

GRBs are giving the first clear evidence for the extraction of energy from black holes during the last phases of their formation process. This new form of energy is unprecedented in the Universe, both for its magnitude and its very high efficiency in transforming matter into radiation, which reaches the 50% limit while the nuclear energy reaches efficiency of 2 – 3% only. These sources, with their energy of $10^{54}$ ergs/pulse, dwarf the corresponding nuclear energy events with their energy of $\sim 10^{22}$ ergs/pulse.

We have shown how the quest for understanding the initial conditions leading to this new gravitational electromagnetic phenomenon has originated a new inquiry
into the properties of nuclear matter in bulk which sheds new light on two classical physical problems: the heavy nuclei and neutron stars. Such an analysis follows a new conceptual unified approach of important heuristic content. It may well lead to the solution of yet unsolved problems in physics and astrophysics and on much different scales, such as the emission of the remnant in supernova phenomena on a macroscopic scale or the limits on the dimension of the constituent of elementary particles on a microscopic scale.

The richness of the experimental data obtained from GRBs, especially thanks to the Swift and INTEGRAL satellites, has been exemplified in a selected number of sources. This allowed the development and for the first time the testing of the theory of ultrarelativistic collisions of baryonic matter with a Lorentz gamma factor up to $\gamma \sim 300$ and involving scales extending up to a few light years. The interpretation of these observations needed the development of a highly self-consistent theoretical framework, which has been tested with high accuracy analyzing both spectra and luminosities in selected energy bands on unprecedented time scales ranging from few milliseconds all the way to $10^6$ seconds. In turn, the CBM structure has been analyzed using GRB light curves as its tomographic image. From all these analyses a “canonical” GRB scenario is emerging, quite independent on their enormous energy ranging from $10^{48}$ erg to $10^{54}$ erg. Such a “canonical” GRB scenario promises to be relevant for the future use of GRBs as cosmological standard candles.

As the comprehension of the GRB phenomenon progresses, so the astrophysical setting where they originate is further clarified. We have given evidence for the GRB observation originating in active star formation region, with $n_{cbm} \sim 1$ particle/cm$^3$, as well as in galactic halos, with $n_{cbm} \sim 10^{-3}$ particles/cm$^3$. There is also mounting evidence that all observed GRBs originate from a variety of binary systems. There are, in this respect, at least three different systems: 1) GRBs associated with supernova originating from an initial binary system formed by a massive star and a neutron star and evolving to a neutron star, 2) originating from the supernova event, and 3) a black hole, originating from the gravitational collapse of the neutron star. The occurrence of these GRBs is strictly linked to the process of induced gravitational collapse. These supernova-related GRBs are therefore the less energetic ones, being formed by the smallest possible black holes just over the critical mass of a neutron star. There are also much more energetic phenomena occurring both in the galactic halos (e.g. GRB 970228) and in star-forming regions (e.g. GRB 050315), which must definitely originate from the collapse of binary systems formed by two neutron stars or a white dwarf and a neutron star or two intermediate mass black holes.

What would appear to be a priori very surprising is that such a large variety of initial conditions leads to a “canonical” GRB scenario consistent over a range of energies spanning 6 orders of magnitude. Again, the crucial explanation for this is due to the uniqueness of the black hole, which was represented in a thought-provoking form in the paper “Introducing the black hole” which we reproduce here in Fig. 61 and we also represent in its updated version for the GRB connection.
Fig. 61. The uniqueness of the black hole. Reproduced from the paper “Introducing the black hole”.

in Fig. 62.

References
Fig. 62. The uniqueness of the black hole picture updated for GRB connection.

15. V. Weisskopf, Zeitschrift fur Physik, 89, 27 (1934).
129. R. Ruffini, L. Vitagliano, S.-S. Xue, in Quantum Aspects of Beam Physics, 28th
130

144. G. Scorza-Dragoni, Rend. Accad. Lincei, 8, 301 (1928).


213. S. D. Barthelmy, L. M. Barbier, J. R. Cummings, E. E. Fenimore, N. Gehrels,


224. T. Sakamoto, L. Barbier, S. Barthelmy, J. Cummings, E. Fenimore, N. Gehrels,


289. E. Pian, P. Giommi, L. Amati, E. Costa, J. Danziger, M. Feroci, M. T. Fiocchi,


Attachment 8
The Blackholic energy and the canonical Gamma-Ray Burst
IV: the “long”, “genuine short” and “fake - disguised short”
GRBs

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Abstract. We report some recent developments in the understanding of GRBs based on the theoretical framework of the
“fireshell” model, already presented in the last three editions of the “Brazilian School of Cosmology and Gravitation”. After
recalling the basic features of the “fireshell model”, we emphasize the following novel results: 1) the interpretation of the
X-ray flares in GRB afterglows as due to the interaction of the optically thin fireshell with isolated clouds in the CircumBurst
Medium (CBM); 2) an interpretation as “fake - disguised” short GRBs of the GRBs belonging to the class identified by Norris
& Bonnell; we present two prototypes, GRB 970228 and GRB 060614; both these cases are consistent with an origin from
the final coalescence of a binary system in the halo of their host galaxies with particularly low CBM density ncbm ∼ 10−3
particles/cm3; 3) the first attempt to study a genuine short GRB with the analysis of GRB 050509B, that reveals indeed still
an open question; 4) the interpretation of the GRB-SN association in the case of GRB 060218 via the “induced gravitational
collapse” process; 5) a first attempt to understand the nature of the “Amati relation”, a phenomenological correlation between
the isotropic-equivalent radiated energy of the prompt emission Eiso and the cosmological rest-frame νFs spectrum peak
energy Eν,p. In addition, recent progress on the thermalization of the electron-positron plasma close to their formation phase, as
well as the structure of the electrodynamics of Kerr-Newman Black Holes are presented. An outlook for possible explanation
of high-energy phenomena in GRBs to be expected from the AGILE and the Fermi satellites are discussed. As an example
of high energy process, the work by Enrico Fermi dealing with ultrarelativistic collisions is examined. It is clear that all the
GRB physics points to the existence of overcritical electrodynamical fields. In this sense we present some progresses on a
unified approach to heavy nuclei and neutron stars cores, which leads to the existence of overcritical fields under the neutron
star crust.

INTRODUCTION

Gamma-Ray Bursts (GRBs) represent very likely “the” most extensive computational, theoretical and observational
effort ever carried out successfully in physics and astrophysics. The extensive campaign of observation from space
based X-ray and γ-ray observatory, such as the Vela, CGRO, BeppoSAX, HETE-II, INTEGRAL, Swift, Agile, GLAST,
R-XTE, Chandra, XMM satellites, have been matched by complementary observations in the radio wavelength (e.g.
by the VLA) and in the optical band (e.g. by VLT, Keck, REM). The very fortunate situation occurs that these data can
be confronted with a mature theoretical development.

We outline how this progress leads to the confirmation of three interpretation paradigms for GRBs we proposed

1 Part I, Part II and Part III of these Lecture notes have been published respectively in COSMOLOGY AND GRAVITATION: Xth Brazilian School of
in COSMOLOGY AND GRAVITATION: Xth Brazilian School of Cosmology and Gravitation, M. Novello, S.E. Perez Bergliaffa (eds.), AIP Conf.
Proc., 782, 42 (2005), see Ref. [2], and in COSMOLOGY AND GRAVITATION: XII Brazilian School of Cosmology and Gravitation, M. Novello,
in July 2001 [4, 5, 6]. The outcome of this analysis points to the existence of a “canonical” GRB, originating from a variety of different initial astrophysical scenarios. The communality of these GRBs appears to be that they all are emitted in the process of formation of a black hole with a negligible value of its angular momentum. The following sequence of events appears to be canonical: the gravitational collapse to a black hole, the vacuum polarization process in the dyadosphere with the creation of the optically thick self accelerating electron-positron plasma; the engulfment of baryonic mass during the plasma expansion; adiabatic expansion of the optically thick “fireshell” of electron-positron-baryon plasma up to the transparency; the interaction of the accelerated baryonic matter with the CircumBurst Medium (CBM). This leads to the canonical GRB composed of a proper GRB (P-GRB), emitted at the moment of transparency, followed by an extended afterglow. The sole parameters in this scenario are the total energy of the dyadosphere $E_{dya}$, the fireshell baryon loading $M_B$ defined by the dimensionless parameter $B \equiv M_B E_{dya}^{-1}$, and the CBM filamentary distribution around the source. In the limit $B \rightarrow 0$ the total energy is radiated in the P-GRB with a vanishing contribution in the extended afterglow. We refer globally to this model as the “fireshell” model.

The increase of observational details on many different GRBs, gained in the last two year observational campaign, is showing very clearly an evolution in the understanding of this basic scenario. This has allowed to explore GRBs with different baryon loading and very different CBM properties. A major new result has been obtained in the understanding of the origin of flares, which is traced back to the interaction of the fireshell with isolated CBM clouds (see below, section GRB 060607A: a complete analysis of the prompt emission and X-ray flares).

Another key result, presented in these lectures, has been the understanding of a new GRB class, which was pioneered by Norris and Bonnell [7], characterized by “an occasional softer extended emission lasting tenths of seconds after an initial spikelike emission”. From the “fireshell” model it clearly follows that these sources explore a new range of parameters and mark a fundamental step in the understanding of the astrophysical nature of the GRB progenitor systems. The new class is shown to be characterized by GRBs occurring in a particularly low density CBM, $n_{\text{cbm}} \sim 10^{-3}$ particles/cm$^3$, typical of galactic halos. The progenitors must therefore necessarily be binary sources which have migrated from their birth location in a star forming region to a low density region within the galactic halo, where the final merging occurs. If such sources did explode in a CBM with average density $n_{\text{cbm}} \sim 1$ particle/cm$^3$, they would look exactly like what is called long GRBs. In this sense, we decided to call them “fake” or “disguised” short GRBs (see below, section The Norris & Bonnel kind of sources: the new class of “fake - disguised” short GRBs).

This leads, in turn, to a novel interpretation of the traditional GRB classification [8, 9] in “short” GRBs (with a $T_{90}$ duration lasting less than $\sim 2$ s) and “long” ones (with a $T_{90}$ duration lasting more than $\sim 2$ s up to $\sim 1000$ s). The duration of long GRBs is shown to be just due to the instrumental noise threshold and not to represent any intrinsic property of the source. On the other hand, there is an increasing evidence that the majority of the sources classified as short GRBs are actually “fake - disguised” short ones. No Supernova can possibly be related to these GRBs (see below, section The “fireshell” model and GRB progenitors). However, some issues remains open (see below, section Open issues in current theoretical models). Moreover, the quest for the first clear identification of a “genuine” short GRB is also still open (see below, section The search for a “genuine short” GRB: the case of GRB 050509B).

The GRBs associated with SN do necessarily form a different class, of weakest and more numerous sources, originating also from binary systems, formed by a neutron star, close to its critical mass, and a companion star, evolved out of the main sequence, via the “induced gravitational collapse” process (see below, section GRBs and SNe: the induced gravitational collapse).

The detection of GRBs up to very high redshifts [up to $z = 6.7$, see Ref. 10], their high observed rate of one every few days, and the progress in the theoretical understanding of these sources all make them useful as cosmological tools, complementary to supernovae Ia, which are observed only up to $z = 1.7$ [11, 12]. One of the hottest topics on GRBs is the possible existence of empirical relations between GRB observables [13, 14, 15, 16, 17, 18], which may lead, if confirmed, to using GRBs as tracers of models of universe. The first empirical relation, discovered when analyzing the BeppoSAX so-called “long” bursts with known redshift, was the “Amati relation” [13]. It was found that the isotropic-equivalent radiated energy of the prompt emission $E_{iso}$ is correlated with the cosmological rest-frame $\nu F_{\nu}$ spectrum peak energy $E_{p,\gamma}$: $E_{(p,\gamma)} \propto E_{iso}^{a}$, with $a = 0.52 \pm 0.06$ [13]. The existence of the Amati relation has been confirmed by studying a sample of GRBs discovered by Swift, with $a = 0.49^{+0.06}_{-0.05}$ [19, 20]. We present a first attempt to understand the nature of such a phenomenological correlation (see below, section Theoretical background for GRBs’ empirical correlations).

All these theoretical and phenomenological approach has motivated significant progresses on:

a) The electron-positron plasma thermalization. We solved numerically relativistic Boltzmann equations with collisional integrals representing two-body as well as three-body interactions, in particular Compton, Moller and Bhabha scattering, pair creation and annihilation, relativistic bremsstrahlung, three photon creation and annihilation,
double Compton scattering and the corresponding processes where protons participate. This allowed determination of characteristic timescales of thermalization as well as clarified the role of binary and triple interactions in reaching thermal equilibrium (see below, section Thermalization process of electron-positron plasma with baryon loading).

b) The electrodynamics of neutron stars and black holes. We present a unified treatment of nuclear density cores recovering the classical results of neutral atoms with heavy nuclei with mass number $A \approx 10^{2}-10^{6}$ and extrapolating these results to massive nuclear density cores with $A \approx (m_{\text{Planck}}/m_{n})^3 \sim 10^{57}$. The treatment is approached by solving the relativistic Thomas-Fermi equation describing a system of $N_n$ neutrons, $N_p$ protons and $N_e$ electrons in beta equilibrium. In order to show the stability of such cores under the competing effects of self gravity and Coulomb repulsion, we have started to take the gravitational field into duly account, and put the issue within the framework of general relativity. We expect that starting from such configuration, gravitational collapse would lead to Dyadosphere (electron-positron-photon plasma) in the Reissner-Nordström geometry, or Dyadotorus in the Kerr-Newman geometry. We present in some details the analysis of the Dyadotorus (see below, section Critical Electric Fields on the Surface of Massive Cores and Dyadotorus of the Kerr-Newman Geometry).

c) In view of the new data from the Fermi and AGILE satellites, an analysis of the GRB radiation over 1 MeV. It is by now clear that our pure thermal emission previously considered, and which has been fundamental in expressing the average CBM density, is not appropriate to the description of this high-energy component. In parallel, we are currently examining how Fermi ideas [21] have been further developed in large data analysis procedures at CERN and other accelerators all over the world (see below, section Selected Processes Originating High-Energy Emission).

**BRIEF REMINDER OF THE FIRESHELL MODEL**

The black hole uniqueness theorem [see Left panel in Fig. 1 and e.g. Ref. 22] is at the very ground of the fact that it is possible to explain the different Gamma-Ray Burst (GRB) features with a single theoretical model, over a range of energies spanning over 6 orders of magnitude. The fundamental point is that, independently of the fact that the progenitor of the gravitational collapse is represented by merging binaries composed by neutron stars and white dwarfs in all possible combinations, or by a single process of gravitational collapse, or by the process of "induced" gravitational collapse, the formed black hole is totally independent from the initial conditions and reaches the standard configuration of a Kerr-Newman black hole (see Right panel in Fig. 1). It is well known that pair creation by vacuum polarization process can occur in a Kerr-Newman black hole [23, 24].

We consequently assume, within the fireshell model, that all GRBs originate from an optically thick $e^\pm$ plasma with total energy $E_{\text{tot}}$ in the range $10^{49}-10^{55}$ ergs and a temperature $T$ in the range 1–4 MeV [26]. Such an $e^\pm$
Figure 2. The “canonical GRB” light curve theoretically computed for GRB 991216. The prompt emission observed by BATSE is identified with the peak of the extended afterglow, while the small precursor is identified with the P-GRB. For this source we have $E_{\text{tot}} \pm = 4.83 \times 10^{53}$ ergs, $B \approx 3.0 \times 10^{-3}$ and $\langle n_{\text{cmb}} \rangle \sim 1.0$ particles/cm$^3$. Details in Ruffini et al. [5, 25, 3].
plasma has been widely adopted in the current literature [see e.g. Refs. 27, 28, and references therein]. After an early expansion, the $e^\pm$-photon plasma reaches thermal equilibrium with the engulfed baryonic matter $M_B$ described by the dimensionless parameter $B = M_B c^2 / E_{tot}^0$, that must be $B < 10^{-2}$ [29, 30]. As the optically thick fireshell composed by $e^\pm$-photon-baryon plasma self-accelerate to ultrarelativistic velocities, it finally reaches the transparency condition. A flash of radiation is then emitted. This is the P-GRB [5]. Different current theoretical treatments of these early expansion phases of GRBs are compared and contrasted in Bianco et al. [31] and Ruffini et al. [32]. The amount of energy radiated in the P-GRB is only a fraction of the initial energy $E_{tot}^0$. The remaining energy is stored in the kinetic energy of the optically thin baryonic and leptonic matter fireshell that, by inelastic collisions with the CBM, gives rise to a multi-wavelength emission. This is the extended afterglow. It presents three different regimes: a rising part, a peak and a decaying tail. We therefore define a “canonical GRB” light curve with two sharply different components (see Fig. 2) [5, 3, 33, 34, 35]: 1) the P-GRB and 2) the extended afterglow. What is usually called “Prompt emission” in the current literature mixes the P-GRB with the raising part and the peak of the extended afterglow. Such an unjustified mixing of these components, originating from different physical processes, leads to difficulties in the current models of GRBs, and can as well be responsible for some of the intrinsic scatter observed in the Amati relation [20, 36, , see also below, section Theoretical background for GRBs’ empirical correlations].

### The optically thick phase

In Fig. 3 we present the evolution of the optically thick fireshell Lorentz $\gamma$ factor as a function of the external radius for 7 different values of the fireshell baryon loading $B$ and two selected limiting values of the total energy $E_{tot}^0$ of the $e^\pm$ plasma. We can identify three different eras:

1. **Era I:** The fireshell is made only of electrons, positrons and photons in thermodynamical equilibrium (the “pair-electromagnetic pulse”, or PEM pulse for short). It self-accelerate and begins its expansion into vacuum, because the environment has been cleared by the black hole collapse. The Lorentz $\gamma$ factor increases with radius and the dynamics can be described by the energy conservation and the condition of adiabatic expansion [29, 31]:

\[
T^{\mu\nu} \gamma = 0
\]

\[
\frac{\epsilon}{\gamma} = \left( \frac{V}{V_c} \right) \Gamma = \left( \frac{\gamma \gamma}{\gamma \gamma} \right) \Gamma
\]

where $T^{\mu\nu}$ is the energy-momentum tensor of the $e^\pm$ plasma (assumed to be a perfect fluid), $\epsilon$ is its internal energy density, $V$ and $\gamma$ are its volumes in the co-moving and laboratory frames respectively, $\Gamma$ is the thermal index and the quantities with and without the $\circ$ subscript are measured at two different times during the expansion.

2. **Era II:** The fireshell impacts the non-collapsed baryonic remnants and engulfs them. The Lorentz $\gamma$ factor drops. The dynamics of this era can be described by imposing energy and momentum conservation during the fully inelastic collision between the fireshell and the baryonic remnant. For the fireshell solution to be still valid, it must be $B \lesssim 10^{-2}$ [30].

3. **Era III:** The fireshell is now made of electrons, positrons, baryons and photons in thermodynamical equilibrium (the “pair-electromagnetic-baryonic pulse”, or PEMB pulse for short). It self-accelerate again and the Lorentz $\gamma$ factor increases again with radius up to when the transparency condition is reached, going to an asymptotic value $\gamma_{asym} = 1 / B$. If $B \sim 10^{-3}$ the transparency condition is reached when $\gamma \sim \gamma_{asym}$. On the other hand, when $B < 10^{-2}$, the transparency condition is reached much before $\gamma$ reaches its asymptotic value [30, 5, , see next section for details]. In this era the dynamical equations are the same of the first one, together with the baryon number conservation:

\[
T^{\mu\nu} \gamma = 0
\]

\[
\frac{\epsilon}{\gamma} = \left( \frac{V}{V_c} \right) \Gamma = \left( \frac{\gamma \gamma}{\gamma \gamma} \right) \Gamma
\]

\[
\frac{n_B^2}{n_B} = \left( \frac{V}{V_c} \right) \left( \frac{\gamma \gamma}{\gamma \gamma} \right)
\]
where \( n_B \) is the baryon number density in the fireshell. In this era it starts to be crucial the contribution of the rate equation to describe the annihilation of the \( e^+ e^- \) pairs:

\[
\frac{\partial}{\partial t} N_{e^\pm} = -N_{e^\pm} \frac{1}{T} \frac{d}{dt} + \frac{1}{2} \left( N_{e^+}^2(T) - N_{e^-}^2 \right)
\]  

where \( N_{e^\pm} \) is the number of \( e^+ e^- \) pairs and \( N_{e^\pm}(T) \) is the number of \( e^+ e^- \) pairs at thermal equilibrium at temperature \( T \).

