Semidegenerate system of fermions as dark matter on galaxies and
A multi-wavelength catalog of TeV candidate blazars

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Abstract

I present here a model of selfgravitating semidegenerate fermions as a unified model for dark matter halos and for compact objects in the center of galaxies, usually thought to be black holes. This model gives naturally flat rotation curves and can reproduce observables for a particle mass of a few keV. We use some universality laws (laws obeyed by galaxies of different magnitudes and masses) to constrain our free parameters. It is shown that these systems reach a maximum core mass, and that this critical mass is reached in different regimes: degenerate systems where changing the temperature has almost no influence on the core and systems where the contribution of the thermal pressure is appreciable. The sterile neutrino is a viable particle candidate for this model.

On a second part it is shown how the 1WHSP, a multi-wavelength catalog of TeV candidate blazars based on the WISE survey was built. It is currently the largest catalog high synchrotron peaked blazars with 713 blazars and blazar candidates. We show that selecting blazars using the WISE blazar strip leads to severe incompleteness and present some evidence against the blazar sequence. Our sample includes many TeV emitters that could already be identified by current telescopes, at least in a flare state, and some that can surely be detected by CTA.
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Chapter 1

Introduction

The fact that there is some unseen matter in clusters was known since the 30s, when the velocities of individual galaxies could not be explained by the visible matter alone. In the beginning it was thought to be just low mass stars and gas that could not be detected. However, with the advances in detection techniques rotation curves of galaxies were shown to be flat far away from the center and it became increasingly clear that some type of matter that doesn’t emit light actually exists and it is distributed to larger distances than visible matter. It is usually said that every galaxy has a dark matter halo which is roughly spherical. Many candidates have been proposed over the years, from the three families of active neutrinos to supersymmetric particles to extra dimensions and modified theories of gravity. So far, dark matter (DM) has escaped detection and there is no preferred candidate.

It is widely believed today that every galaxy harbors a SMBH, not just AGNs, even if not in an active phase. Observations of orbits and emission from the central parts of galaxies seem to corroborate this view. However, observations don’t go deep enough (near the Schwarzschild radius) to rule out completely a compact object with a radius similar to the BH radius, like it was proposed by [1, 2].

On a yet completely unrelated field, Active Galactic Nuclei (AGN) remain one of the most challenging astrophysical objects. Its emission appears to be powered by a supermassive black hole (SMBH) and observations indicate a dust
torus around the accretion disk that obscures the emission at some angles. The
exact mechanism of emission remains a hot topic until today despite the massive
amount of data we have.

With this in mind, this thesis is divided in two parts: in chapter 2 I give a brief
introduction to the discovery of DM and the evidence we have for it today, describe
how neutrinos were thought as the first DM candidate and describe the different
classifications for DM. In chapter 3, I present a unified model of a semidegenerate
system of fermions that tries to account for the DM in galaxies but it is also
an alternative to the BH in the center of galaxies, concluding the first part. In
chapter 4 I give a brief review of AGNs observational properties and classification
and finally in chapter 5 I describe the building and properties of the 1WHSP, a
catalog of over 700 TeV candidate blazars, finishing the second part. The last
chapter is for conclusions and future work.
Chapter 2

Dark matter

2.1 Evidence

In 1933, the swiss astronomer Fritz Zwicky, observing the Coma cluster, noticed that the velocities of the galaxies ranged over 1000 km/s. By using the virial theorem, he was able to estimate the mass of the cluster at $3 \times 10^{14} M_\odot$, about a hundred times larger than the mass that could be accounted for by the galaxies. This lead him to conclude that a considerable amount of non-radiating matter should be present in the cluster. Three years later, Sinclair Smith repeated the same analysis for the Virgo cluster, and obtained a similar conclusion: there was considerable more mass than light, i.e., the mass-to-light ratio was more than 100, which is hundreds of time larger than in the solar neighbourhood. Since both these conclusions are based on the use of the virial theorem, considering that the system is not in equilibrium could in principle solve the problem. However, if a system was dominated by kinetic energy, it should dissipate in a few billion years, making it impossible for so many clusters to be present. If the system is dominated by its potential energy, the problem gets worse, since there should be even more unseen mass. Another possibility advanced by Zwicky to explain the missing mass was that this mass was not yet detected in the intergalactic medium. Although it was not realized at the time, this was the discovery of dark matter in clusters.

One year before Zwicky, the dutch astronomer Jan Oort already had seen some
discrepancies between observed and inferred mass in the milky way. He calculated that only one third of the dynamically inferred mass was present in bright stars. He suggested, however, that this missing mass was present in low-mass stars and in the interstellar medium. This was the first hint of DM in individual galaxies. In the mid-thirties, some rotation curves for M31 were obtained, and the expected keplerian behaviour far from the center was not observed, further strengthening the idea of some missing mass. With the advent of radio astronomy and the discovery of a new window of observation (the 21cm line of hydrogen), the rotation curve of galaxies could be measured beyond the optical image, and it was found to have a very slight decrease. This in turn implies that the mass-to-light ratio in the outer parts of galaxies should increase. This problem was considered an observational issue (better detection techniques would clear the issue by showing an additional component of the interstellar medium) and not given much attention.

In the seventies, Ostriker and Peebles showed that the milky way would be an unstable system if not for a spherical dark halo, with a mass-to-light ratio a lot larger than that of a stellar population. Later, with Yahill, they showed that the mass enclosed in the milky way increases linearly with the radius for regions far from the center. More dark matter is required to bind larger systems, so that the dark region extends farther than the inner regions, and the interior mass keeps increasing with distance. This would make the mass-to-light ratio of the outer portions of a spiral galaxy comparable to the average value estimated by Zwicky to the galaxies in the Coma cluster. They speculated that the component of this dark halo, being the dominant component is the galaxies, could make the universe flat; this was a huge extrapolation at the time, that the matter that composes galaxies could have also cosmological significance. However, at this point, no one knew what it was composed of.