In the “fireball” model in the current literature the baryons are usually considered present in the plasma since the beginning. In other words, in the fireball dynamics there is only one era corresponding to the Era III above [37, 38, 39, 31]. Moreover, the rate equation is usually neglected, and this affects the reaching of the transparency condition. A detailed comparison between the different approaches is reported in Bianco et al. [31].

**The transparency point**

At the transparency point, the value of the \( B \) parameter rules the ratio between the energetics of the P-GRB and the kinetic energy of the baryonic and leptonic matter giving rise to the extended afterglow. It rules as well the time separation between the corresponding peaks [5, 32].

We have recently shown [40] that a thermal spectrum still occurs in presence of \( e^\pm \) pairs and baryons. By solving the rate equation we have evaluated the evolution of the temperature during the fireshell expansion, all the way up to when the transparency condition is reached [29, 30]. In the upper panel of Fig. 4 we plot, as a function of \( B \), the fireshell temperature \( T_\gamma \) at the transparency point, i.e. the temperature of the P-GRB radiation. The plot is drawn for four different values of \( E_{\gamma, \text{tot}} \) in the interval \([10^{49}, 10^{55}] \) ergs, well encompassing GRBs’ observed isotropic energies. We plot both the value in the co-moving frame \( T_\gamma^{\text{com}} \) and the one Doppler blue-shifted toward the observed \( T_\gamma^{\text{obs}} = (1 + \beta) T_\gamma^{\text{com}} \), where \( \beta \) is the fireshell speed at the transparency point in units of \( c \) [30].

In the middle panel of Fig. 4 we plot, as a function of \( B \), the fireshell Lorentz gamma factor at the transparency point \( \gamma_\circ \). The plot is drawn for the same four different values of \( E_{\gamma, \text{tot}} \) of the upper panel. Also plotted is the asymptotic value \( \gamma_\circ = 1/B \), which corresponds to the condition when the entire initial internal energy of the plasma \( E_{\gamma, \text{tot}}^{\text{obs}} \) has been converted into kinetic energy of the baryons [30]. We see that such an asymptotic value is approached for \( B \rightarrow 10^{-2} \). We see also that, if \( E_{\gamma, \text{tot}}^{\text{obs}} \) increases, the maximum values of \( \gamma_\circ \) are higher and they are reached for lower values of \( B \).

In the lower panel of Fig. 4 we plot, as a function of \( B \), the total energy radiated at the transparency point in the P-GRB and the one converted into baryonic and leptonic kinetic energy and later emitted in the extended afterglow. The plot is drawn for the same four different values of \( E_{\gamma, \text{tot}} \) of the upper panel and middle panels. We see that for \( B \lesssim 10^{-5} \) the total energy emitted in the P-GRB is always larger than the one emitted in the extended afterglow. In the limit \( B \rightarrow 0 \) it gives rise to a “genuine” short GRB (see also Fig. 5). On the other hand, for \( 3.0 \times 10^{-4} \lesssim B < 10^{-2} \) the total energy emitted in the P-GRB is always smaller than the one emitted in the extended afterglow. If it is it not below the instrumental threshold and if \( n_{\text{obs}} \sim 1 \) particle/cm\(^2\) (see Fig. 14), the P-GRB can be observed in this case as a small pulse preceding the main GRB event (which coincides with the peak of the extended afterglow), i.e. as a GRB “precursor” [5, 1, 35, 32, 33, 41].

Particularly relevant for the new era of the *Agile* and *GLAST* satellites is that for \( B < 10^{-3} \) the P-GRB emission has an observed temperature up to \( 10^3 \) keV or higher. This high-energy emission has been unobservable by the *Swift* satellite.
Figure 3. The Lorentz $\gamma$ factor of the expanding fireshell is plotted as a function of its external radius for 7 different values of the fireshell baryon loading $B$, ranging from $B = 10^{-8}$ and $B = 10^{-2}$, and two selected limiting values of the total energy $E_{\text{tot}}^\pm$ of the $e^\pm$ plasma: $E_{\text{tot}}^\pm = 1.17 \times 10^{49}$ ergs (upper panel) and $E_{\text{tot}}^\pm = 1.47 \times 10^{53}$ ergs (lower panel). The asymptotic values $\gamma \to 1/B$ are also plotted (dashed horizontal lines). The lines are plotted up to when the fireshell transparency is reached.
Figure 4. At the fireshell transparency point, for 4 different values of $E_{\text{tot}}^e$, we plot as a function of $B$: (Above) The fireshell temperature in the co-moving frame $T_{\text{com}}^e$ (thicker lines) and the one Doppler blue-shifted along the line of sight toward the observer in the source cosmological rest frame $T_{\text{obs}}^e$ (thinner lines); (Middle) The fireshell Lorentz gamma factor $\gamma^e$ together with the asymptotic value $\gamma^e = 1/B$; (Below) The energy radiated in the P-GRB (thinner lines, rising when $B$ decreases) and the one converted into baryonic kinetic energy and later emitted in the extended afterglow (thicker lines, rising when $B$ increases), in units of $E_{\text{tot}}^e$. 

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Here the energies emitted in the P-GRB (solid line) and in the extended afterglow (dashed line), in units of the total energy of the plasma, are plotted as functions of the $B$ parameter for a typical value of $E_{\text{tot}} \approx 10^{53}$ erg (see lower panel of Fig. 4). When $B \lesssim 10^{-5}$, the P-GRB becomes predominant over the extended afterglow, giving rise to a “genuine” short GRB. In the figure are also marked the values of the $B$ parameters corresponding to some GRBs we analyzed, all belonging to the class of long GRBs.

The optically thin phase

The dynamics of the optically thin fireshell of baryonic matter propagating in the CBM can be obtained from the relativistic conservation laws of energy and momentum [see e.g. Ref. 42]:

\[
\begin{align*}
    dE_{\text{int}} &= (\gamma - 1) dM_{\text{cbm}} c^2 \\
    d\gamma &= -\gamma^2 - 1 dM_{\text{cbm}} \\
    dM &= \frac{1 - \varepsilon}{\gamma} dE_{\text{int}} + dM_{\text{cbm}} \\
    dM_{\text{cbm}} &= 4\pi m_p n_{\text{cbm}}^2 r^2 dr
\end{align*}
\]

(7)

where $\gamma$, $E_{\text{int}}$, and $M$ are the pulse Lorentz gamma factor, internal energy and mass-energy respectively, $n_{\text{cbm}}$ is the CBM number density, $m_p$ is the proton mass, $\varepsilon$ is the emitted fraction of the energy developed in the collision with the CBM and $M_{\text{cbm}}$ is the amount of CBM mass swept up within the radius $r$: $M_{\text{cbm}} = m_p n_{\text{cbm}} (4\pi/3)(r^3 - r_0^3)$, where $r_0$ is the starting radius of the shock front.

In both our approach and in the other ones in the current literature [see e.g. Refs. 43, 44, 42, 45, 3] such conservations laws are used. The main difference is that in the current literature an ultra-relativistic approximation, following the Blandford and McKee [46] self-similar solution, is widely adopted, leading to a simple constant-index power-law relations between the Lorentz $\gamma$ factor of the optically thin “fireshell” and its radius:

\[
\gamma \propto r^{-a}
\]

(8)
with \(a = 3\) in the fully radiative case and \(a = 3/2\) in the adiabatic case [43, 45]. On the contrary, we use the exact solutions of the equations of motion of the fireshell [47, 42, 45, 48, 3]. In the adiabatic regime (\(\varepsilon = 0\)) we get:

\[
\gamma^2 = \frac{\gamma^2 + 2\gamma \left( M_{\text{cbm}} / M_b \right) + \left( M_{\text{cbm}} / M_b \right)^2}{1 + 2\gamma \left( M_{\text{cbm}} / M_b \right) + \left( M_{\text{cbm}} / M_b \right)^2},
\]

(9)

where \(\gamma\) and \(M_b\) are respectively the values of the Lorentz gamma factor and of the mass of the accelerated baryons at the beginning of the afterglow phase. In the fully radiative regime (\(\varepsilon = 1\)), instead, we have:

\[
\gamma = \frac{1 + \left( M_{\text{cbm}} / M_b \right) \left( 1 + \gamma^{-1} \right) \left( 1 + (1/2) \left( M_{\text{cbm}} / M_b \right) \right)}{\gamma^{-1} + \left( M_{\text{cbm}} / M_b \right) \left( 1 + \gamma^{-1} \right) \left( 1 + (1/2) \left( M_{\text{cbm}} / M_b \right) \right)}.
\]

(10)

A detailed comparison between the equations used in the two approaches has been presented by Bianco and Ruffini [47, 42, 45, 48]. In particular, Bianco and Ruffini [45] show that the regime represented in Eq.(8) is reached only asymptotically when \(\gamma \gg 1\) in the fully radiative regime and \(\gamma \gg 1\) in the adiabatic regime, where \(\gamma\) the initial Lorentz gamma factor of the optically thin fireshell.

In Fig. 6 we show the differences between the two approaches. In the upper panel there are plotted the exact solutions for the fireshell dynamics in the fully radiative and adiabatic cases. In the lower panel we plot the corresponding “effective” power-law index \(a_{\text{eff}}\), defined as the index of the power-law tangent to the exact solution [45]:

\[
a_{\text{eff}} = -\frac{d \ln \gamma}{d \ln r}
\]

(11)

Such an “effective” power-law index of the exact solution smoothly varies from 0 to a maximum value which is always smaller than 3 or 3/2, in the fully radiative and adiabatic cases respectively, and finally decreases back to 0 (see Fig. 6).

**Extended afterglow luminosity and spectrum**

The extended afterglow luminosity in the different energy bands is governed by two quantities associated to the environment. Within the fireshell model, these are the effective CBM density profile, \(n_{\text{cbm}}\), and the ratio between the effective emitting area \(A_{\text{eff}}\) and the total area \(A_{\text{tot}}\) of the expanding baryonic and leptonic shell, \(\delta = A_{\text{eff}} / A_{\text{tot}}\). This last parameter takes into account the CBM filamentary structure [49, 50] and the possible occurrence of a fragmentation in the shell [51].

Within the “fireshell” model, in addition to the determination of the baryon loading, it is therefore possible to infer a detailed description of the CBM, its average density and its porosity and filamentary structure, all the way from the black hole horizon to distance \(r \lesssim 10^{17}\) cm. This corresponds to the prompt emission. This description is lacking in the traditional model based on the synchrotron emission. The attempt to use the internal shock model for the prompt emission [see e.g. Refs. 52, 27, 28, and references therein] only applies to regions where \(r > 10^{17}\) cm [53].

In our hypothesis, the emission from the baryonic and leptonic matter shell is spherically symmetric. This allows us to assume, in a first approximation, a modeling of thin spherical shells for the CBM distribution and consequently to consider just its radial dependence [25]. For simplicity, and in order to have an estimate of the energetics, the emission process is postulated to be thermal in the co-moving frame of the shell [49]. We are currently examining a departure from this basic mechanism by taking into account inverse Compton effects due to the electron collisions with the thermal photons. The observed GRB non-thermal spectral shape is due to the convolution of an infinite number of thermal spectra with different temperatures and different Lorentz and Doppler factors. Such a convolution is to be performed over the surfaces of constant arrival time of the photons at the detector [EQUiTemporal Surfaces, EQTSs; see e.g. Ref. 42] encompassing the whole observation time [54].

In Fig. 7 we plot, as a function of \(B\), the arrival time separation \(\Delta t_a\) between the P-GRB and the peak of the extended afterglow measured in the cosmological rest frame of the source. Such a time separation \(\Delta t_a\) is the “quiescent time” between the precursor (i.e. the P-GRB) and the main GRB event (i.e. the peak of the extended afterglow). The plot is drawn for the same four different values of \(E^{90}_{\text{iso}}\) of Fig. 4. The arrival time of the peak of the extended afterglow emission depends on the detailed profile of the CBM density. In this plot it has been assumed a constant CBM density \(n_{\text{cbm}} = 1.0\) particles/cm\(^3\). We can see that, for \(3.0 \times 10^{-4} \lesssim B < 10^{-2}\), which is the condition for P-GRBs to be “precursors” (see above), \(\Delta t_a\) increases both with \(B\) and with \(E^{90}_{\text{iso}}\). We can have \(\Delta t_a > 10^{2}\) s and, in some extreme cases

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Figure 6. In the upper panel, the analytic behavior of the Lorentz $\gamma$ factor during the afterglow era is plotted versus the radial coordinate of the expanding optically thin fireshell in the fully radiative case (solid line) and in the adiabatic case (dotted line) starting from $\gamma = 10^2$ and the same initial conditions as GRB 991216 [45]. In the lower panel are plotted the corresponding values of the “effective” power-law index $a_{\text{eff}}$ (see Eq. (11)), which is clearly not constant, is highly varying and systematically lower than the constant values 3 and $3/2$ purported in the current literature (horizontal thin dotted lines).

even $\Delta t_a \sim 10^3$ s. For $B \lesssim 3.0 \times 10^{-4}$, instead, $\Delta t_a$ presents a behavior which qualitatively follows the opposite of $\gamma$, (see middle panel of Fig. 4).

Finally, in Fig. 8 we present three theoretical extended afterglow bolometric light curves together with the corresponding P-GRB peak luminosities for three different values of $B$. The duration of the P-GRBs has been assumed to be the same in the three cases (i.e. 5 s). The computations have been performed assuming the same $E_{\text{tot}}$ and the same detailed CBM density profile of GRB 991216 [1]. In this picture we clearly see how, for $B$ decreasing, the extended afterglow light curve “squeezes” itself on the P-GRB and the P-GRB peak luminosity increases.

The “prompt emission” light curves of many GRBs present a small pulse preceding the main GRB event, with a lower peak flux and separated by this last one by a quiescent time. The nature of such GRB “precursors” is one of the most debated issues in the current literature [see e.g. Refs. 55, 32]. Already in 2001 [5], within the “fireshell” model, we proposed that GRB “precursors” are the P-GRBs emitted when the fireshell becomes transparent, and we gave precise estimates of the time sequence and intensities of the P-GRB and of the extended afterglow, recalled above.

The radiation viewed in the co-moving frame of the accelerated baryonic matter is assumed to have a thermal spectrum and to be produced by the interaction of the CBM with the front of the expanding baryonic shell [49]. In Bernardini et al. [54] it is shown that, although the instantaneous spectrum in the co-moving frame of the optically thin
For 4 different values of $E_{\text{tot}}^e$, we plot as a function of $B$ the arrival time separation $\Delta t_a$ between the P-GRB and the peak of the extended afterglow (i.e. the "quiescent time between the "precursor" and the main GRB event), measured in the source cosmological rest frame. This computation has been performed assuming a constant CBM density $n_{\text{cbm}} = 1.0$ particles/cm$^3$. The points represents the actually numerically computed values, connected by straight line segments.

The time-integrated observed GRB spectra show a clear power-law behavior. Within a different framework [see e.g. Ref. 60, and references therein] it has been argued that it is possible to obtain such power-law spectra from a convolution of many non power-law instantaneous spectra monotonically evolving in time. This result was recalled and applied to GRBs [61] assuming for the instantaneous spectra a thermal shape with a temperature changing with time. It was shown that the integration of such energy distributions over the observation time gives a typical power-law shape possibly consistent with GRB spectra.

Our specific quantitative model is more complicated than the one considered by Blinnikov et al. [61]: the instantaneous spectrum here is not a black body. Each instantaneous spectrum is obtained by an integration over the corresponding EQTS: [47, 42] it is itself a convolution, weighted by appropriate Lorentz and Doppler factors, of $\sim 10^6$ thermal spectra with variable temperature. Therefore, the time-integrated spectra are not plain convolutions of thermal spectra: they are convolutions of convolutions of thermal spectra [49, 54]. In Fig. 9 we present the photon number spectrum $N(E)$ time-integrated over the 20 s of the whole duration of the prompt event of GRB 031203 observed by INTEGRAL [62]: in this way we obtain a typical non-thermal power-law spectrum which results to be in good agreement with the INTEGRAL data [62, 54] and gives a clear evidence of the possibility that the observed GRBs spectra are originated from a thermal emission [54].
Before closing, we like to mention that, using the diagrams represented in Figs. 4-7, in principle one can compute the two free parameters of the fireshell model, namely $E_{\text{tot}}^e \pm$ and $B$, from the ratio between the total energies of the P-GRB and of the extended afterglow and from the temporal separation between the peaks of the corresponding bolometric light curves. None of these quantities depends on the cosmological model. Therefore, one can in principle use this method to compute the GRBs’ intrinsic luminosity and make GRBs the best cosmological distance indicators available today. The increase of the number of observed sources, as well as the more accurate knowledge of their CBM density profiles, will possibly make viable this procedure to test cosmological parameters, in addition to the Amati relation [18, 36].

**GRB 060607A: A COMPLETE ANALYSIS OF THE PROMPT EMISSION AND X-RAY FLARES.**

GRB 060607A is a very distant [$z = 3.082$, see Ref. 63] and energetic event [$E_{\text{iso}} \sim 10^{53}$ erg, see Ref. 64]. Its BAT light curve shows a double-peaked structure with a duration of $T_{90} = (100 \pm 5)$ s [65]. The time-integrated spectrum over the $T_{90}$ is best fit with a simple power-law model with an index $\Gamma = 1.45 \pm 0.08$ (Guidorzi, private communication). The XRT light curve shows a prominent flaring activity (at least three flares) superimposed to the normal afterglow decay [66].

The GRB 060607A main peculiarity is that the peak of the near-infrared (NIR) afterglow has been observed with the REM robotic telescope [64]. Interpreting the NIR light curve as corresponding to the afterglow onset as predicted by the fireball forward shock model [67, 28], it is possible to infer the initial Lorentz gamma factor of the emitting system that results to be $\Gamma_0 \sim 400$ [64, 68, 69]. Moreover, these measurements seem to be consistent with an interstellar medium environment, ruling out the wind-like medium [64, 69].

We analyze GRB 060607A within the fireshell model [4, 5, 3]. We show that within this interpretation the N(E) spectrum of the prompt emission can be fitted in a satisfactory way by a convolution of thermal spectra as predicted.
Figure 9. Three theoretically predicted time-integrated photon number spectra $N(E)$, computed for GRB 031203 [54], are here represented for $0 \leq t_d \leq 5$ s, $5 \leq t_d \leq 10$ s and $10 \leq t_d \leq 20$ s (dashed and dotted curves), where $t_d$ is the photon arrival time at the detector [4, 54]. The hard to soft behavior is confirmed. Moreover, the theoretically predicted time-integrated photon number spectrum $N(E)$ corresponding to the first 20 s of the “prompt emission” (black bold curve) is compared with the data observed by INTEGRAL [62]. This curve is obtained as a convolution of 108 instantaneous spectra, which are enough to get a good agreement with the observed data. Details in Bernardini et al. [54].

by the model we applied [49, 50, 54]. The theoretical spectrum and light curve in the BAT energy band obtained are in good agreement with the observations, enforcing the plausibility of our approach. Moreover, in analogy with the case of GRB 011121 [70], we propose an interpretation of the observed X-ray flares as produced by overdense CBM clouds, in analogy with the gamma-ray light curve.

In this preliminary analysis we deal only with the BAT and XRT observations, which are the basic contribution to the afterglow emission according to the fireshell model. We do not deal with the infrared emission that, on the contrary, is used in the current literature to estimate the dynamical quantities of the fireball in the forward external shock regime. Nevertheless, the initial value of Lorentz gamma factor we predict is compatible with the one deduced from the REM observations even under very different assumptions.

**GRB 060607A prompt emission**

*Light curves*

In Fig. 10 we present the theoretical fit of Swift BAT light curves in different energy bands (15–25 keV, 25–50 keV, 50–100 keV, 100–150 keV) of GRB 060607A. We identify the whole prompt emission with the peak of the extended
Figure 10. *Swift* BAT (15–25 keV, 25–50 keV, 50–100 keV, 100–150 keV) light curves (points) compared with the theoretical ones (solid lines).

afterglow emission, and the remaining part of the light curve with the decaying tail of the extended afterglow, according to our “canonical GRB” scenario [5, 3]. The temporal variability of the light curves has been reproduced assuming overdense spherical CBM regions [25]. The detailed structure of the CBM adopted is presented in Fig. 11.

We therefore obtain for the two parameters characterizing the source in our model $E_{i}^{\text{tot}} = 2.5 \times 10^{53}$ erg and $B = 3.0 \times 10^{-3}$. This implies an initial $e^\pm$ plasma with a total number of $e^\pm$ pairs $N_{e^\pm} = 2.6 \times 10^{58}$ and an initial temperature $T = 1.7$ MeV. The theoretically estimated total isotropic energy emitted in the P-GRB is $E_{i}^{\text{P-GRB}} = 1.9\% E_{i}^{\text{tot}} = 4.7 \times 10^{51}$ erg, hence the P-GRB results to be undetectable if we assume a duration $\Delta T_{p-\text{grb}} \gtrsim 10$ s.

After the transparency point at $r_{0} = 1.4 \times 10^{14}$ cm from the progenitor, the initial Lorentz gamma factor of the fireshell is $\gamma_{i} = 328$. This value has been obtained adopting the exact solutions of the equations of motions of the fireshell [45] and using as initial conditions the two free parameters ($E_{i}^{\text{tot}}$ and $B$) estimated from the simultaneous analysis of the BAT and XRT light curves.

**Time-integrated spectra**

We turn now to the analysis of the GRB 060607A prompt emission time-integrated spectrum. As discussed in previous works [49, 50, 54], even if the fireshell model assumes that the GRB spectrum is thermal in the comoving frame, the shape of the final spectrum in the laboratory frame is clearly non-thermal. In fact each single instantaneous spectrum is the result of a convolution of thermal spectra. In fact photons observed at the same arrival time are emitted at different comoving time [the so called EQTS, see Refs. 47, 42], hence with different temperatures. This calculation produces a non-thermal instantaneous spectrum in the observer frame. This effect is enhanced if we calculate the time-integrated spectrum: we perform two different integrations, one on the observation time and one on the EQTS, and what we get is a typical non-thermal power-law spectrum which results to be in good agreement with the observations (see Fig. 12).
**Attachments**

![Graph](image)

**Figure 11.** Detailed structure of the CBM adopted: particle number density $n_{cbm}$ (upper panel) and fraction effective emitting area $R$ (lower panel) versus distance from the progenitor. The two X-ray flares corresponds in the upper panel to the huge increases in the CBM density that departs from the roughly power-law decrease observed. In the lower panel, the X-ray flares produce an increase of the emitting area which is not real but due to the lack of a complete 3-dimensional treatment of the interaction between the fireshell and the CBM (see text).

**The X-ray flares.**

We analyze now the X-ray flares observed by *Swift* XRT (0.2 – 10 keV) in the early part of the decaying phase of the extended afterglow. According to the fireshell model these flares have the same origin of the prompt emission, namely they are produced by the interaction of the fireshell with overdense CBM. As we can see in the upper panel in Fig. 13, the result obtained is compatible with the observations only for the second flare but not for the first one since its duration is longer. This discrepancy is due to the simple modeling adopted, namely the CBM is arranged in spherical shells [25]. This approximention fails when the visible area of the fireshell is comparable with the size of the
Figure 12. Theoretically predicted time-integrated photon number spectrum $N(E)$ corresponding to the $0 - 15$ s (upper panel), $15 - 50$ s (middle panel), and to the whole duration ($T_{90} = 100$ s, lower panel) of the prompt emission (solid lines) compared with the observed spectra integrated in the same intervals.
Figure 13. Swift XRT (0.2–10 keV) light curve compared with, respectively, the theoretical one obtained assuming the CBM distributed in spherical shells (upper panel), the theoretical one obtained imposing a finite transverse dimension for the CBM cloud (middle panel) and the same theoretical curve in logarithmic scale (lower panel).
CBM clouds.

To solve this problem, following the results obtained for GRGB 011121 [70], we tried to account for the three-dimensional structure of the CBM clouds by “cutting” the emission at a certain angle \( \theta_{\text{cloud}} \) from the line of sight, corresponding to the transverse dimension of the CBM cloud, until the duration of the flare \( \delta t / t_{\text{tot}} \) is compatible with the observation (see Fig. 13 middle and lower panels). It is worth to observe that with this procedure we keep the value of \( R \) constant during the flare. Hence the increase in \( R \) that we obtained in our previous analysis (see Fig. 11) is not real but it compensates the fact that spherical approximation is not valid at this stage.

This procedure affects the dynamics of the fireshell, so the light curve after the “cut” is meaningless. Nevertheless, it is a confirmation that it is possible to obtain arbitrarily short flares by the interaction with the CBM.

**Conclusions**

We presented the analysis of GRB 060607A within the fireshell model [4, 5, 3]. According to the “canonical GRB” scenario [5, 3] we interpreted the whole prompt emission as the peak of the extended afterglow emission, and the remaining part of the light curve with the decaying tail of the extended afterglow. We found in this second case that the P-GRB is too faint to be detected, as we expected from our interpretation. The theoretical light curves obtained are well in agreement with the observations in all the Swift BAT energy bands.

Furthermore, the initial Lorentz gamma factor of the fireshell, obtained adopting the exact solutions of its equations of motions [45] and as initial condition the free parameters of the fireshell estimated by the simultaneous analysis of the BAT and XRT light curves, is \( \gamma_i = 328 \). In this preliminary analysis we deal only with the BAT and XRT observations, which are the basic contribution to the afterglow emission according to the fireshell model. We do not deal with the infrared emission that, on the contrary, is used in the current literature to estimate the dynamical quantities of the fireball in the forward external shock regime. Nevertheless, the initial value of Lorentz gamma factor we predict is compatible with the one deduced from the REM observations even under very different assumptions [64, 68, 69].

We investigated also the GRB 060607A prompt emission spectra integrated in different time intervals assuming a thermal spectrum in the comoving frame. The results obtained show clearly that, after the correct space-time transformations, both the instantaneous and the time-integrated spectra in the observer frame have nothing to do with a Planckian distribution, but they have a power-law shape, thus confirming our previous analyses [54, 71].

Finally we analyzed the X-ray flares observed by Swift XRT (0.2 – 10 keV) in the early part of the decaying phase of the extended afterglow. According to the fireshell model these flares have the same origin of the prompt emission, namely they are produced by the interaction of the fireshell with overdense CBM. We found that our theoretical light curve is not compatible with the observations since in such regime the one-dimensional approximation fails. Following the results obtained for GRB 011121 [70], we tried to account for the three-dimensional structure of the CBM clouds by “cutting” the emission at a certain angle \( \theta_{\text{cloud}} \) from the line of sight, corresponding to the transverse dimension of the CBM cloud. We obtain in this way a flare whose duration \( \delta t / t_{\text{tot}} \) is compatible with the observation.

**THE NORRIS & BONNEL KIND OF SOURCES: THE NEW CLASS OF “FAKE-DISGUISED” SHORT GRBS**

We now present the theoretical understanding, within the fireshell model, of a new class of sources, pioneered by Norris and Bonnell [7]. This class is characterized by an occasional softer extended emission after an initial spikelike emission. The softer extended emission has a peak luminosity smaller than the one of the initial spikelike emission. This has misled the understanding of the correct role of the extended afterglow. As shown in the prototypical case of GRB 970228 [see below and Ref. 33], the initial spikelike emission can be identified with the P-GRB and the softer extended emission with the peak of the extended afterglow. Crucial is the fact that the time-integrated extended afterglow luminosity is much larger than the P-GRB one, and this fact unquestionably identifies GRB 970228 as a canonical GRB with \( B > 10^{-4} \). The consistent application of the fireshell model allowed to compute the CBM porosity, filamentary structure and average density which, in that specific case, resulted to be \( n_{\text{cm}} \sim 10^{-7} \) particles/cm\(^3\) [33]. This explained the peculiarity of the low extended afterglow peak luminosity and of its much longer time evolution. These features are not intrinsic to the progenitor nor to the black hole, but they uniquely depend on the peculiarly low value of the CBM density, typical of galactic halos. If one takes the same total energy, baryon loading and CBM
distribution as in GRB 970228, and rescales the CBM density profile by a constant numerical factor in order to raise its average value from $10^{-3}$ to 1 particles/cm$^3$, he obtains a GRB with a much larger extended afterglow peak luminosity and a much reduced time scale. Such a GRB would appear a perfect traditional “long” GRB following the current literature [see below and Ref. 33]. This has led us to expand the traditional classification of GRBs to three classes: “genuine” short GRBs, “fake” or “disguised” short GRBs, and all the remaining “canonical” ones [see Fig. 14 and Ref. 72].

**GRB 970228 and a class of GRBs with an initial spikelike emission**

GRB 970228 was detected by the Gamma-Ray Burst Monitor (GRBM, 40–700 keV) and Wide Field Cameras (WFC, 2–26 keV) on board BeppoSAX on February 28, 123620 UT [73]. The burst prompt emission is characterized by an initial 5 s strong pulse followed, after 30 s, by a set of three additional pulses of decreasing intensity [73]. Eight hours after the initial detection, the NFIs on board BeppoSAX were pointed at the burst location for a first target of opportunity observation and a new X-ray source was detected in the GRB error box: this is the first “afterglow” ever detected [74]. A fading optical transient has been identified in a position consistent with the X-ray transient [75], coincident with a faint galaxy with redshift $z = 0.695$ [76]. Further observations by the Hubble Space Telescope clearly showed that the optical counterpart was located in the outskirts of a late-type galaxy with an irregular morphology [77].

The BeppoSAX observations of GRB 970228 prompt emission revealed a discontinuity in the spectral index between the end of the first pulse and the beginning of the three additional ones [74, 73, 57]. The spectrum during the first 3 s of the second pulse is significantly harder than during the last part of the first pulse [73, 57], while the spectrum of the last three pulses appear to be consistent with the late X-ray afterglow [73, 57]. This was soon recognized by Frontera et al. [73, 57] as pointing to an emission mechanism producing the X-ray afterglow already taking place after the first pulse.

The simultaneous occurrence of an extended afterglow with total time-integrated luminosity larger than the P-GRB one, but with a smaller peak luminosity, is indeed explainable in terms of a peculiarly small average value of the CBM density and not due to the intrinsic nature of the source. In this sense, GRBs belonging to this class are only “fake” or “disguised” short GRBs. We show that GRB 970228 is a very clear example of this situation. We identify the initial spikelike emission with the P-GRB, and the late soft bump with the peak of the extended afterglow. GRB 970228 shares the same morphology and observational features with the sources analyzed by Norris and Bonnell [7] as well as with e.g. GRB 050709 [78], GRB 050724 [79] and GRB 060614 [see next section and Ref. 80]. Therefore, we propose GRB 970228 as a prototype for this new GRB class.
Figure 15. The “canonical GRB” light curve theoretically computed for the prompt emission of GRB 970228. BeppoSAX GRBM (40–700 keV, above) and WFC (2–26 keV, below) light curves (data points) are compared with the extended afterglow peak theoretical ones (solid lines). The onset of the extended afterglow coincides with the end of the P-GRB (represented qualitatively by the dotted lines). For this source we have \( B \simeq 5.0 \times 10^{-3} \) and \( \langle n_{\text{cmb}} \rangle \sim 10^{-3} \) particles/cm\(^3\). Details in Bernardini et al. [33, 81].

The analysis of GRB 970228 prompt emission

In Fig. 15 we present the theoretical fit of BeppoSAX GRBM (40–700 keV) and WFC (2–26 keV) light curves of GRB 970228 prompt emission [75]. Within our “canonical GRB” scenario we identify the first main pulse with the P-
GRB and the three additional pulses with the extended afterglow peak emission, consistently with the above mentioned observations by Costa et al. [74] and Frontera et al. [73]. Such last three pulses have been reproduced assuming three overdense spherical CBM regions (see Fig. 16) with a very good agreement (see Fig. 15).

We therefore obtain for the two parameters characterizing the source in our model $E_{\text{tot}}^e = 1.45 \times 10^{54}$ erg and $B = 5.0 \times 10^{-3}$. This implies an initial $e^+$ plasma created between the radii $r_1 = 3.52 \times 10^7$ cm and $r_2 = 4.87 \times 10^8$ cm with a total number of $e^+$ pairs $N_{e^+} = 1.6 \times 10^{58}$ and an initial temperature $T = 1.7$ MeV. The theoretically estimated total isotropic energy emitted in the P-GRB is $E_{P-\text{GRB}} = 1.1\% E_{\text{tot}}^e = 1.54 \times 10^{52}$ erg, in excellent agreement with the one observed in the first main pulse ($E_{P-\text{GRB}} \sim 1.5 \times 10^{52}$ erg in 2 – 700 keV energy band, see Fig. 15), as expected due to their identification. After the transparency point at $r_0 = 4.37 \times 10^{13}$ cm from the progenitor, the initial Lorentz gamma factor of the fireshell is $\gamma_0 = 199$. On average, during the extended afterglow peak emission phase we have for the CBM $\langle R \rangle = 1.5 \times 10^{-7}$ and $\langle n_{\text{cbm}} \rangle = 9.5 \times 10^{-4}$ particles/cm$^3$. This very low average value for the CBM density is compatible with the observed occurrence of GRB 970228 in its host galaxy’s halo [77, 75, 82] and it is crucial in explaining the light curve behavior.

The values of $E_{\text{tot}}^e$ and $B$ we determined are univocally fixed by two tight constraints. The first one is the total energy emitted by the source all the way up to the latest extended afterglow phases (i.e. up to $\sim 10^6$ s). The second one is the ratio between the total time-integrated luminosity of the P-GRB and the corresponding one of the whole extended afterglow (i.e. up to $\sim 10^6$ s). In particular, in GRB 970228 such a ratio results to be $\sim 1.1\%$ (see Fig. 5). However, the P-GRB peak luminosity actually results to be much more intense than the extended afterglow one (see Fig. 15). This is due to the very low average value of the CBM density $\langle n_{\text{cbm}} \rangle = 9.5 \times 10^{-4}$ particles/cm$^3$, which produces a less intense extended afterglow emission. Since the extended afterglow total time-integrated luminosity is fixed, such a less intense emission lasts longer than what we would expect for an average density $\langle n_{\text{cbm}} \rangle \sim 1$ particles/cm$^3$.

**Rescaling the CBM density**

We present now an explicit example in order to probe the crucial role of the average CBM density in explaining the relative intensities of the P-GRB and of the extended afterglow peak in GRB 970228. We keep fixed the basic
The theoretical fit of the BeppoSAX GRBM observations (solid line, see Fig. 15) is compared with the extended afterglow light curve in the 40–700 keV energy band obtained rescaling the CBM density to $\langle n_{\text{cbm}} \rangle = 1$ particle/cm$^3$ keeping constant its shape and the values of the fundamental parameters of the theory $E_{\text{tot}}^{\text{e}}$ and $B$ (double dotted line). The P-GRB duration and luminosity (dotted line), depending only on $E_{\text{tot}}^{\text{e}}$ and $B$, are not affected by this process of rescaling the CBM density.

We notice a clear enhancement of the extended afterglow peak luminosity with respect to the P-GRB one in comparison with the fit of the observational data presented in Fig. 15. The two light curves actually crosses at $t_{\text{d}} \approx 1.8 \times 10^4$ s since their total time-integrated luminosities must be the same. The GRB “rescaled” to $\langle n_{\text{ism}} \rangle = 1$ particle/cm$^3$ appears to be totally similar to, e.g., GRB 050315 [71] and GRB 991216 [1, 49, 2].

It is appropriate to emphasize that, although the two underlying CBM density profiles differ by a constant numerical factor, the two extended afterglow light curves in Fig. 17 do not. This is because the absolute value of the CBM density at each point affects in a non-linear way all the following evolution of the fireshell due to the feedback on its dynamics [45]. Moreover, the shape of the surfaces of equal arrival time of the photons at the detector (EQTS) is strongly elongated along the line of sight [42]. Therefore photons coming from the same CBM density region are observed over a very long arrival time interval.

**GRB 970228 and the Amati relation**

We turn now to the “Amati relation” [13, 20] between the isotropic equivalent energy emitted in the prompt emission $E_{\text{iso}}$ and the peak energy of the corresponding time-integrated spectrum $E_{\text{p}, i}$ in the source rest frame. It has been shown by Amati et al. [13], Amati [20] that this correlation holds for almost all the “long” GRBs which have a redshift and an $E_{\text{p}, i}$ measured, but not for the ones classified as “short” [20]. If we focus on the “fake” or “disguised” short GRBs, namely the GRBs belonging to this new class, at least in one case [GRB 050724, see Ref. 79] it has been shown that the correlation is recovered if also the extended emission is considered [83].

It clearly follows from our treatment that for the “canonical GRBs” with large values of the baryon loading and high...
Figure 18. The estimated values for $E_{p,i}$ and $E_{iso}$ obtained by our analysis (black dot) compared with the “Amati relation” [13]: the solid line is the best fitting power law [20] and the dashed lines delimit the region corresponding to a vertical logarithmic deviation of 0.4 [20]. The uncertainty in the theoretical estimated value for $E_{p,i}$ has been assumed conservatively as 20%.

$\langle n_{cbm} \rangle$, which presumably are most of the GRBs for which the correlation holds, the leading contribution to the prompt emission is the extended afterglow peak emission. The case of the “fake” or “disguised” short GRBs is completely different: it is crucial to consider separately the two components since the P-GRB contribution to the prompt emission in this case is significant.

To test this scenario, we evaluated from our fit of GRB 970228 $E_{iso}$ and $E_{p,i}$ only for the extended afterglow peak emission component, i.e. from $t_d^P = 37$ s to $t_d^P = 81.6$ s. We found an isotropic energy emitted in the 2–400 keV energy band $E_{iso} = 1.5 \times 10^{52}$ erg, and $E_{p,i} = 90.3$ keV. As it is clearly shown in Fig. 18, the sole extended afterglow component of GRB 970228 prompt emission is in perfect agreement with the Amati relation. If this behavior is confirmed for other GRBs belonging to this new class, this will enforce our identification of the “fake” or “disguised” short GRBs. This result will also provide a theoretical explanation for the apparent absence of such correlation for the initial spikelike component in the different nature of the P-GRB.

Conclusions

We conclude that GRB 970228 is a “canonical GRB” with a large value of the baryon loading quite near to the maximum $B \sim 10^{-2}$ (see Fig. 5). The difference with e.g. GRB 050315 [71] or GRB 991216 [1, 49, 2] is the low average value of the CBM density $\langle n_{cbm} \rangle \sim 10^{-3}$ particles/cm$^3$ which deflates the extended afterglow peak luminosity. Hence, the predominance of the P-GRB, coincident with the initial spikelike emission, over the extended afterglow is just apparent: 98.9% of the total time-integrated luminosity is indeed in the extended afterglow component. Such
a low average CBM density is consistent with the occurrence of GRB 970228 in the galactic halo of its host galaxy [77, 75], where lower CBM densities have to be expected [82].

We propose GRB 970228 as the prototype for the new class of GRBs comprising GRB 060614 and the GRBs analyzed by Norris and Bonnell [7]. We naturally explain the hardness and the absence of spectral lag in the initial spikelike emission with the physics of the P-GRB originating from the gravitational collapse leading to the black hole formation. The hard-to-soft behavior in the extended afterglow is also naturally explained by the physics of the relativistic fireshell interacting with the CBM, clearly evidenced in GRB 031203 [54] and in GRB 050315 [71]. Also justified is the applicability of the Amati relation to the sole extended afterglow component [see Refs. 20, 83].

This class of GRBs with $z \sim 0.3$ appears to be nearer than the other GRBs detected by Swift [z $\sim$ 2.3, see Ref. 84]. This may be explained by the extended afterglow peak luminosity deflation. The absence of a jet break in those afterglows has been pointed out [79, 85], consistently with our spherically symmetric approach. Their association with non-star-forming host galaxies appears to be consistent with the merging of a compact object binary [86, 87]. It is here appropriate, however, to caution on this conclusion, since the association of GRB 060614 and GRB 970228 with the explosion of massive stars is not excluded [88, 89].

Most of the sources of this class appear indeed not to be related to bright “Hypermovac”, to be in the outskirts of their host galaxies [87, see above] and a consistent fraction of them are in galaxy clusters with CBM densities $(n_{CBM}) \sim 10^{-3}$ particles/cm$^3$ [see e.g. Ref. 90, 91]. This suggests a spiraling out binary nature of their progenitor systems [92] made of neutron stars and/or white dwarfs leading to a black hole formation.

Moreover, we verified the applicability of the Amati relation to the sole extended afterglow component in GRB 970228 prompt emission, in analogy with what happens for some of the GRBs belonging to this new class. In fact it has been shown by Amati [20, 83] that the “fake” or “disguised” short GRBs do not fulfill the $E_{iso} - E_{iso}$ correlation when the sole spikelike emission is considered, while they do if the long soft bump is included. Since the spikelike emission and the soft bump contributions are comparable, it is natural to expect that the soft bump alone will fulfill the correlation as well.

Within our “canonical GRB” scenario the sharp distinction between the P-GRB and the extended afterglow provide a natural explanation for the observational features of the two contributions. We naturally explain the hardness and the absence of spectral lag in the initial spikelike emission with the physics of the P-GRB originating from the gravitational collapse leading to the black hole formation. The hard-to-soft behavior in the extended afterglow is also naturally explained by the physics of the relativistic fireshell interacting with the CBM, clearly evidenced in GRB 031203 [54] and in GRB 050315 [71]. Therefore, we expect naturally that the $E_{iso} - E_{iso}$ correlation holds only for the extended afterglow component and not for the P-GRB. Actually we find that the correlation is recovered for the extended afterglow peak emission of GRB 970228.

In the original work by Amati et al. [13], Amati [20] only the prompt emission is considered and not the late afterglow one. In our theoretical approach the extended afterglow peak emission contributes to the prompt emission and continues up to the latest GRB emission. Hence, the meaningful procedure within our model to recover the Amati relation is to look at a correlation between the total isotropic energy and the peak of the time-integrated spectrum of the whole extended afterglow. A first attempt to obtain such a correlation has already been performed using GRB 050315 as a template, giving very satisfactory results (see section Theoretical background for GRBs’ empirical correlations).

**THE “FIRESHELL” MODEL AND GRB PROGENITORS**

“Long” GRBs are traditionally related in the current literature to the idea of a single progenitor, identified as a “collapsar” [93]. Similarly, short GRBs are assumed to originate from binary mergers formed by white dwarfs, neutron stars, and black holes in all possible combinations [see e.g. Refs. 94, 95, 96, 97, 27, 28, and references therein]. It has been also suggested that short and long GRBs originate from different galaxy types. In particular, short GRBs are proposed to be associated with galaxies with low specific star forming rate [see e.g. Ref. 98]. Some evidences against such a scenario have been however advanced, due to the small sample size and the different estimates of the star forming rates [see e.g. Ref. 99]. However, the understanding of GRB structure and of its relation to the CBM distribution, within the fireshell model, leads to a more complex and interesting perspective than the one in the current literature.

The first general conclusion of the “fireshell” model [5] is that, while the time scale of “short” GRBs is indeed intrinsic to the source, this does not happen for the “long” GRBs: their time scale is clearly only a function of the instrumental noise threshold. This has been dramatically confirmed by the observations of the Swift satellite [see Fig. 19 and Ref. 101]. Among the traditional classification of “long” GRBs we distinguish two different sub-classes of
events, none of which originates from collapsars.

The first sub-class contains “long” GRBs particularly weak ($E_{\text{iso}} \sim 10^{50}$ erg) and associated with Supernovae (SNe) Ib/c. In fact, it has been often proposed that such GRBs, only observed at smaller redshift $0.0085 < \text{z} < 0.168$, form a different class, less luminous and possibly much more numerous than the high luminosity GRBs at higher redshift [102, 103, 104, 105]. Therefore in the current literature they have been proposed to originate from a separate class of progenitors [106, 107]. Within our “fireshell” model, they originate in a binary system formed by a neutron star, close to its critical mass, and a companion star, evolved out of the main sequence. They produce GRBs associated with SNe Ib/c, via the “induced gravitational collapse” process [6]. The low luminosity of these sources is explained by the formation of a black hole with the smallest possible mass: the one formed by the collapse of a just overcritical neutron star [108, 51].

A second sub-class of “long” GRBs originates from merging binary systems, formed either by two neutron stars or a neutron star and a white dwarf. A prototypical example of such systems is GRB 970228. The binary nature of the source is inferred by its migration from its birth location in a star forming region to a low density region within the galactic halo, where the final merging occurs [33]. The location of such a merging event in the galactic halo is indeed confirmed by optical observations of the GRB 970228 afterglow [77, 75]. The crucial point is that, as recalled above, GRB 970228 is a “canonical” GRB with $B > 10^{-4}$ “disguised” as a short GRB. We are going to see in the following that GRB 060614 also comes from such a progenitor class.

If the binary merging would occur in a region close to its birth place, with an average density of 1 particle/cm$^3$, the GRB would appear as a traditional high-luminosity “long” GRB, of the kind currently observed at higher redshifts (see above, Fig. 17), similar to, e.g., GRB 050315 [71].

Figure 19. The theoretical light curves in the 15 – 150 keV (solid line) and 0.2 – 10 keV (dotted line) energy bands compared with XRT observations of GRB 050315 [100]. The horizontal dashed lines correspond to different possible instrumental thresholds. It is clear that long GRB durations are just functions of the observational threshold. Details in Ruffini et al. [101].
Within our approach, therefore, there is the distinct possibility that all GRB progenitors are formed by binary systems, composed by neutron stars, white dwarfs, or stars evolved out of the main sequence, in different combinations.

The case of the “genuine” short GRBs is currently being examined within the “fireshell” model.

**GRB 060614: a “fake” or “disguised” short GRB from a merging binary system**

GRB 060614 [80, 109] has imposed to the general attention of the Gamma-Ray Burst’s (GRB’s) scientific community because it is the first clear example of a nearby ($z = 0.125$), long GRB not associated with a bright Ib/c Supernova (SN) [88, 110]. It has been estimated that, if present, the SN-component should be about 200 times fainter than the archetypal SN 1998bw associated to GRB 980425; moreover, it would also be fainter (at least 30 times) than any stripped-envelope SN ever observed [111].

Within the standard scenario, long duration GRBs ($T_{90} > 2$ s) are thought to be produced by SN events during the collapse of massive stars in star forming regions [“collapsar”, see Ref. 93]. The observations of broad-lined and bright type Ib/c SNe associated with GRBs are often reported to favor this scenario [see Ref. 112, and references therein]. The ansatz has been advanced that every long GRB should have a SN associated with it [113]. Consequently, in all nearby long GRBs ($z \leq 1$) the SN emission should be observed.

For these reasons the case of GRB 060614 is indeed revolutionary. Some obvious hypothesis have been proposed and ruled out: the chance superposition with a galaxy at low redshift [110] and the strong dust obscuration and extinction [114]. Appeal has been made to the possible occurrence of an unusually low luminosity stripped-envelope core-collapse SN [88].

The second novelty of GRB 060614 is that it challenges the traditional separation between Long Soft GRBs and Short Hard GRBs. Traditionally [8, 9], the “short” GRBs have $T_{90} < 2$ s, present a harder spectrum and negligible spectral lag, and are assumed to originate from merging of two compact objects, i.e. two neutron stars or a neutron star and a black hole [see e.g. Ref. 94, 95, 96, 97, 27, 28, and references therein]. GRB 060614 lasts about one hundred seconds ($T_{90} = (102 \pm 5)$ s; see Ref. 80), it fulfills the $E_{iso} - E_{peak}$ correlation [115], and therefore it should be traditionally classified as a “long” GRB. However, its morphology is different from typical long GRBs, similar to the one of GRB 050724, traditionally classified as a short GRB [113, 116]. Its optical afterglow luminosity is intermediate between the traditional long and short ones [117]. Its host galaxy has a moderate specific star formation rate ($M_{*} \approx 2 \times 10^{9} L_{\odot}$) [88]; see Refs. 114, 88]. The spectral lag in its light curves is very small or absent [80]. All these features are typical of the short GRBs.

A third peculiarity of GRB 060614 is that its 15–150 keV light curve presents a short, hard and multi-peaked episode (about 5 s). Such an episode is followed by a softer, prolonged emission that manifests a strong hard to soft evolution in the first 400 s of data [109]. The total fluence in the 15–150 keV energy band is $F = (2.17 \pm 0.04) \times 10^{-5}$ erg/cm$^2$, the 20% emitted during the initial spikelike emission, where the peak luminosity reaches the value of 300 keV before decreasing until 8 keV during the BAT-XRT overlap time (about 80 s).

These apparent contradictions find a natural explanation in the framework of the “fireshell” model. Within the fireshell model, the occurrence of a GRB-SN is not a necessity. The origin of all GRBs is traced back to the formation of a black hole, either occurring in a single process of gravitational collapse, or in a binary system composed by a neutron star and a companion star evolved out of the main sequence, or in a merging binary system composed by neutron stars and/or white dwarfs in all possible combinations. The occurrence of a GRB-SN is indeed only one of the possibilities, linked, for example, to the process of “induced gravitational collapse” [6, 108, 51].

We here show how the “fireshell” model can explain all the above mentioned GRB 060614 peculiarities and solve the apparent contradictions. In doing so, we also infer constraints on the astrophysical nature of the GRB 060614 progenitors. In turn, these conclusions lead to a new scenario for all GRBs. We can confirm a classification of GRBs in “genuine” short, “fake” or “disguised” short, and, finally, all the remaining “canonical” GRBs. The connection between this new classification and the nature of GRB progenitors is quite different from the traditional one in the current literature.

*The fit of the observed luminosity*

In this scenario, GRB 060614 is naturally interpreted as a “disguised” short GRB. We have performed the analysis of the observed light curves in the 15–150 keV energy band, corresponding to the γ-ray emission observed by the BAT
instrument on the Swift satellite, and in the 0.2–10 keV energy band, corresponding to the X-ray component from the XRT instrument on Swift satellite. We do not address in this paper the issue of the optical emission, that represent less than 10% of the total energy of the GRB. From this fit (see Figs. 20, 22) we have derived the total initial energy $E_{\text{tot}}$^cbm, the value of $B$ as well as the effective CBM distribution (see Fig. 21). We find $E_{\text{tot}}$^cbm = $2.94 \times 10^{51}$ erg, that accounts for the bolometric emission of both the P-GRB and the extended afterglow. Such a value is compatible with the observed $E_{\text{bol}} \approx 2.5 \times 10^{51}$ erg [80]. The value of $B$ is $B = 2.8 \times 10^{-3}$, that corresponds to the lowest one of all the GRBs we have examined (see Fig. 5). It corresponds to a canonical GRB with a very clear extended afterglow predominance over the P-GRB. From the model, having determined $E_{\text{tot}}$^cbm and $B$, we can compute the theoretical expected P-GRB energetics $E_{\text{P-GRB}}$ [5]. We obtain $E_{\text{P-GRB}} \approx 1.15 \times 10^{50}$ erg, that is in good agreement with the observed one $E_{\text{iso},p} \approx 1.18 \times 10^{50}$ erg [80]. The Lorentz Gamma Factor at the transparency results to be $\gamma_0 = 346$, one of the highest of all the GRBs we have examined.

In Fig. 20 we plot the comparison between the BAT observational data of the GRB 0606014 prompt emission in the 15–150 keV energy range and the P-GRB and extended afterglow light curves computed within our model. The temporal variability of the extended afterglow peak emission is due to the inhomogeneities in the effective CBM density (see Figs. 20, 21). Toward the end of the BAT light curve, the good agreement between the observations and the fit is affected by the Lorentz gamma factor decrease and the corresponding increase of the maximum viewing angle. The source visible area becomes larger than the typical size of the filaments. This invalidates the radial approximation we use for the CBM description. To overcome this problem it is necessary to introduce a more detailed three-dimensional CBM description, in order to avoid an over-estimated area of emission and, correspondingly, to describe the sharpness of some observed light curves. We are still working on this issue [25, 118, 70, 119].

We turn now to the crucial determination of the CBM density, which is derived from the fit. At the transparency point it resulted to be $n_{\text{cbm}} = 4.8 \times 10^{-3}$ particles/cm$^3$ (see Fig. 21). This density is compatible with the typical values of the galactic halos. During the peak of the extended afterglow emission the effective average CBM density decreases reaching $\langle n_{\text{cbm}} \rangle = 2.25 \times 10^{-1}$ particles/cm$^3$, possibly due to an occurring fragmentation of the shell [51] or due to a fractal structure in the CBM. The $\langle n \rangle$ value resulted to be on average $\langle n \rangle = 1.72 \times 10^{-3}$. It is interesting to emphasize the striking analogy of the numerical value and the overall radial dependence of the CBM density in the present case of GRB 060614 when compared and contrasted with the ones of GRB 970228 [33].

Concerning the 0.2–10 keV light curve of the decaying phase of the afterglow, observed by the XRT instrument, we have also reproduced very satisfactorily both the hard decrease in the slope and the plateau of the light curve keeping constant the effective CBM density and changing only $\langle n \rangle$. The result of this analysis is reported in Fig. 22. We assume in this phase $n_{\text{cbm}} = 4.70 \times 10^{-6}$ particles/cm$^3$. The average value of the $\langle n \rangle$ parameter is $\langle n \rangle = 1.27 \times 10^{-2}$. The drastic enhancement in the $\langle n \rangle$ parameter with respect to the values at the peak of the extended afterglow is consistent with similar features encountered in other sources we have studied: GRB 060218 presents a bump of five orders of magnitude [51], in GRB 060710 the bump is of about four orders of magnitude (see Izzo et al., in preparation) while in GRB 050315 there is a three orders of magnitude bump [71]. In these last two cases, we find the occurrence of the enhancement of $\langle n \rangle$ between $r=2 \times 10^{17}$ cm and $r=3 \times 10^{17}$ cm, just like for GRB 060614, for which we have the bump at $r=3.5 \times 10^{17}$ cm. The time of the bump approximately corresponds to the appearance of the optical emission observed in GRB 060614 and, more in general, to the onset of the second component of the Willingale et al. [120] scheme for GRBs.

**Conclusions**

GRB 060614 presents three major novelties, which challenges the most widespread theoretical models and which are strongly debated in the current literature. The first one is that it challenges the traditional separation between Long Soft GRBs and Short Hard GRBs [80]. The second one is that it presents a short, hard and multi-peaked episode, followed by a softer, prolonged emission with a strong hard to soft evolution [80, 109]. The third one is that it is the first clear example of a nearby, long GRB not associated with a bright SN Ibc [88, 110]. All these three issues are naturally explained within our “fireshell” model, which allows a detailed analysis of the temporal behavior of the signal originating up to a distance $r \sim 10^{17}$–$10^{18}$ cm from the black hole, and relates, with all the relativistic transformations, the arrival time to the CBM structure and the relativistic parameters of the fireshell.

One of the major outcome of the Swift observation of, e.g., GRB 050315 [100, 71] has been the confirmation that long GRB duration is not intrinsic to the source but it is merely a function of the instrumental noise threshold [101]. GRB 060614 represents an additional fundamental progress in clarifying the role of the CBM density in determining the arrival time to the CBM structure and the relativistic parameters of the fireshell.
the GRB morphology. It confirms the results presented in GRB 970228 [33], that is the prototype of the new class of “fake” or “disguised” short GRBs. They correspond to canonical GRBs with an extended afterglow emission energetically predominant with respect to the P-GRB one and a baryon loading $B > 10^{-4}$. The sharp spiky emission corresponds to the P-GRB. As recalled above, a comparison of the luminosities of the P-GRB and of the extended afterglow is indeed misleading: it follows from the low average CBM density inferred from the fit of the fireshell model, which leads to $n_{\text{cbm}} \sim 10^{-3}$ particles/cm$^3$. Therefore such a feature is neither intrinsic to the progenitor nor to the black hole, but it is only indicative of the CBM density at the location where the final merging occurs. GRB 060614 is a canonical GRB and it is what would be traditionally called a “long” GRB if it had not exploded in a specially low CBM density environment. GRB 060614 must necessarily fulfill, and indeed it does, the Amati relation. This happens even taking into account the entire prompt emission mixing together the P-GRB and the extended afterglow [115], due to the above recalled energetic predominance of the extended afterglow [see also Ref. 36]. These results justify the occurrence of the above mentioned first two novelties.

The low value of the CBM density is compatible with a galactic halo environment. This result points to an old binary system as the progenitor of GRB 060614 and it justifies the above mentioned third novelty: the absence of an associated SN IIb/c [see also Ref. 121]. Such a binary system departed from its original location in a star forming region and spiraled out in a low density region of the galactic halo [see e.g. Ref. 92]. The energetic of this GRB is about two orders of magnitude smaller than the one of GRB 970228 [33]. A natural possible explanation is that instead of a neutron star - neutron star merging binary system we are in presence of a white dwarf - neutron star binary. We
Figure 21. Here are the plots of the effective CBM density (solid line) and of the $R$ parameter (dotted line) versus the radial coordinate of the shell. The CBM density peaks are labeled to match them with the corresponding extended afterglow light curve peaks in Fig. 20. They correspond to filaments of characteristic size $\Delta r \sim 10^{15}$ cm and density contrast $\Delta n_{cbm}/\langle n_{cbm} \rangle \sim 20$ particles/cm$^3$.

Therefore agree, for different reasons, with the identification proposed by Davies et al. [121] for the GRB 060614 progenitor. In principle, the nature of the white dwarf, with typical radius on the order of $10^3$ km, as opposed to the one of the neutron star, typically on the order of 10 km, may manifest itself in characteristic signatures in the structure of the P-GRB (see Fig. 20).

It is interesting that these results lead also to three major new possibilities:

- The majority of GRBs declared as shorts [see e.g. Ref. 116] are likely “disguised” short GRBs, in which the extended afterglow is below the instrumental threshold.

- The observations of GRB 060614 offer the opportunity, for the first time, to analyze in detail the structure of a P-GRB lasting 5 s. This feature is directly linked to the physics of the gravitational collapse originating the GRB. Recently, there has been a crucial theoretical physics result, showing that the characteristic time constant for the thermalization for an $e^\pm$ plasma is on the order of $10^{-13}$ s [122]. Such a time scale still applies for an $e^\pm$ plasma with a baryon loading on the order of the one observed in GRBs [40]. The shortness of such a time scale, as well as the knowledge of the dynamical equations of the optically thick phase preceding the P-GRB emission [31], implies that the structure of the P-GRB is a faithful representation of the gravitational collapse process leading to the formation of the black hole [123]. In this respect, it is indeed crucial that the Swift data on the P-GRB observed in GRB 060614 (see Fig. 20) appear to be highly structured all the way to time scale of 0.1 s. This opens a new field of research: the study of the P-GRB structure in relation to the process of gravitational collapse leading to the GRB.

- If indeed the binary nature of the progenitor system and the peculiarly low CBM density $n_{cbm} \sim 10^{-3}$ particles/cm$^3$ will be confirmed for all “fake” or “disguised” GRBs, then it is very likely that the traditionally “long” high luminosity GRBs at higher redshift also originates from the merging of binary systems formed by neutron stars and/or white dwarfs occurring close to their birth location in star forming regions with $n_{cbm} \sim 1$ particle/cm$^3$ (see Fig. 17).


Figure 22. The XRT 0.2–10 keV light curve (points) is compared with the corresponding theoretical extended afterglow light curve we compute (dotted line). Also in this case we have a good correspondence between data and theoretical results. For completeness, the solid line shows again the theoretical extended afterglow light curve in the 15–150 keV energy range presented in Fig. 20.

OPEN ISSUES IN CURRENT THEORETICAL MODELS

The “fireshell” model addresses mainly the $\gamma$ and X-ray emission, which are energetically the most relevant part of the GRB phenomenon. The model allows a detailed identification of the fundamental three parameters of the GRB source: the total energy, the baryon loading, as well as the CBM density, filamentary structure and porosity. The knowledge of these phenomena characterizes the region surrounding the black hole up to a distance which in this source reaches $\sim 10^{17}$–$10^{18}$ cm. When applied, however, to larger distances, which corresponds to the latest phases of the X-ray afterglow, since the beginning of the “plateau” phase, the model reveals a different regime which has not yet been fully interpreted in its astrophysical implications. To fit the light curve in the soft X-ray regime for $r \gtrsim 4 \times 10^{17}$ cm, we must appeal to an enhancement of about six orders of magnitude in the $\beta$ factor (see above, and Fig. 21). This would correspond to a more diffuse CBM structure, with a smaller porosity, interacting with the fireshell. This points to a different leading physical process during the latest X-ray afterglow phases. When we turn to the optical, IR and radio emission, the fireshell model leads to a much smaller flux than the observed one, especially for $r \sim 10^{17}$–$10^{18}$ cm. Although the optical, IR, and radio luminosities have a minority energetic role, they may lead to the identification of crucial parameters and new phenomena occurring in the source, and they deserve maximum attention.

In these latest phases for $r \gtrsim 10^{17}$ cm it is currently applied the treatment based on synchrotron emission pioneered by Meszaros and Rees [124] even before the discovery of the afterglow [74]. Such a model has been further developed [see Refs. 125, 27, 28, and references therein]. Also in this case, however, some difficulties remain since it is necessary to invoke the presence of an unidentified energy injection mechanism [126]. Such a model appears to be quite successful in explaining the late phases of the X-ray emission of GRB 060614, as well as the corresponding optical emission, in terms of different power-law indexes for the different parts of the afterglow light curves [109, 127]. However, also in this case an unidentified energy injection mechanism between $\sim 0.01$ days and $\sim 0.26$ days appears to be necessary [127].

The attempt to describe the prompt emission via the synchrotron process by the internal shock scenario [see e.g. Refs. 52, 27, 28, and references therein] also encounters difficulties: Kumar and McMahon [53] have shown that the traditional synchrotron model can be applied to the prompt emission only if it occurs at $r > 10^{17}$ cm. A proposed
way-out of this problem, via the inverse Compton process, suffers of an "energy crisis" [see e.g. Ref. 128]. Interestingly, the declared region of validity of the traditional synchrotron model ($r > 10^{17}$ cm) is complementary to the one successfully described by our model ($r < 10^{17}$–$10^{18}$ cm). Astrophysically, Xu et al. [127] have reached, within the framework of the traditional synchrotron model, two conclusions which are consistent with the results of our analysis of GRB 060614. First, they also infer from their numerical fit a very low density environment, namely $n_{\text{cmb}} \sim 0.04$ particles/cm$^3$. Second, they also mention the possibility that the progenitor of GRB 0606014 is a merging binary system formed by two compact objects.

THE SEARCH FOR A "GENUINE SHORT" GRB: THE CASE OF GRB 050509B

As we already discussed above, within the fireshell model the baryon loading is the key parameter to classify GRBs: if $B \lesssim 10^{-5}$ we have what we call "genuine" short GRBs. In order to investigate if this is indeed the case of GRB 050509B [129] we performed two different analyses, respectively with $B = 3.7 \times 10^{-3}$ and with $B = 1.1 \times 10^{-4}$ (see Fig. 23).

Analysis 1. We identify the prompt emission of this GRB [see Ref. 129] with our P-GRB. Consequently, the extended afterglow corresponds to the observed X-ray afterglow (see Fig. 24). In this case, we have the total energy of the GRB estimated in $E_{\text{tot}} = 2.11 \times 10^{49}$ erg (which is a low energy for GRBs) and the baryon loading is $B = 3.7 \times 10^{-3}$. With this choice of the fireshell parameters, we obtain that the P-GRB energy is $E_{\text{P-GRB}} = 1.6 \times 10^{47}$ erg. More than 90% of the total energy is released in the extended afterglow, hence GRB 050509B cannot be classified as "genuine" short GRB. The reason for the non observability of the peak of this extended afterglow is that it results under the BAT threshold in the gamma-ray energy band, and before the beginning of the XRT observations (which sets around 100 s).

Analysis 2. We performed an alternative analysis interpreting GRB 050509B as a "genuine" short GRB. We fit the BAT observations as the peak of the extended afterglow (see Fig. 25). In this case the total energy is $E_{\text{tot}} = 5.07 \times 10^{49}$ erg, the baryon loading is $B = 1.1 \times 10^{-4}$, and this implies that the energy emitted on the P-GRB is almost 60% of the total one, $E_{\text{P-GRB}} = 3.30 \times 10^{49}$ erg. Differently with the previous case, it was not observable by BAT since its peak energy would be about 817 keV. According this second interpretation, GRB 050509B is a "genuine" short GRB.
Theoretical fit in the 0.3-10 keV band
Theory fit in the 15-350 keV band
Flux in the bolometric band
Flux in 0.3-10 keV band
Flux in 15-350 keV band
2.0e-14
XRT threshold
BAT threshold
PGRB

Figure 24. Analysis 1: the P-GRB corresponds to the BAT observations and the extended afterglow, that has a total energy that is much greater than the P-GRB one, to the XRT observations.

Figure 25. Analysis 2: both the BAT observations and the XRT ones are identified with the extended afterglow emission. The P-GRB flux is more than twice the extended afterglow one, but it is too hard to be observed by BAT.

GRB 050509B within the Amati relation. In order to discriminate between the above analyses we checked if their results are compatible with the Amati relation [13, 20]. According to the fireshell model, only the extended afterglow component should satisfy the Amati relation, while the P-GRB component should not. This accounts for the fact that the “short” GRBs are outliers of the correlation. Therefore, according to the first interpretation the P-GRB which coincides with the BAT observation should be out from the correlation: this is indeed true.

On the other hand, the second analysis reveals that the BAT observation should satisfy the correlation. Hence, this possibility have to be ruled out. We continue investigating the first possibility in order to obtain further information that from the astrophysical setting of this GRB constraint better the fit of the extended afterglow.

Apart from this result, the new generation of high energy satellites are very important for the observation and study of the P-GRB, and for the identification of “genuine” short GRBs.
Figure 26. The Amati relation with our predictions about GRB 050509B in the two analyses. In the first one, the P-GRB results out from the correlation, as it should be. In the second one, the peak of the extended afterglow emission should satisfy the correlation, but it does not happen. Hence, the first analysis turns to be more correct.

GRBS AND SNE: THE INDUCED GRAVITATIONAL COLLAPSE

The Collapsar model [93, 130, 131, 132] proposes that GRBs arise from the collapse of a single Wolf-Rayet star endowed with fast rotation. This idea is purported by the evidence that many GRBs are close to star-forming regions and that this suggests that GRBs are linked to cataclysmic deaths of massive stars ($M > 30 M_\odot$). In such a model very massive stars are able to fuse material in their centers all the way to iron, at which point a star cannot continue to generate energy by fusion and collapses, in this case, immediately forming a black hole. Matter from the star around the core rains down toward the center and (for rapidly rotating stars) swirls into a high-density accretion disk. The mass of the accretion disk is around $0.1 M_\odot$. The infall of this material into the black hole is assumed to drive a pair of jets (with opening angles < 10 degrees) out along the rotational axis, where the matter density is much lower than in the accretion disk, toward the poles of the star at velocities approaching the speed of light, creating a relativistic shock wave at the front [46]. The processes of core collapse and of accretion along the polar column and the jet propagation through the stellar envelope all together last $\sim 10$ sec [131]. The jet, as it passes through the star, is modulated by its interaction with the surrounding medium. In this way the Collapsar model attempts to explain the time structure of GRB prompt emission and to produce the variable Lorentz factor necessary for the internal shocks occurrence [112]. Moreover it is a prediction of this model that the central engine remains active for a long time after the principal burst is over, potentially contributing to the GRB afterglow [133].

Three very special conditions are required for a star to evolve all the way to a gamma-ray burst according to this theory: the star should be very massive ($25 M_\odot$ Woosley [93], $35 - 40 M_\odot$ on the main sequence Fryer et al. [134]) to form a central black hole, the star rapidly rotates to develop an accretion torus capable of launching jets, and the star should have low metallicity in order to strip off its hydrogen envelope so the jets can reach the surface. As a result, gamma-ray bursts are far rarer than ordinary core-collapse supernovae, which only require the star to be massive enough to fuse all the way to iron.

The consensus and the difficulties for the Collapsar model can be simply summarized:

- Langer [135] asserts that long gamma-ray bursts are found in systems with abundant recent star formation, low metallicity environment.
- The second evidence in favor of the Collapsar model is that there are several observed cases where a supernova is practical coeval with GRBs.
- However, strong evidence against the Collapsar model comes from the fact that there were recently discovered two nearby long gamma-ray bursts which lacked a signature of any type Ib/c supernova: both GRB 060614 [88, 110, see also above] and GRB 060505 [114] defied predictions that a supernova would emerge despite intense scrutiny.
Within our fireshell model, we recall, the approach is drastically different, as already introduced in Ruffini et al. [6]. In fact in this framework, the SN which is often observed in temporal and spatial coincidence with the GRB cannot be interpreted as its progenitor because of the high quantity of ejected matter from the supernova explosion would prevent the GRB occurrence. Moreover:

- It is very unlikely that a core collapse SN produces directly a black hole.
- GRBs originate from gravitational collapses to black holes (see above). The possible explanation for the GRB-SN connection proposed in Ruffini et al. [6] is that both the GRB and the supernova progenitors belong to a binary system. Under special conditions it is possible that the GRB emission triggers the supernova explosion of the companion star. Alternatively, it is possible that the process of gravitational collapse to a black hole producing the GRB is “induced” by the supernova Ib/c on a companion neutron star [see Fig. 27 and Refs. 108, 51]. The faintness of this GRB class could be in this case naturally explained by the formation of the smallest possible black hole, just over the critical mass of the neutron star [136]. Moreover these systems occur in a low density CBM ($10^{-2}–1$ particle/cm$^3$).
- Also the observation of the occurrence of “long” GRBs in star forming regions is explained by identifying the progenitor with a binary system formed by a neutron star and a star evolved out of the main sequence.
Application to GRB 060218

GRB 060218 triggered the BAT instrument of Swift on 18 February 2006 at 03:36:02 UT and has a $T_{90} = (2100 \pm 100)$ s [137]. The XRT instrument [138, 137] began observations $\sim 153$ s after the BAT trigger and continued for $\sim 12.3$ days [139]. The source is characterized by a flat $\gamma$-ray light curve and a soft spectrum [140]. It has an X-ray light curve with a long, slow rise and gradual decline and it is considered an X-Ray Flash (XRF) since its peak energy occurs at $E_{\gamma} = 4.9^{+0.4}_{-0.3}$ keV [141]. It has been observed by the Chandra satellite on February 26.78 and March 7.55 UT ($t \geq 8.8$ and 17.4 days) for 20 and 30 ks respectively [142]. The spectroscopic redshift has been found to be $z = 0.033$ [143, 144]. The corresponding isotropic equivalent energy is $E_{iso} = (1.9 \pm 0.1) \times 10^{50}$ erg [139] which sets this GRB as a low luminous one, consistent with most of the GRBs associated with SNe [106, 107, 145].

GRB 060218 is associated with SN2006aj whose expansion velocity is $v \sim 0.1c$ [102, 146, 147, 107]. The host galaxy of SN2006aj is a low luminosity, metal poor star forming dwarf galaxy [148] with an irregular morphology [149], similar to the ones of other GRBs associated with SNe [150, 143].

The fit of the observed data

In this section we present the fit of our fireshell model to the observed data (see Figs. 28, 31). The fit leads to a total energy of the $e^\pm$ plasma $E_{\text{tot}}^\text{cf} = 2.32 \times 10^{50}$ erg, with an initial temperature $T = 1.86$ MeV and a total number of pairs $N_{e^\pm} = 1.79 \times 10^{35}$. The second parameter of the theory, $B = 1.0 \times 10^{-2}$, is the highest value ever observed and is close to the limit for the stability of the adiabatic optically thick acceleration phase of the fireshell [for further details see Ref. 30]. The Lorentz gamma factor obtained solving the fireshell equations of motion [42, 45] is $\gamma \approx 99.2$ at the beginning of the extended afterglow phase at a distance from the progenitor $r_0 = 7.82 \times 10^{12}$ cm. It is much larger than $\gamma \sim 5$ estimated by Kaneko et al. [151] and Toma et al. [152].

In Fig. 28 we show the extended afterglow light curves fitting the prompt emission both in the BAT (15–150 keV) and in the XRT (0.3–10 keV) energy ranges, as expected in our “canonical GRB” scenario [51]. Initially the two luminosities are comparable to each other, but for a detector arrival time $t_\text{d} > 1000$ s the XRT curves becomes dominant. The displacement between the peaks of these two light curves leads to a theoretically estimated spectral lag greater than 500 s in perfect agreement with the observations [153]. We obtain that the bolometric luminosity in this early part coincides with the sum of the BAT and XRT light curves (see Fig. 31) and the luminosity in the other energy ranges is negligible.

We recall that at $t_\text{d} > 10^4$ s there is a sudden enhancement in the radio luminosity and there is an optical luminosity dominated by the SN2006aj emission [141, 142, 154]. Although our analysis addresses only the BAT and XRT observations, for $r > 10^{13}$ cm corresponding to $t_\text{d} > 10^4$ s the fit of the XRT data implies two new features: 1) a sudden increase of the $\tilde{R}$ factor from $\tilde{R} = 1.0 \times 10^{-11}$ to $\tilde{R} = 1.6 \times 10^{-6}$, corresponding to a significantly more homogeneous effective CBM distribution (see Fig. 32b); 2) an XRT luminosity much smaller than the bolometric one (see Fig. 31). These theoretical predictions may account for the energetics of the enhancement of the radio and possibly optical and UV luminosities. Therefore, we identify two different regimes in the extended afterglow, one for $t_\text{d} < 10^4$ s and the other for $t_\text{d} > 10^4$ s. Nevertheless, there is a unifying feature: the determined effective CBM density decreases with the distance $r$ monotonically and continuously through both these two regimes from $n_{\text{cbm}} = 1$ particle/cm$^3$ at $r = r_0$ to $n_{\text{cbm}} = 10^{-6}$ particle/cm$^3$ at $r = 6.0 \times 10^{15}$ cm: $n_{\text{cbm}} \propto r^{-\alpha}$, with $1.0 \leq \alpha \leq 1.7$ (see Fig. 32a).

Our assumption of spherical symmetry is supported by the observations which set for GRB 060218 an opening beaming angle larger than $\sim 37^\circ$ [106, 141, 142, 145].

The procedure of the fit

The arrival time of each photon at the detector depends on the entire previous history of the fireshell [4]. Moreover, all the observables depends on the EQTS [47, 42] which, in turn, depend crucially on the equations of motion of the fireshell. The CBM engulfment has to be computed self-consistently through the entire dynamical evolution of the fireshell and not separately at each point. Any change in the CBM distribution strongly influences the entire dynamical evolution of the fireshell and, due to the EQTS structure, produces observable effects up to a much later time. For example if we change the density mask at a certain distance from the black hole we modify the shape of the lightcurve and consequently the evolution changes at larger radii corresponding to later times. Anyway the change of the density
is not the only problem to face in the fitting of the source, in fact first of all we have to choose the energy in order to have Lorentz gamma factor sufficiently high to fit the entire GRB. In order to show the sensitivity of the fitting procedure I also present two examples of fits with the same value of $E$ and different value of $E_{\text{tot}}$.

The first example has an $E_{\text{tot}} = 1.36 \times 10^{50}$ erg. This fit resulted unsuccessfully as we see from the Fig. 29, because the bolometric lightcurve is under the XRT peak of the extended afterglow. This means that the value of the energy chosen is too small to fit any data points after the peak of the extended afterglow. So we have to increase the value of the Energy to a have a better fit. In fact the parameters values have been found with various attempt in order to obtain the best fit.

The second example is characterized by $E_{\text{tot}} = 1.61 \times 10^{50}$ erg and the all the data are fitted except for the last point from $2.0 \times 10^2$ s to the end (see Fig. 30). I attempt to fit these last points trying to diminishes the $R$ values in order to enhance the energy emission, but again the low value of the Lorentz gamma factor, that in this case is 3 prevent the fireshell to expand. So again in this case the value of the Energy chosen is too small, but it is better than the previous attempt. In this case we increased the energy value of the 24%, but it is not enough so we decide to increase 16%.

Figure 28. GRB 060218 prompt emission: a) our theoretical fit (dotted line) of the BAT observations in the 15–150 keV energy band (dotted points); b) our theoretical fit (solid line) of the XRT observations in the 0.1–10 keV energy band (solid points) [Data from Ref. 141].
Figure 29. GRB 060218 light curves with $E_{\text{tot}}^{\text{obs}} = 1.36 \times 10^{50}$ erg: our theoretical fit (dotted line) of the 15–150 keV BAT observations and our theoretical fit (solid line) of the 0.3–10 keV XRT observations are represented (Data from: Campana et al. [141], Soderberg et al. [142]).

So the final fit is characterized by the $B = 1.0 \times 10^{-2}$ and by the $E_{\text{tot}}^e = 2.32 \times 10^{50}$ erg. With this value of the energy we are able to fit all the experimental points.

The fireshell fragmentation

GRB 060218 presents different peculiarities: the extremely long $T_{90}$, the very low effective CBM density decreasing with the distance and the largest possible value of $B = 10^{-2}$. These peculiarities appear to be correlated. Following Ruffini et al. [108], we propose that in the present case the fireshell is fragmented. This implies that the surface of the fireshell does not increase any longer as $r^2$ but as $r^\beta$ with $\beta < 2$. Consequently, the effective CBM density $n_{\text{cbm}}$ is linked to the actual one $n_{\text{act}}^{\text{cbm}}$ by:

$$n_{\text{cbm}} = R_{\text{shell}} n_{\text{act}}^{\text{cbm}},$$

where $r^\ast$ is the starting radius at which the fragmentation occurs and $\alpha = 2 - \beta$ (see Fig. 32a). For $r^\ast = r$, we have $n_{\text{cbm}}^{\text{act}} = 1$ particles/cm$^3$, as expected for a “canonical GRB” [3] and in agreement with the apparent absence of a massive stellar wind in the CBM [142, 154, 155].

The $R$ parameter defined in Eq.(13) has to take into account both the effect of the fireshell fragmentation ($R_{\text{shell}}$) and of the effective CBM porosity ($R_{\text{cbm}}$):

$$R \equiv R_{\text{shell}} \times R_{\text{cbm}}.$$
The phenomenon of the clumpiness of the ejecta, whose measure is the filling factor, is an aspect well known in astrophysics. For example, in the case of Novae the filling factor has been measured to be in the range $10^{-2}$–$10^{-5}$ [156]. Such a filling factor coincides, in our case, with $R_{\text{shell}}$.

**Binaries as progenitors of GRB-SN systems**

The majority of the existing models in the literature appeal to a single astrophysical phenomenon to explain both the GRB and the SN ["collapsar", see e.g. Ref. 112]. On the contrary, a distinguishing feature of our theoretical approach is to differentiate between the SN and the GRB process (see above). The GRB is assumed to occur during the formation process of a black hole. The SN is assumed to lead to the formation of a neutron star (NS) or to a complete disruptive explosion without remnants and, in no way, to the formation of a black hole. In the case of SN2006aj the formation of such a NS has been actually inferred by Maeda et al. [104] because of the large amount of $^{58}\text{Ni}$ (0.05 $M_\odot$). Moreover the significantly small initial mass of the SN progenitor star $M \approx 20M_\odot$ is expected to form a NS rather than a black hole when its core collapses [104, 148, 157, 158]. In order to fulfill both the above requirement, we assume that the progenitor of the GRB and the SN consists of a binary system formed by a NS close to its critical mass collapsing to a black hole, and a companion star evolved out of the main sequence originating the SN. The temporal coincidence between the GRB and the SN phenomenon is explained in term of the concept of "induced" gravitational collapse [6, 108]. There is also the distinct possibility of observing the young born NS out of the SN [see e.g. Ref. 108, and references therein].

It has been often proposed that GRBs associated with SNe Ib/c, at smaller redshift $0.0085 < z < 0.168$ [see e.g.?
Figure 31. GRB 060218 complete light curves: our theoretical fit (dotted line) of the 15–150 keV BAT observations, our theoretical fit (solid line) of the 0.3–10 keV XRT observations and the 0.3–10 keV Chandra observations are represented together with our theoretically computed bolometric luminosity (double dotted line) [Data from Refs. 141, 142].

Ref. 105, and references therein], form a different class, less luminous and possibly much more numerous than the high luminosity GRBs at higher redshift [102, 103, 104, 105]. Therefore they have been proposed to originate from a separate class of progenitors [106, 107]. In our model this is explained by the nature of the progenitor system leading to the formation of the black hole with the smallest possible mass: the one formed by the collapse of a just overcritical NS [159, 108].

Conclusions

GRB 060218 presents a variety of peculiarities, including its extremely large $T_{90}$ and its classification as an XRF. Nevertheless, a crucial point of our analysis is that we have successfully applied to this source our “canonical GRB” scenario.

Within our model there is no need for inserting GRB 060218 in a new class of GRBs, such as the XRFs, alternative to the “canonical” ones. This same point recently received strong observational support in the case of GRB 060218 [153] and a consensus by other models in the literature [151].

The anomalously long $T_{90}$ led us to infer a monotonic decrease in the CBM effective density giving the first clear evidence for a fragmentation in the fireshell. This phenomenon appears to be essential in understanding the features of also other GRBs [see e.g. GRB 050315 in Refs. 108, 33].

Our “canonical GRB” scenario originates from the gravitational collapse to a black hole and is now confirmed over a $10^6$ range in energy [see e.g. Ref. 3, and references therein]. It is clear that, although the process of gravitational collapse is unique, there is a large variety of progenitors which may lead to the formation of black holes, each one with precise signatures in the energetics. The low energetics of the class of GRBs associated with SNe, and the necessity of the occurrence of the SN, naturally leads in our model to identify their progenitors with the formation of the smallest possible black hole originating from a NS overcoming his critical mass in a binary system. For GRB 060218 there is no need within our model for a new or unidentified source such as a magnetor or a collapsar.

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GRB 060218 is the first GRB associated with SN with complete coverage of data from the onset all the way up to $\sim 10^6$ s. This fact offers an unprecedented opportunity to verify theoretical models on such a GRB class. For example, GRB 060218 fulfills the Amati et al. [13] relation unlike other sources in its same class. This is particularly significant, since GRB 060218 is the only source in such a class to have an excellent data coverage without gaps. We are currently examining if the missing data in the other sources of such a class may have a prominent role in their non-fulfillment of the Amati et al. [13] relation [Dainotti et al., in preparation; see also Ref. 160].

**THEORETICAL BACKGROUND FOR GRBS’ EMPIRICAL CORRELATIONS**

The detection of GRBs up to very high redshifts [up to $z = 6.7$, see Ref. 10], their high observed rate of one every few days, and the progress in the theoretical understanding of these sources all make them useful as cosmological tools, complementary to supernovae Ia, which are observed only up to $z = 1.7$ [11, 12]. One of the hottest topics on GRBs is the possible existence of empirical relations between GRB observables [13, 14, 15, 16, 17, 18], which may lead, if
confirmed, to using GRBs as tracers of models of universe. The first empirical relation, discovered when analyzing the BeppoSAX so-called “long” bursts with known redshift, was the “Amati relation” [13]. It was found that the isotropic-equivalent radiated energy of the prompt emission $E_{iso}$ is correlated with the cosmological rest-frame $\nu F_{\nu}$ spectrum peak energy $E_{p,iso}$, $E_{p,iso} \propto (E_{iso})^a$, with $a = 0.52 \pm 0.06$ [13]. The existence of the Amati relation has been confirmed by studying a sample of GRBs discovered by Swift, with $a = 0.49 \pm 0.06$ [19, 20].

Swift has for the first time made it possible to obtain high quality data in selected energy bands from the GRB trigger time all the way to the latest extended afterglow phases [161]. This has given us the opportunity to apply our theoretical “fireshell” model, thereby obtaining detailed values for its two free parameters, namely for the total energy $E_{tot}$, and the baryon loading $B$ of the fireshell, as well as for the effective density and filamentary structure of the CBM. From this we were able to compute multi-band light curves and spectra, both instantaneous and time-integrated, compared with selected GRB sources, such as GRB 050315.

In the “fireshell” model, $E_{50}^{iso}$ comprises two different components: (i) the P-GRB with energy $E_{p,GRB}$, emitted at the moment when the $e^+ e^-$-driven accelerating baryonic matter reaches transparency, and (ii) the following extended afterglow phase with energy $E_{a,f}$, with the decelerating baryons interacting with the CBM [5]. These two phases are clearly distinguishable by their relative intensity and temporal separation in arrival time. We have

$$E_{50}^{iso} = E_{p,GRB} + E_{a,f}.$$  \hspace{1cm} (14)

What is usually called the “prompt emission” corresponds within the fireshell model to the P-GRB together with the peak of the extended afterglow [see below, e.g. Ref. 5, 71, 3, 51, 33, 162, 34, and references therein].

Among the crucial issues raised by the Amati relation, there are its theoretical explanation and its possible dependence on the assumed cosmological model. We examined a set of “gedanken” GRBs, all at the same cosmological redshift of GRB 050315. Such a set assumes the same fireshell baryon loading and effective CBM distribution as GRB 050315 [71] and each “gedanken” GRB differs from the others uniquely by the value of its total energy $E_{50}^{iso}$. We then considered a second set of “gedanken” GRBs, differing from the previous one by assuming a constant effective CBM density instead of the one inferred for GRB 050315. In both these sets, we looked for a relation between the isotropic-equivalent radiated energy of the entire extended afterglow $E_{a,f}$, and the corresponding time-integrated $\nu F_{\nu}$ spectrum peak energy $E_{p}$:

$$E_{p} \propto (E_{a,f})^a.$$  \hspace{1cm} (15)

In this chapter, after briefly recalling the various spectral-energy correlations mentioned above, we present the derivation of the $E_{p} - E_{a,f}$ relation for the two sets of “gedanken” GRBs.

Spectral-energy correlations

Many empirical spectral-energy correlations exist, some are purely phenomenological and assumption free while others are based on assumptions and are dependent on model, basically the standard fireball model [43]. Some correlations assume spherical symmetry while others assume collimated (jet) emission. This last case was triggered by the observation by Frail et al. [163] that the collimation corrected energetics of those GRBs of know jet aperture angles clustered into a narrow distribution, $E_{p} = (1 - \cos \theta_j)E_{iso} \approx 10^{51}$ erg. The opening angle of the jet is estimated within the standard model as

$$\theta_j = 0.161 \left( \frac{t_{jet,d}}{1 + z} \right)^{3/8} \left( \frac{n \eta_{p}}{E_{iso,52}} \right)^{1/8}; \text{ H}$$

$$\theta_j = 0.2016 \left( \frac{t_{jet,d}}{1 + z} \right)^{1/4} \left( \frac{\eta_{p} A_{w}}{E_{iso,52}} \right)^{1/4}; \text{ W}$$  \hspace{1cm} (16)

where $t_{jet,d}$ is the break time measured in days and $z$ is the redshift. The efficiency $\eta_{p}$ relates the isotropic kinetic energy of the fireball $E_{k,iso}$ to the prompt emitted energy $E_{iso}$: $E_{k,iso} = E_{iso}/\eta_{p}$. Usually, it is assumed a constant value for all bursts, i.e. $\eta_{p} = 0.2$ (after its first use by Frail et al. [163], following the estimate of this parameter in GRB 970508 [164]). In the homogeneous (H) case, $n$ is the CircumBurst density, independent from the radial coordinate; for the wind (W) case, the density is a function of the radial coordinate, $n(r) = A r^{-2}$ and $A_{w}$ is the value of $A$ ($A = M_{e}/(4\pi
\nu_{w}) = 5 \times 10^{11} A_{w} \text{ g cm}^{-3}$) when setting the wind mass loss rate to $M_{w} = 10^{-8} M_{\odot} \text{yr}^{-1}$ and the wind velocity to $\nu_{w} = 10^{4} \text{ km s}^{-1}$. Usually, a constant value (i.e. $A_{w} = 1$) is adopted for all bursts.
The most important spectral-energy correlations are:

- **The Amati relation**: It was historically the first correlation discovered, considering *BeppoSAX* bursts [13]. It was found that the isotropic-equivalent radiated energy of the prompt emission $E_{\text{iso}}$ is correlated with the cosmological rest-frame $V_F$ spectrum peak energy $E_{\nu,p}$: $E_{\text{iso}} \propto E_{\nu,p}^{0.5}$. This correlation, recently updated [20] to a larger sample, holds for all but two long bursts, while no short burst satisfies it. The long burst outliers are GRB 980425 and the debated GRB 031203 [165]. As far as short bursts are concerned, there are two cases: the burst with an initial spike-like emission followed by a soft bump (short burst with afterglow) and the short burst with no afterglow. The burst belonging to the first class are what we named [33] “fake” or “disguised” short GRBs, while the ones belonging to the second case are the “genuine” short GRBs. Both classes, as already said above, does not follow the Amati relation, but, if one excludes the initial spike-like emission and considers only the soft later part of the bursts in the first class, then the Amati relation is recovered [83, 81].

- **The Yonetoku correlation**: Yonetoku et al. [15] showed that also the peak luminosity $L_{\nu,\text{iso}}$ of the prompt emission correlates with $E_{\nu,\text{p}}$, in the same way as $E_{\text{iso}}$: $E_{\nu,\text{p}} \propto L_{\nu,\text{iso}}^{1/2}$. The scatter is similar to the scatter of the Amati correlation, and the outliers are the same as well.

- **The Ghirlanda correlation**: Assuming a collimated emission, Ghirlanda et al. [14] found that the collimation corrected (by a factor $(1 - \cos \theta_j)$) energy, $E_{\gamma, j}$, is tightly correlated with $E_{\nu, p}$. The correlation is $E_{\gamma, j} \propto E_{\nu, p}^{0.7}$. As outlined above, this relation is based on a theoretical model needed to calculate $\theta_j$, that in turns relies on the assumptions adopted for the efficiency and the CircumBurst density and profile.

- **The Liang & Zhang correlation**: To find the jet angle $\theta_j$, as explained above, one needs a model and some assumptions; the Liang and Zhang [16] correlation instead is entirely phenomenological, so model independent and assumptions free. It involves three observables (plus the redshift) and it is of the form $E_{\text{iso}} \propto E_{\nu, p}^{2/3} L_{\nu, p}$. It is consistent [166] with the Ghirlanda correlation, and has similar spread.

- **The Firmani correlation**: The Firmani et al. [17] correlation links three quantities of the prompt emission: the bolometric isotropic peak luminosity $L_{\nu, \text{iso}}$, the peak energy $E_{\nu, \text{p}, \text{iso}}$ of the time integrated spectrum, and a characteristic time $T_{\text{aft}}$, which is the time interval spanned by the brightest 45% of the total light curve counts above the background. This time is used to characterize the variability properties of the prompt emission [167]. The correlation is of the form: $L_{\nu, \text{iso}} \propto E_{\nu, \text{p}, \text{iso}}^{0.3} T_{\text{aft}}^{-1/2}$. Also this relation is model independent and assumption free.

**The $E_{\nu, p} - E_{\text{aft}}$ relation**

In our approach, only the *entire* extended afterglow emission is considered in establishing our $E_{\nu, p} - E_{\text{aft}}$ relation. From this assumption one derives, in a natural way, that the Amati relation holds only for long GRBs, where the P-GRB is negligible, and not for short GRBs [115].

We can compute the “instantaneous” spectrum of GRB 050315 at each value of the detector arrival time during the entire extended afterglow emission. Such a spectrum sharply evolves in the arrival time, presenting a typical hard–soft behavior [71]. We then computed the $V_F$ time-integrated spectrum over the total duration of our extended afterglow phase, that is, from the end of the P-GRB up to when the fireball reaches a Lorentz gamma factor close to unity. We can then define the energy $E_{\nu, p}$ as the energy of the peak of this $V_F$-time integrated spectrum, and we look at its relation with the total energy $E_{\text{aft}}$ of the extended afterglow.

We construct two sets of “gedanken” GRBs at a fixed cosmological redshift, therefore independently of the cosmological model. The first set assumes the same fireball baryon loading and effective CBM distribution as GRB 050315 (see Fig. 33) and each “gedanken” GRB differs from the others uniquely by the value of its total energy $E_{\text{tot}}$. The second set assumes a constant effective CBM density $\sim 1 \text{ particle/cm}^3$ instead of the one inferred for GRB 050315.

In our model, $E_{\text{aft}}$ is a fixed value determined by $E_{\text{iso}}^{\text{str}}$ and $B$, so clearly there are no errors associated to it. Instead, $E_{\nu, p}$ is evaluated from the numerically calculated spectrum, and its determination is therefore affected by the numerical resolution. Choosing a 5% error on $E_{\nu, p}$, which is consistent with our numerical resolution, we checked that this value is reasonable looking at each spectrum. Figure 34 shows the time-integrated spectrum corresponding to $E_{\text{iso}}^{\text{str}} = 3.40 \times 10^{51} \text{ erg}$ with the error around $E_{\nu, p}$.
Results and discussion

Figure 35 shows the $E_p - E_{\text{a f t}}$ relation of the “gedanken” GRBs belonging to the first set (red points). It extends over two orders of magnitude in energy, from $10^{51}$ to $10^{53}$ erg, and is well-fitted by a power law $E_p \propto (E_{\text{a f t}})^{a}$ with $a = 0.45 \pm 0.01$. We emphasize that such a power-law slope strictly agrees with the Amati relation, namely...
Figure 35. The $E_p - E_{a f t}$ relation: the results of the simulations of the first set of “gedanken” GRBs (points marked as crosses) are well-fitted by a power law (solid line) $E_p \propto (E_{a f t})^a$ with $a = 0.45 \pm 0.01$. The points marked as “X” are the results of the extension of the first set above $10^{53}$ erg.

Figure 36. The $vF_v$ time-integrated spectrum over the total duration of our extended afterglow phase for the “gedanken” GRB of the extended first set with total energy $E'_{e \sigma} = 6.95 \times 10^{53}$ erg. The vertical lines constrain the 5% error region around each peak.

$E_{p,i} \propto (E_{iso})^a$, with $a = 0.49^{+0.06}_{-0.05}$ [20]. We recall that $E_p$ is the observed peak energy; i.e., it is not rescaled for the cosmological redshift, because all the “gedanken” GRBs of the set are at the same redshift of GRB 050315, namely $z = 1.949$ [100]. The normalization is clearly different from the Amati one.

If we try to extend the first sample of “gedanken” GRBs below $10^{51}$ erg, the relevant CBM distribution would be for $r \lesssim 10^{16}$ cm, where no data are available from the GRB 050315 observations. If we try to extend the first set...
Attachments

Figure 37. The second set of “gedanken” GRBs. Clearly, in this case there is no relation between $E_p$ and $E_{\text{aft}}$.

The high-energy spectral peak is due to the emission at the peak of the extended afterglow, and therefore due to the so-called “prompt emission”. The low-energy one is due to late-time soft X-ray emission. Therefore, the high-energy spectral peak is the relevant one for the Amati relation. We find indeed that such a high-energy spectral peak still fulfills the $E_p - E_{\text{aft}}$ relation for $E_{\text{tot}}^\infty \sim 10^{54}$ erg, with a possible saturation for $E_{\text{tot}}^\infty > 10^{54}$ erg (see Fig. 35).

Figure 37 clearly shows that in the second set of “gedanken” GRBs, built assuming a constant effective CBM density $\sim 1$ particle/cm$^3$, instead of the one specifically inferred for GRB 050315, there is no relation between $E_p$ and $E_{\text{aft}}$.

Conclusions

The high-quality Swift data, for the first time giving gapless and multiwavelength coverage from the GRB trigger all the way to the latest extended afterglow phases, have led to a complete fit of the GRB 050315 multiband light curves based on our fireshell model. We fixed the free parameters describing the source and determined the instantaneous and time-integrated spectra during the entire extended afterglow.

Starting from this, we examined two sets of “gedanken” GRBs, constructed at a fixed cosmological redshift. The first set assumes the same fireshell baryon loading and effective CBM distribution as GRB 050315, and each “gedanken” GRB differs from the others uniquely by the value of its total energy $E_{\text{tot}}^\infty$. The second set assumes a constant effective CBM density $\sim 1$ particle/cm$^3$, instead of the one inferred for GRB 050315.

Recalling that the “canonical” GRB light curve in the fireshell model is composed of two well-separated components, the P-GRB, and the entire extended afterglow, we looked for a relation in both sets between the isotropic-equivalent radiated energy of the entire extended afterglow $E_{\text{tot}}^\infty$ and the corresponding time-integrated $\nu F_\nu$ spectrum peak energy $E_p$: $E_p \propto (E_{\text{tot}}^\infty)^x$. In doing so, we assumed that the Amati relation is directly linked to the interaction between the accelerated baryons and the CBM. The P-GRBs, which originate from the fireshell transparency, do not fulfill the Amati relation in our approach. Consequently, the short GRBs, which have a vanishing extended afterglow...
with respect to the P-GRB, should also not fulfill the Amati relation. This last point is supported by the observational evidence [20].

We notice that the first set of “gedanken” GRBs fulfills the $E_p \propto (E_{iso})^a$ relation very well with $a = 0.45 \pm 0.01$. This slope strongly agrees with the Amati relation. In contrast, no relation between $E_p$ and $E_{iso}$ seems to hold for the second set. We conclude that the Amati relation originates from the detailed structure of the effective CBM.

Turning now to the analogies and differences between our $E_p - E_{iso}$ relation and the Amati one, our analysis excludes the P-GRB from the prompt emission, extends all the way to the latest extended afterglow phases, and is independent of the assumed cosmological model, since all “gedanken” GRBs are at the same redshift. The Amati relation, on the other hand, includes the P-GRB, focuses only on the prompt emission, being therefore influenced by the instrumental threshold that fixes the end of the prompt emission, and depends on the assumed cosmology. This might explain the intrinsic scatter observed in the Amati relation [20]. Our theoretical work is a first unavoidable step toward supporting the use of the empirical Amati relation for measuring the cosmological parameters.

**THERMALIZATION PROCESS OF ELECTRON-POSITRON PLASMA WITH BARYON LOADING**

Initial evolution of electron-positron-photon plasma in the source of a GRB has a key role in the subsequent dynamics of the fireshell. In particular, particle spectra, temperatures, chemical potentials all need to be known in order to describe acceleration of the fireshell and in general its expansion in terms of hydrodynamics. Since quite different theoretical arguments existed on the initial state of optically thick electron-positron-plasma in GRBs [see e.g. Refs. 168, 96] we turned to analysis of kinetic properties of nonequilibrium electron-positron pairs.

Having this goal in mind Aksenov et al. [122] solved numerically relativistic Boltzmann equations for distribution functions of electrons, positrons and photons, assuming their uniform spatial distribution. Considering energy density in the range, typical for GRBs, the relevant thermalization timescales were determined. It turns out that particles reach kinetic equilibrium on a timescale $t < 10^{-14}$ sec, when distribution functions of electrons/positrons (photons) acquire Fermi-Dirac (Bose-Einstein) form, all particles have a common temperature but nonzero chemical potentials. Further, on a timescale $t < 10^{-12}$ sec chemical potentials vanish and particles reach thermal distribution.

Since in many bursts baryon loading is dynamically significant Aksenov et al. [40] considered proton admixture parametrized by the parameter $B = n_p m_p c^2 / \rho$, where $n_p$ is the number density of protons, $m_p$ is their mass, $\rho$ is the radiative energy density (including the energy density of electron-positron pairs). Independent on the baryon loading the thermalization timescale was found to be $t < 10^{-11}$ sec for such a plasma, despite thermalization process is more complicated.

As example, we show energy densities (Fig. 38), number densities (Fig. 39), temperatures and chemical potentials (Fig.40, 41 respectively) depending on time. Initial and final spectra of particles are shown in Fig. 42. Initial conditions were chosen with flat spectral densities and total energy density $\rho = 10^{24}$ erg/cm$^3$. This initial state is clearly far from equilibrium. Interactions between particles change distribution functions such that at the moment $t_1 = 4 \times 10^{-14}$ sec, shown by the vertical line on the left in Fig. 40 and 41, distribution functions acquire an equilibrium form with temperature of photons and pairs is $T_\nu = k_B T_\nu / (m_e c^2) \approx 1.5$, while the chemical potentials of these particles are $\mu_\nu = \mu_\nu / (m_e c^2) \approx -7$, where $k_B$ is Boltzmann’s constant, $m_e$ is electron mass, and $c$ is the speed of light. These changes are essentially due to binary interactions, which do not change number of particles. Further evolution of the system is due to triple interactions. Only triple interactions, which do not conserve the number of particles, are able to change chemical potentials of particles. The chemical potential of photons reaches zero at $t_2 = 10^{-12}$ sec, shown by the vertical line on the right in Fig. 40 and 41, which means that electrons, positrons and photons come to thermal equilibrium. Protons join thermal equilibrium the last. Simulations with different initial conditions show that thermalization timescale depends essentially on total energy density and the baryonic loading parameter $B$ [for details see Ref. 169].

Such short timescales, compared to a typical expansion timescale $t_{ex} \sim R_0/c \sim 10^{-3}$ sec where $R_0$ is initial size of the plasma allow to speak of completely thermalized plasma long before expansion starts [for details see Ref. 169].
Figure 38. Dependence on time of energy densities of electrons (green), positrons (red), photons (black) and protons (blue). Total energy density is shown by dotted black line. Interaction between pairs and photons operates on very short timescales up to $10^{-23}$ sec. Quasi-equilibrium state is established at $t_k \simeq 10^{-14}$ sec which corresponds to kinetic equilibrium for pairs and photons. Protons start to interact with them as late as at $t_h \simeq 10^{-13}$ sec.

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CRITICAL ELECTRIC FIELDS ON THE SURFACE OF MASSIVE CORES AND DYADOTORUS OF THE KERR-NEWMAN GEOMETRY

Critical electric Fields on the surface of massive cores

One of the most active field of research has been to analyse a general approach to Neutron Stars based on the Thomas-Fermi ultrarelativistic equations amply adopted in the study of superheavy nuclei. The aim is to have a unified approach both to superheavy nuclei, up to atomic numbers of the order of $10^5$–$10^6$, and to what we have called "Massive Nuclear Cores", which are

- characterized by atomic number of the order of $10^{57}$;
- composed by neutrons, protons and electrons in $\beta$–equilibrium;
- expected to be kept at nuclear density by self gravity.

The analysis of superheavy nuclei has historically represented a major field of research [170, 171, 172, 173, 174], guided by Prof. V. Popov and Prof. W. Greiner and their schools. This same problem was studied in the context of the relativistic Thomas-Fermi equation also by R. Ruffini and L. Stella [175, 176], already in the 80s. The recent numerical approach has shown the possibility to extrapolate this treatment of superheavy nuclei to the case of Massive Nuclear Cores [177]. The very unexpected result has been that also around these massive cores there is the distinct possibility
Figure 39. Dependence on time of concentrations of electrons (green), positrons (red), photons (black) and protons (blue). Total number density is shown by dotted black line. In this case kinetic equilibrium between electrons, positrons and photons is reached at $t_k \approx 10^{-14}$ sec. Protons join thermal equilibrium with other particles at $t_{th} \approx 4 \times 10^{-12}$ sec.

of having an electromagnetic field close to the critical value $E_c = \frac{m_e^2 c^3}{\hbar}$, although localized in a very narrow shell of the order of the electron Compton wavelength (see Fig. 43, 44).

The welcome result has been that all the analytic work [178] developed by Prof. Popov and his Russian collaborators can be straightforwardly applied to the case of massive cores, and the $\beta$–equilibrium condition is properly taken into account. In Ref. [179], we show that globally neutral massive cores can be gravitationally bound and the value of the charge-to-mass ratios predicted at the surface of massive cores coincides with the range of values expected in astrophysical scenarios for Kerr-Newman black holes, in addition to a further verification of the over critical electric field on the massive core surface, numerically obtained by Ruffini, Xue and Rotondo already in 2007 [177]. A large variety of problems has emerged, and working progress in solving these problems have been going on in the direct discussion with or participation by Prof. Greiner, Prof. Popov, and Prof. ’t Hooft at ICRANet center located at Pescara. The crucial issue to be debated is the stability of such cores under the competing effects of self gravity and Coulomb repulsion. In order to probe this stability, we have started a new approach to the problem within the framework of general relativity. The object of the work by Patricelli and Rueda is the generalization of the Tolman-Oppenheimer-Volkoff equation duly taking into account the electrodynamical contribution. The major scientific issue here is to have a unified approach solving the coupled system of the general relativistic self gravitating electrodynamical problem with the corresponding formulation of the Thomas-Fermi equation in the framework of general relativity. Prof. ’t Hooft, in a series of lectures at Pescara, has forcefully expressed the opinion that necessarily, during the process of gravitational collapse, it should occur a more extended distribution of the electromagnetic field to the entire core of the star and not only confined to a thin shell. This is a necessary condition in order to transmit the gravitational energy of the collapse to the electrodynamical component of the field giving possibly rise to large pair creation processes. This crucial idea
Figure 40. Dependence on time of dimensionless temperature of electrons (green), positrons (red), photons (black) and protons (blue). The temperature for pairs and photons acquires physical meaning only in kinetic equilibrium at \( t_k \approx 10^{-14} \) sec. Protons are cooled by the pair-photon plasma and acquire common temperature with it as late as at \( t_{th} \approx 4 \times 10^{-12} \) sec.

is currently being pursued by the application to this system of a classical work of Feynmann-Metropolis and Teller, who considered in relativistic Thomas-Fermi the crucial role of non-degeneracy.

**On the Dyadotorus of the Kerr-Newman Geometry**

In the merging process of two neutron stars and in the final process of gravitational collapse of a black hole it is possible that very large electromagnetic field strength larger than the critical value of vacuum polarization \( E_c \) do occur [3]. The description of the time evolution of the gravitational collapse and the associated electrodynamical process (occurring on characteristic time scales \( \tau = GM/c^3 \approx 5 \times 10^{-5} M/M_\odot \) s) are too complex for a direct description. A more confined problem is the case of an already formed Kerr-Newman black hole.

This deserves analysis in itself as a theoretical problem and may represent a physical condition asymptotically reached in the process of gravitational collapse. Such an asymptotic configuration will be reached when all the multi-poles departing from the Kerr-Newman geometry have been radiated away either by process of vacuum polarization or electromagnetic and gravitational waves. This simplified problem may lead to a direct evaluation of the energetics as well as of the created \( e^-e^+ \) pairs occurring on time scales \( \Delta t = \hbar/(m_ec^2) \approx 10^{-21} \) s.

Therefore we explore the initial condition for such a process by the definition of the spatial extent of a “dyadotorus” which generalizes to the Kerr-Newman geometry the concept of the “dyadosphere” previously introduced in the case
of the spherically symmetric Reissner-Nordström geometry [180, 26]. Damour and Ruffini [23] showed that vacuum polarization processes à la Sauter-Heisenberg-Euler-Schwinger [181] can occur in the field of a Kerr-Newman black hole endowed with a mass ranging from the maximum critical mass for neutron stars ($3.2M_\odot$) all the way up to $7.2 \times 10^6M_\odot$. It is an almost perfectly reversible process in the sense defined by Christodoulou and Ruffini [182], leading to a very efficient mechanism of extracting energy from the black hole.

In the case of absence of rotation in spacetime, we have a Reissner-Nordström black hole as the background geometry. The region where vacuum polarization processes take place is a sphere centered about the hole, and has been called dyadosphere [180, 26]. We investigate how the presence of rotation in spacetime modifies the shape of the surface containing the region where electron-positron pairs are created.

Due to the axial symmetry we call that region as dyadotorus and we give the conditions for its existence. We have defined the dyadotorus as the locus of points where $E = kE_c$ with $k$ some positive constant which can be less than one [see Ref. 183, for details]. We have found that the geometry of the dyadotorus is indeed torus-like when

$$k \geq \frac{\xi}{8E_c M \mu \alpha^2} \approx 6.6 \times 10^4 \frac{\xi}{\mu \alpha^2},$$

(17)

where $\mu = M/M_\odot$, $\xi = Q/M$ and $\alpha = J/M^2$ being $M$, $Q$ and $J$ the mass, the charge and angular momentum of the black hole. Otherwise, it becomes ellipsoid-like. This can be seen from Fig. 45.

An estimate of the electromagnetic energy contained in the dyadotorus can be calculated by using, for example, three different definitions of it commonly adopted in the literature, i.e. the standard definition in terms of the timelike Killing
vector [see e.g. Ref. 184], the one recently suggested by Katz, Lynden-Bell and Bičák [185, 186] for axially symmetric asymptotically flat spacetimes, which is an observer dependent definition of energy, and the last one involving the theory of pseudotensors [see e.g. 187]. All these approaches are shown to give the same results.

From this, we find that in addition to the topological differences between the dyadotorus and the dyadosphere, larger field strengths and electromagnetic energy are allowed in the case of a Kerr-Newman geometry close to the horizon, when compared with a Reissner-Nordström black hole of the same mass energy and charge to mass ratio.

**SELECTED PROCESSES ORIGINATING HIGH-ENERGY EMISSION**

The knowledge of the radiation mechanisms is crucial for the correct understanding of many astronomical observations. In particular in gamma ray astronomy the observational data can be often explained by two or more production mechanisms. It is therefore important to model correctly the different interactions producing gamma rays. Among all mechanisms producing gamma rays hadronic interactions between nucleons which produce pions, which in turn decay into photons and neutrinos, are one of the most studied models. High energy collisions of nucleons cannot be treated perturbatively because of the large value of the interaction constant in nuclear interactions. Many authors have occupied themselves with this problem and already in 1950 Fermi developed an elegant statistical method for computing the multiple production of particles in collisions of high energetic protons. In the meantime a very large set of data has been acquired from high energy accelerators. Our aim here is 1. to rederive the Fermi theoretical equations, 2. to

**Figure 42.** Spectral density as function of particle energy for electrons (green), positrons (red), photons (black) and protons (blue) in initial and final time moments of the computation. Fits of the spectra with chemical potentials and temperatures corresponding to thermal equilibrium state are also shown by yellow (electrons and positrons), gray (photons) and light blue (protons) thick lines. The final photon spectrum is black body one.
Figure 43. Number density of electrons, protons and neutrons.

Figure 44. Electric Field in units of the critical field.
compare them to the experimental data and 3. to explore possibilities of observing such phenomena.

Fermi’s approach to the study of hadronic interactions

In treating high energy collisions of nucleons, Fermi made the assumption that the possible final configurations of the system are determined by the statistical weights of the various possible final configurations and accordingly developed a statistical method to determine the final particles produced [21]. First of all one might think of many different final configurations for the system after the collision, but conservation laws of charge and of momentum, as well as the feasibility of the processes, have to be taken into account. Transitions in Yukawa’s theory, in which charged and neutral pions are created, are therefore the most probable processes taking place. So during the collisions of high energetic hadrons a large amount of energy is released in a small volume around the hadrons and used to form pions. In view of the strong interactions between these pions, one can imagine that the energy available in the small volume will be rapidly distributed to the different pions having different energies. In other words the energy will be statistically distributed among all degrees of freedom of the system. Fermi himself said: “When two nucleons collide with very great energy in their center of mass system this energy will be suddenly released in a small volume surrounding the two nucleons. The event is a collision in which the nucleons with their surrounding retinue of pions hit against each other so that all the portion of space occupied by the nucleons and by their surrounding pion field will be suddenly loaded with a very great amount of energy. Since the interactions of the pion field are strong we may expect that rapidly this energy will be distributed among the various degrees of freedom present in this volume according to statistical laws. One can then compute statistically the probability that in this tiny volume a certain number of pions will be created with a given energy distribution. It is then assumed that the concentration of energy will rapidly dissolve and that the particles into which the energy has been converted will fly out in all directions.”

Fermi’s method resembles somehow Heisenberg’s approach [188] to treat high energy collisions of nucleons, with the difference that Heisenberg used qualitative ideas of turbulence whereas, Fermi believed that in high energetic processes statistical equilibrium is reached.

According to Fermi the process proceeds as follows: in the laboratory frame a very energetic proton scatters off a proton target. For convenience the process is examined in the center of mass of the system. The only parameter which has to be tuned in Fermi’s theory is the volume in which the energy is dumped. The value of this parameter can be modified to improve the agreement with the experiments. Fermi defined $\Omega$ being the volume at rest in the laboratory frame that contains the energy of the colliding particles. Since the particles mediating the Yukawa interactions are the
pions, the volume is taken as a sphere with radius of order of the pion Compton wavelength $\lambda_c = h/m_\pi c = 1.4 \times 10^{-13}$ cm.

We will use the subscript “0” to indicate quantities in the c.m. frame, and no subscript to indicate the same quantities in the laboratory frame. Analogously, we use the superscript ′ to indicate quantities after the collision, and no superscript to indicate the same quantities before the collision.

Let’s focus for the moment on the target proton, which is at rest in the laboratory frame. Its associated volume $\Omega$ is:

$$\Omega = \frac{4}{3} \pi R^3,$$

with

$$R = a \lambda_c,$$

where $a$ is a free parameter of the order of unity which Fermi leaves free to better fit the experimental data. The same volume measured in the c.m. frame, where the target proton is moving with Lorentz factor $\gamma$, is given by:

$$\Omega_0 = \frac{1}{\gamma} \Omega.$$

In order to determine the Lorentz factor $\gamma$, we consider the case in which, in the initial reference frame, the laboratory frame, a particle with mass $m_1$ and energy $E_1$ collides with a particle with mass $m_2$ which is at rest. The total energy of the two particles is:

$$E = E_1 + E_2 = E_1 + m_2 c^2.$$

Their total momentum is given by:

$$\vec{P} = \vec{p}_1.$$

We can observe the same process in the c.m. frame, where the two colliding particles have zero total momentum. The square $s$ of the 4-momentum in the c.m. frame is given by:

$$s = P_\alpha P_\alpha.$$

Since $s$ is Lorentz invariant, it must be:

$$s = P_\alpha P_\alpha = (E_1 + m_2 c^2)^2 - \vec{p}_1^2 c^2 = (E_1^2 + m_1^2 c^4) - (E_1^2 - m_1^2 c^4) = m_1^2 c^4 + m_2^2 c^4 + 2E_1 m_2 c^2.$$

If the two particles are protons, $m_1 = m_2 = m_p$. We then have

$$s = 2m_p c^2 (E_1 + m_p c^2).$$

The energy of each proton in the c.m. frame can be written as:

$$E_0 = \gamma m_p c^2,$$

in fact, in the c.m. frame both of them moves with the same Lorentz factor $\gamma$. Since

$$s = (2E_0)^2,$$

we have:

$$\gamma = \frac{\sqrt{s}}{2m_p c^2} = \frac{\sqrt{2m_p c^2 (E_1 + m_p c^2)}}{2m_p c^2} = \sqrt{\frac{E_1 + m_p c^2}{2m_p c^2}}.$$

An alternative derivation of Eq.(28) starts from the fact that $\gamma$ is also the Lorentz factor of the motion of the c.m. in the laboratory frame, since, we recall, the target proton is at rest in the laboratory frame. The speed $v$ of the c.m. in the laboratory frame is given by (see e.g. Eq.(11.4) in Landau and Lifshitz [189]):

$$v = \frac{|\vec{P}_1| c^2}{E} = \frac{|\vec{P}_1| c^2}{E_1 + m_p c^2},$$
where we used Eqs. (21)-(22) together with the fact that for two protons \( m_1 = m_2 = m_p \). From Eq. (29), by definition of \( \gamma \), we have:

\[
\gamma = \sqrt{\frac{1}{1 - (\nu/c)^2}} = \sqrt{\frac{(E_1 + m_p c^2)^2}{(E_1 + m_p c^2)^2 - p_1^2 c^2}} = \sqrt{\frac{(E_1 + m_p c^2)^2}{E_1^2 + m_p^2 c^4 + 2 E_1 m_p c^2 - p_1^2 c^2}} = \frac{E_1 + m_p c^2}{2 m_p c^2},
\]

(30)

where we used the fact that \( E_1^2 = p_1^2 c^2 + m_p^2 c^4 \). As we expected, Eq. (30) is identical to Eq. (28).

Substituting Eq. (28) in Eq. (20), we get the expression of Fermi for the Lorentz contraction of the volume \( \Omega_0 \) in the c.m. frame with respect to \( \Omega \):

\[
\Omega_0 = \sqrt{\frac{2 m_p c^2}{E_1 + m_p c^2} \Omega},
\]

(31)

Note that if the energy \( E_1 \) increase, the volume \( \Omega_0 \) decreases, as is predicted in special relativity. The parameter volume will be therefore energy dependent. Fermi calculated also the cross-section as the area available for collisions around the pion cloud

\[
\sigma_{tot} = \pi R^2.
\]

(32)

Substituting \( \lambda_0^2 \) of Eq. (19) in Eq. (32), we get \( \sigma_{tot} = 6 \times 10^{-26} \, \text{cm}^2 \), where \( h = 1.054 \times 10^{-27} \, \text{erg.s} \) and \( c = 2.9979 \times 10^{10} \, \text{cm/s} \) a value close to the modern experimental value.

In treating the collisions of extremely high energy nucleons Fermi make use of thermodynamic laws, instead of considering a detailed statistical treatment. The energy density around the colliding nucleons is so high that multiple pions as well as antiprotons will be produced.

From Planck law, the spectral intensity (dimension \( I_\nu(v,T) \rightarrow dE/\omega dA d\Omega d\nu \)) of the black body is given as

\[
I_\nu(v,T) = \frac{2 h v^3}{c^2} \frac{1}{e^{\pi k T/2} - 1},
\]

(33)

where \( k \) is the Boltzmann constant, \( v \) the frequency and \( T \) the temperature.

From Stefan-Boltzmann law of the black-body (radiation flux \( R(T) = \sigma T^4 \), dimension \( dE/dt dA \)), the energy density \( \rho(T) \) (dimension \( dE/dV \)) is

\[
I(T) = \int_0^\infty I_\nu(v,T) d\nu = \frac{c}{4 \pi} \rho(T) = \frac{1}{\pi} R(T) \Rightarrow
\]

\[
\rho(T) = \frac{4 \pi}{c} \int_0^\infty I_\nu(v,T) d\nu = \frac{4}{c} \sigma T^4,
\]

(34)

\[
\rho(T) = \left( \frac{\pi^2}{15 \pi^3 h^3} \right) (kT)^4 = \left( \frac{6.494}{\pi^2 \cdot 3^3 h^3} \right) (kT)^4,
\]

(35)

where \( \pi^4/15 = 6 \sum 1/\pi^4 = 6.494 \) (from Gamma and Riemann Zeta functions, respectively, \( \Gamma(z) \) and \( \zeta(s) \), \( \sigma \) is the Stefan-Boltzmann constant and \( I(T) \) the spectral intensity integral in all frequency of the black-body. According to Fermi: “Consequently the Stefan’s law for the pions will be quite similar to the ordinary Stefan’s law of the black-body radiation. The difference is only in a statistical weight factor. For the photons the statistical wight is the factor 2, because of the two polarization directions. If we assume that the pions have spin zero and differ only by their charge \( \pm e \) or 0, their statistical weight will be 3. Consequently, the energy density of the pions will be obtained by multiplying the energy density of the ordinary Stefan’s law by the factor 3/2.” Then, multiplying the energy density (35) by 3/2, the energy density via pions is

\[
\rho_\pi(T) = \frac{3}{2} \rho(T) = \frac{3 \times 6.494}{2 \pi^2 h^3 c^3} (kT)^4,
\]

(36)

The total energy of the system (Eq. 25) is divided among pions, protons and anti-protons. Then it is necessary to get the energy density via protons and anti-protons. In this case is used Planck law modified (for fermions) as

\[
I_{\text{current}}(E,T) = \frac{2}{\hbar^2 c^2} \frac{E^3}{e^{E/kT} - 1},
\]

(37)
and
\[ \rho_{\text{fermion}}(T) = \frac{4\pi}{c} \int_{0}^{\infty} I_{\text{fermion}}(E, T) \frac{dE}{\hbar}. \] (38)

It is necessary to use the Planck law modified because the protons and anti-protons are fermions, where they obey Fermi-Dirac statistical. According to Fermi: “The contribution of the nucleons and anti-nucleons to the energy density is given by a similar formula. The differences are that the statistical weight of the nucleons is eight since we have four different types of nucleons and anti-nucleons and for each, two spin orientations. A further difference is due to the fact that these particles obey the Pauli principle.” Then it is necessary to multiply the Stefan-Boltzmann law by fermions to 8/2. The process is \( pp \rightarrow \pi + X \), where “X” represent the protons and anti-protons, then the energy density via protons and anti-protons is
\[ \rho_x(T) = \frac{8}{2} \rho_{\text{fermion}}(T) = \frac{4 \times 5.682}{\pi^2 \hbar^3 c^3} (kT)^4 \] (39)

where \( 6 \sum (-1)^{n+1}/n^4 = 5.682 \).

The total energy density \( \rho_{\text{tot}} \) of the system during the collision is given by sum \( \rho_{\text{tot}} = \rho_\pi + \rho_x \), but also it is the energy of c.m. divided per volume, \( \rho_{\text{tot}} = \sqrt{\frac{s}{\Omega}}. \) Then
\[ \rho_{\text{tot}} = \frac{\sqrt{s}}{\Omega_0} = \rho_\pi + \rho_x. \] (40)

Substituting Eqs. (36) and (39) in Eq. (40)
\[ \rho_{\text{tot}} = \frac{\sqrt{s}}{\Omega_0} = \frac{3 \times 6.494}{2 \pi^2 \hbar^3 c^3} (kT)^4 \]
\[ + \frac{4 \times 5.682}{\pi^2 \hbar^3 c^3} (kT)^4, \]
\[ (kT)^4 = 0.152 \frac{n^3 c^3}{m_p c^2 \Omega}. \] (41)

Note that the energy density is frame invariant, \( \rho_{\text{tot}} = E_{\text{tot}}/\Omega_0 = E_{\text{tot}}/\Omega \), because cancel the Lorentz factors.

Analogous to the Eqs. (33) and (34) it has the definition of numerical density of the black-body radiation is [see 190]
\[ n(T) = \frac{4\pi}{c} \int_{0}^{\infty} I_{\nu}(\nu, T) \frac{d\nu}{\hbar \nu} = \frac{8\pi}{c^3} \int_{0}^{\infty} \frac{\nu^2 d\nu}{e^{\nu/kT} - 1}, \]
\[ n(T) = \frac{1}{\pi^2} \left( \frac{kT}{\hbar} \right)^3 \int_{0}^{\infty} \frac{x^2 dx}{e^x - 1} = \frac{\Gamma(3) \zeta(3)}{\pi^2} \left( \frac{kT}{\hbar} \right)^3, \]
\[ n(T) = 0.243576 \frac{(kT)^3}{\hbar^3 c^3}, \] (42)

where \( x = h\nu/kT. \) In the case of pions, \( \frac{\text{num}}{\text{tot}} \), it is necessary multiply the last equation by 3/2 (analogous Eq. 36), as
\[ n_{\pi}(T) = \frac{3}{2} n(T) = 0.365 \frac{(kT)^3}{\hbar^3 c^3}, \] (43)

where Fermi [21] got also the last expression.

Substituting Eq. (41) in Eq. (42), we get
\[ n_{\pi}(\sqrt{s}) = 0.0888 \left( \frac{s}{\hbar c m_p c^2 \Omega} \right)^{1/4}. \] (44)

We can define the multiplicity of pions per collision \( N_{\pi}^{\text{num}} \rightarrow \# \text{pions} \), for high energy, being (using Eq. 31)
\[ n_{\pi}^{\text{num}} = \frac{N_{\pi}^{\text{num}}}{\Omega_0} \Rightarrow N_{\pi}^{\text{num}} = \frac{2 m_p c^2}{\sqrt{s}} \Omega n_{\pi}^{\text{num}}. \] (45)
Substituting Eq. (44) in Eq. (45)

\[ N_{HE}^{\pi} (\sqrt{s}) = 0.1777 \left( \frac{m_{\pi} c^2 \Omega \sqrt{s}}{h c^3} \right)^{1/4}. \]  

(46)

According Eqs. (18) and (19),

\[ \Omega = \frac{4\pi a^3 h^3 c^3}{3(m_{\pi} c^2)^3}. \]  

(47)

Substituting Eq. (47) in Eq. (46)

\[ N_{HE}^{\pi} (\sqrt{s}) = 0.2542 \left[ \frac{m_{\pi} c^2 a^3}{(m_{\pi} c^2)^3} \right]^{1/4}. \]  

(48)

The pion rest masses are different, \( m_{\pi^0} c^2 = 0.135 \text{GeV} \) and \( m_{\pi^+} c^2 = 0.139 \text{GeV} \). Fermi got that when considerer the conservation of angular momentum, it has the effect of reduction the numbers of pions and nucleons, then he found a factor obtained numerically of 0.51. The total energy via pions is divided approximately equal among \( \pi^0, \pi^- \) and \( \pi^+ \), then multiplying and dividing Eq. (48) per, respectively, 0.51 and 3, it gets the \( \pi^0 \) and \( \pi^\pm \) multiplicities

\[ N_{HE}^{\pi^0} (\sqrt{s}) = 0.2542 \left[ \frac{m_{\pi^0} c^2 a^3}{(m_{\pi^0} c^2)^3} \right]^{1/4}. \]  

(49)

Doing \( m_{\pi^0} c^2 = 0.14385 m_{\mu} c^2 \) and \( m_{\pi^\pm} c^2 = 0.14875 m_{\mu} c^2 \) in the least two equations,

\[ N_{HE}^{\pi^0} (\sqrt{s}) = 0.185 a^{3/4} \sqrt{s} / m_{\mu} c^2, \]  

(51)

\[ N_{HE}^{\pi^\pm} (\sqrt{s}) = 0.180 a^{3/4} \sqrt{s} / m_{\mu} c^2, \]  

(52)

\[ N_{HE}^{\pi_{total}} (\sqrt{s}) = 0.546 a^{3/4} \sqrt{s} / m_{\mu} c^2. \]  

(53)

where \( N_{HE}^{\pi_{total}} = 2N_{HE}^{\pi^0} + N_{HE}^{\pi^\pm} \), can note that the lest value is the same in Fermi [21].

The center of mass energy is according Eqs.(25)-(27). Substituting in Eq. (48), we get the equation of Fermi to the \( \pi^0 \) multiplicity for extreme high energies

\[ N_{HE}^{\pi^0} (E_p) = 0.777 a^{3/4} \left( 1 + \frac{E_p}{m_{\mu} c^2} \right)^{1/4}. \]  

(54)

Doing the same procedure from Eq. (45) to (51),

\[ N_{HE}^{\pi^0} (\sqrt{s}) = 0.6533 a^{3/4} \left( \sqrt{s}/m_{\mu} c^2 - 2 \right)^{3/2}. \]  

(55)

Substituting Eq. (25) in Eq. (55), we get the equation of Fermi to the multiplicity of pions in intermediate energy range,

\[ N_{HE}^{\pi^0} (E_p) = 0.777 a^{3/4} \left( \sqrt{1 + E_p/2m_{\mu} c^2} - \sqrt{2} \right)^{3/2}. \]  

(56)

**Modern approach**

Currently the modeling of pp interactions is done through computational codes, Monte Carlo codes such as SIBYLL, PHYTHIA, Dpmjet. Kelner [191] presented new parameterizations of energy spectra of secondary particles, \( \pi \) and \( \eta \).
mesons, gamma rays, electrons, and neutrinos produced in inelastic proton-proton collisions based on the SIBYLL code by Lipari [192]. These parameterizations have very good accuracy in the energy range above 100 GeV (see figures 46 and 47).

The fit to the pp cross section obtained by Kelner et al. [191] is

\[
\sigma_{pp}(E_p) = (34.3 + 1.88L + 0.25L^2) \left(1 - \frac{E_{\text{th}}}{E_p}\right)^2 \text{mb},
\]

where \(E_p\) is the incident proton energy in laboratory frame (the same that \(E_p\) in the last section), \(L = \ln\left[E_p/(\text{TeV})\right]\) and \(E_{\text{th}}\) is the minimum threshold energy of the incident proton for production of a pion (\(E_{\text{th}} = 1.22\text{GeV}\)) and \(1\text{mb} = 1\text{mbarn} = 10^{-27}\text{cm}^2\).

The function that Kelner got for the multiplicity of pions “\(N_\pi\)” is given as

\[
dN_\pi(x, E_p) = F_\pi(x, E_p) dx,
\]

\[
F_\pi(x, E_p) = d \phi(x, E_p) dx,
\]

\[
\phi_{\text{SIBYLL}} = -B_\pi \left(1 - \frac{x^{\beta \gamma}}{1 + k_\gamma x^{\beta \gamma}(1-x^{\beta \gamma})}\right)^4,
\]

where \(E_{\pi}\) is the total energy via neutral pions productions (secondary pions mesons), \(x = E_{\pi}/E_p\), and \(B_\pi\), \(\beta \gamma\) and \(k\gamma\) are functions dependent only of \(E_p\) in the Kelner parametrization [191, see graphic 47]. Figure 47 shows the spectrum distribution \(xF_\pi(x, E_p)\) of \(\pi^0\) production as a function of \(x = E_{\pi}/E_p\), which is the percentage of incident proton energy transferred to the pions. Note in Fig. 47 shows the small probabilities of the \(\pi^0\) productions at small energy transformation \(x \leq 0.0015\) and large energy transformation \(1 \geq x \geq 0.7\ E_p\). The maximum probability is at \(x = 0.03\), indicating the approximate 3% of incident proton energy is transferred via neutral pion \(\pi^0\). Since the experimental data show that the production of neutral pions \(\pi^0\) is practically the same as the productions of positive \(\pi^+\) and negative \(\pi^-\) charged pions, then we get that approximate 10% of incident proton energy \(E_p\) is transferred via pions (\(E_\pi\)).

Integrating Eq. (58) over \(x\), Kelner obtained [191] the multiplicity (number) of \(\pi^0\) per pp collision as a function of \(E_p\)

\[
N_{\pi^0}^p(E_p) = 3.92 + 0.83L + 0.075L^2.
\]
Comparison between Fermi’s and Kelner-SIBYLL’s approaches

We compared the multiplicities of neutral pions $\pi^0$ obtained the Fermi theoretical approach (Eqs. 54 and 56) and the Kelner analytical parametrization (Eq. 61) to the SIBYLL code [192]. The analytical parametrization of Kelner has good agreement with the SIBYLL code for energy $E_p > 100\, \text{GeV}$.

In the figure 48 we compare the Fermi results in high-energy region (Eq. 54) and median-energy region (Eq. 56) with the description of Kelner-SIBYLL in intermediate energy range 50 – 300 GeV. We used parameter $a = 5$, which give a very good agreement between Fermi result in high energy and Kelner-SIBYLL result in the range 100 – 300 GeV. But Fermi result in median energy Eq. (56) gives lower multiplicity of $\pi^0$ in this energy range.

In the figure 49, we use $a = 5$ and compare the Fermi result for high energy (54) with Kelner-SIBYLL result in the energy range 300 – 1000 GeV. We find that Fermi result is about 8% larger than Kelner-SIBYLL result, this indicates that the validity of the Fermi approach in extremal high-energy range is in question. Because the cross-section grows with $E_p$ (see Figure 1), and the $a$ parameter by its definition should become larger with larger as the cross-section grows in this energy range. This makes that Fermi result further deviate from Kelner-SIBYLL result. One of reasons could probably be the fact that more particles, e.g., gluons and quarks, are exited and participate the thermalization in the Fermi volume $\Omega$, as a result, the energy transferred to $\pi$-productions is smaller than that estimated by Fermi with
three particles proton, neutron and pion.

**Maximum and minimum energy of the Pions**

In this section we will show the energy limits of the pions created via pp interactions and we will apply the limits in the work of Blattnig et al. [193].

The energy of a pion in the laboratory frame (LF) in function of its energy in center of mass (c.m.) frame, is as

\[ E_\pi = \gamma (E_{\pi 0} + v p_{\pi 0} \cos \theta) \tag{62} \]

where \( \gamma \) is the Lorentz factor, “v” the velocity of the pion, \( E_{\pi 0} \) the pion energy in the c.m. frame and \( p_{\pi 0} \) the pion momentum in the c.m. frame (the index “0” inform c.m. frame and without the index “0” give in the LF). We will consider \( c = 1 \).

If \( \cos \theta = 1 \) the pion energy is maximum (\( E_{\pi \text{max}} \)), and if \( \cos \theta = -1 \) the pion energy is minimum (\( E_{\pi \text{min}} \)). Then the maximum and minimum energy of the pion is,

\[ E_{\pi \text{max}} = \gamma (E_{\pi 0} + v p_{\pi 0}) \tag{63} \]

Developing the calculus we obtain the maximum and minimum energy of the pions,

\[ E_{\pi \text{max}} = \frac{1}{4m_p} (2m_p E_p - 2m_p^2 + m_\pi + R_p) \tag{64} \]

\[ E_{\pi \text{min}} = \begin{cases} \frac{1}{4m_p} (2m_p E_p - 2m_p^2 - m_\pi^2 - m_\pi^2) & \text{if } E_p > E_p^* \\ m_\pi & \text{if } E_p \leq E_p^* \end{cases} \tag{65} \]

where

\[ R_p = \sqrt{(2m_p E_p - 2m_p^2 - m_\pi^2 - m_\pi^2 - m_\pi^2)} \tag{66} \]

\[ E_p^* = \frac{2m_p^2 + 2m_p m_\pi - m_\pi^2}{2(m_p - m_\pi)} \approx 1.242 \text{ GeV} \tag{67} \]

where \( E_p^* \) is the limit (threshold) of energy in the case that \( E_\pi = m_\pi \).

In Fig. 50a, we can see the behavior of the Eqs. (64) and (65), where they are the maximum and minimum limits of the neutral pion energy created. We can note that the minimum limit is approximated of \( E_{\pi \text{min}} = 0.478 \text{ GeV} \) for energy of incident proton \( E_p = 10^8 \text{ GeV} \), where \( E_{\pi \text{min}} \) increase very softly. It is possible note also that the maximum limit tends to \( E_{\pi \text{max}} = E_p - 3m_p/2 \).
Blattnig et al. [193] obtains the spectral distribution of $\pi^0$ $[d\sigma/dE_\pi (\text{mb/GeV})]$ in function of the kinetic energy of $\pi^0$ created ($T_p$), for energy of incident proton $E_p \leq 50$ GeV. Blattnig obtain the spectral distribution and total cross section of $\pi^0$ for seven different kinetic energies of incident protons, $T_p = 0.5, 1.0, 1.9, 5.0, 9.5, 20, 50$ GeV. The analytical function that Blattnig got is

$$
\frac{d\sigma}{dE_\pi} = \exp \left( K_1 + \frac{K_2}{E_p} + \frac{K_3}{T_p^2} + \frac{K_4}{T_p^4} \right),
$$

(68)

where $K_1 = -5.8$, $K_2 = -1.82$, $K_3 = 13.5$, and $K_4 = -4.5$. The last expression is the analytical fit of a numerical integration [193]. In Fig. 50b we apply the maximum (64) and minimum (65) limit energies of the $\pi^0$ in the spectrum distribution of Blattnig et al. [193] (Fig. 50b), and we obtained results that contradict their limits of energy, where it shows problems in minimum and maximum energies.

Similar conclusions are being explored using the PYTHIA code [194], and results will be presented soon [195].

**CONCLUSIONS**

Our current understanding of GRBs is based on a general picture which was presented in a set of letters [4, 5, 6]. On that basis a canonical GRB scenario has emerged, with three distinct phases:

1. The vacuum polarization process occurring in the gravitational collapse to a black hole, and the consequent creation of an electron-positron plasma.
2. An optically thick fireshell characterized by the self-acceleration of such an optically thick electron-positron plasma, with the engulfed baryon loading. This phase ends with the reaching of transparency, when the P-GRB is emitted.
3. An optically thin fireshell characterized by an accelerated beam of protons and electrons with a Lorentz $\gamma$ factor roughly inversely proportional to the baryon loading, interacting with the CBM.

This basic scenario is currently evolving in a large number of theoretical details, ranging from a) the thermalization process of electrons and positrons after their production, to b) the dynamics of the electron-positron pairs in the optically thick phase and their instabilities, to c) the probing and determination of the CBM distribution around the gravitationally collapsed object by the interaction of the ultrarelativistic baryons and electrons colliding with the CBM.

Many of the properties of the observed extended afterglow X- and $\gamma$-ray emission below $\sim 1$ MeV has been obtained by postulating a thermal spectrum of the emission process in the co-moving frame. This treatment is particularly appealing since it allowed to derive explicit analytic formulas to compute all the relativistic transformations between the co-moving, the laboratory frame and the arrival time of the signal. This approach has been very satisfactory in...
explaining the overall bolometric luminosity of the sources, its evolution as a function of the arrival time, and the CBM filamentary structure. With the improvement of the observational techniques, more and more time resolved spectra have been observed and our approach shows some discrepancy in the low and high energy tails of the time resolved spectra, although the time integrated spectral distribution is very well recovered. We are currently evolving this basic mechanism by assuming some departures from a pure thermal spectral shape in the co-moving frame.

This analysis has led to a clear identification of sources occurring in CBM with average density \( \rho_{\text{cbm}} \sim 1 \text{ particle/cm}^3 \) and with \( \rho_{\text{cbm}} \sim 10^{-3} \text{ particles/cm}^3 \). The first ones correspond to sources occurring in star forming regions in the host galaxy, and the second ones to sources occurring in the galactic halo. This problematic has led to a new understanding of the traditional separation between long and short GRBs, as exemplified in these lectures.

We are now approaching, in view of the new data from the Fermi and AGILE satellites, an analysis of the GRB radiation over 1 MeV. It is by now clear that the emission process previously considered of a purely thermal spectrum is not appropriate to the description of this high-energy component.

In parallel, we are currently examining how Fermi ideas [21] have been further developed in large data analysis procedures at CERN and other accelerators all over the world (see the last section of this paper). We are going to probe, by fitting the observational data, an higher density nodule component in the CBM, which will add to the understanding of the CBM itself and will lead very likely to the explanation of GRB sources.

The major effort now is also directed to the understanding of the process of the electrodynamics of gravitational collapse.

REFERENCES


Bibliography


Bernardini, M. G., Bianco, C. L., Chardonnet, P., et al., 2005a, World Scientific, 2459


BIBLIOGRAPHY

Blandford, R.D., 2002, Lighthouses of the Universe, ed. R. Sunyaev Berlin:Springer-Verlag
Burd, A., Cwiok, M., Czyrkowsky, H., et al., 2005, NewA, 10, 409
N. Chamel, P. Haensel, 2008, LRR, 11, 10


Couderc, P. 1939, Ann. Astroph., 2, 271

Covino, S., Vergani, S., Malesani, D., et al., 2008, ChJAS, 8, 356


Cummings, J., Barthelmy, S.D., Fenimore, E., et al., 2008, GCN Circular 7462


de Barros, G., Vereshchagin, G. V., Ruffini, R., submitted to A&A


Della Valle, M., Chincarini, G., Panagia, N., 2006, Nature, 444, 1050


Fermi, E., 1927, Rend. Accad. Lincei, 6, 602


Golenetskii, S., Aptekar, R., Mazets, E., et al., 2008, GCN Circular 7482

W. Heisenberg and H. Euler, 1936, ZPhy, 98, 714
Kawai, N., Yamada, T., Kosugi, G., et al., 2005, GCN Circular 3937
Krenrich, F., Blaylock, G., Bradbury, S. M., et al., 2007, JPhCS, 60, 34
Lyutikov, M., 2002, Phys. Fluids, 13, 963
Lyutikov, M., 2006, NJPh, 8, 119
Lyutikov, M., Blandford, R. D., 2002, “Electromagnetic Outflows, GRBs” in
Beaming, Jets in Gamma Ray Bursts, ed. R. Ouyed, J. Hjorth, A. Nordlund
Masetti, N., Palazzi, E., Pian, E., Patat, F., 2006, GCN Circular 4803
Mészáros, P., 2006, RPPh, 69, 2259
Migdal, A. B., Voskresenskii, D. N., Popov, and V. S., 1976, JETPL, 24, 186
Müller, B., Peitz, H., Rafelski, J., Greiner, W., 1972, Ph. Rev. Lett., 28, 1235
BIBLIOGRAPHY


Page, K., Goad, M., Beardmore, A., 2006, GCN Circular 5240


Pieper, W., Greiner, W., 1969, Z. Phys., 218, 327


Piran, T., 2004, Rev. Mod. Phys. 76, 1143


Popov, V. S., 1971, JETP, 32, 526


412
Ruderman, M., 1975, NYASA, 262, 164
Ruffini, R., Bernardini, M. G., Bianco, C. L., et al., 2004a, Advances in Space Research, 34, 2715
Ruffini, R., Bianco, C.L., Chardonnet, P., et al., 2004, IJMPD, 13, 843
Ruffini, R., Bianco, C.L., Chardonnet, P., et al., 2005a, IJMPD, 14, 97
BIBLIOGRAPHY

Ruffini, R., Rotondo, M., Xue, S. -S., 2007, IJMPD, 16, 1
F. Sauter, 1931, ZPhy, 69, 742
J. Schwinger, 1951, PhRv, f82, 664
J. Schwinger, 1954, PhRv, 93, 615
J. Schwinger, 1954, PhRv, 94, 1362


Thomas, L. H., 1927, Proc. CPS, 23, 542


Tueller, J., Barbier, L., Barthelmy, S., et al., 2006, GCN Circular 5242


Usov, V.V., 1992, Nature, 357, 472


BIBLIOGRAPHY


Vreeswijk, P.M., Smette, A., Malesani, D., et al., 2008, GCN Circular 7444


Yost, S. A., Aharonian, F., Akerlof, C. W., et al., 2006, AN, 327, 803


Zel’dovich, Ya B., Popov, V. S., 1972, Sov. Phys. USP, 14, 673