Another independent evidence for DM comes from big bang nucleosynthesis (BBN) and the cosmic microwave background (CMB). Standard BBN predicts a relation between the abundance of light elements (mainly Helium and Deuterium) and the barion density \( \omega_b \equiv \Omega_b h^2 \), with \( H_0 = 100 h \text{km/s/Mpc} \) the Hubble parameter and \( \Omega_b \equiv \frac{8 \pi G \rho_m}{3 H_0^2} \) (see Ref.[3] for a review on standard BBN). With this relation,
Planck (Ref. [4]) finds a baryon density of

\[ w_b = 0.022 \pm 0.00027 \]

which gives for the abundances of Deuterium and Helium

\[ \frac{4n_{He}}{n_b} = 0.24725, \quad \frac{10^5 n_D}{n_H} = 2.656 \]  

(2.1.1)

in agreement with the data from other sources, such as clouds absorbing the light of distant quasars. This strongly supports standard BBN. However, the total matter content in the CMB as measured by Planck is

\[ w_m = 0.143 \pm 0.003 \]

which is larger than the baryon fraction. This implies that there is still some unseen matter permeating the universe, and that this matter is most likely non-baryonic, giving more strength to the DM hypothesis.

### 2.2 The first candidate: the neutrino

The first true DM candidate, i.e., matter that does not emit light and not yet unobserved low-brightness objects, was the neutrino. Since it interact very weakly with matter via the weak force it could escape detection. Also, with the discovery of the cosmic microwave background radiation giving more strength to the fact that the universe was very hot a long time ago, there should be also a background of neutrinos, with a slightly lower temperature but with a considerable number density. This also means that cosmology could put contraints on the mass of the neutrinos.

The main reaction that maintain the neutrinos coupled to the plasma is

\[ n \to p + e^- + \bar{\nu}_e. \]  

(2.2.2)

This reaction is innefective below \( kT \approx 2 \) MeV, so the neutrinos decouple from the plasma while still relativistic and their momenta redshifts as \((1+z)\). However, after the decoupling, the temperature drops below the electron mass and the plasma is
reheated by the annihilation of positrons and electrons. Since the neutrinos are
decoupled they don’t share this increase in entropy; therefore, the temperature of
the neutrino background is different from the CMB by

\[
\frac{T_{\nu}}{T_0} = \left( \frac{4}{11} \right)^{1/3}
\]

which gives us \(T_0 \approx 1.9 \text{ K}\). With this, it’s easy to calculate the density of neutrinos
today. Using the argument that the total density of the neutrinos cannot be larger
than the critical density of the universe \(\frac{3H_0^2}{8\pi G}\), Zeldovich and Gershtein ([5]) found
that

\[
m_{\nu_{\mu}}(m_{\nu_e}) \leq 400 \text{ eV}.
\]

which, while not an improvement for the electron neutrino, reduced the upper
limit on the muonic neutrino mass by two orders of magnitude.

Later on Cowsik and McClelland ([6]), considering that the energy of the neu-
trinos is necessary to close the universe, provided a more stringent upper limit on
the mass of the neutrino of \(8 \text{ eV}\).

All these works conclude that if neutrinos are to be the DM particle, it should
have a mass of a few eV. This, however, is only valid for neutrinos with masses
well below 1 MeV, the decoupling temperature; for a neutrino heavier than this
they would already be much rarer than photons at the time they go out of thermal
equilibrium, and so their density would be smaller than the critical density. Lee
and Weinberg ([7]) considered such heavy neutrino (called \(L^0\)) that is stable and can
only annihilate with its own antiparticle through

\[
L^0 \bar{L}^0 \rightarrow \nu \bar{\nu}, e^- e^+, \mu^+ \mu^-, \pi^+ \pi^-, \text{ etc.}
\]

This collisions keep the energy distribution of the heavy neutrino thermalized down
to a temperature of 1 MeV, but at this time they are rare enough so that the rate
of collisions is less than the expansion rate. So the heavy neutrinos go out of
chemical equilibrium at a temperature \(T_f > 1 \text{ MeV}\). They consider temperatures
around 10-100 MeV, so that the only degrees of freedom are photons, electrons,
positrons and neutrinos and the annihilation processes are governed by Fermi-
like interactions. In this regime the cross section is inversely proportional to the
2.2. \textsc{The First Candidate: The Neutrino}

velocity, so that $\langle \sigma v \rangle$ is a constant independent of the temperature. To make the problem quantitative, it is useful to use the rate equation

$$\frac{dn}{dt} = -3H - \langle \sigma v \rangle (n^2 - n_0^2),$$

(2.2.5)

where \(H\) is the Hubble parameter, \(n\) is the number density and \(n_0\) is the number density of the neutrinos in chemical and thermal equilibrium.

By solving the equation and imposing that the density of the heavy neutrinos today do not exceed the critical density, it is found that

$$m_L \geq 2 \text{ GeV}.$$  

This was the birth of the WIMPs, Weakly Interacting Massive Particles, even though the name was coined later. The freezing temperature of these neutrinos should be at least 100 MeV.

Later on, Tremaine and Gunn ([8]), using the same arguments as [5, 6] estimated the contribution of neutrinos to the density as

$$\Omega_{\nu} = 0.04 \left( \frac{50}{H} \right)^2 (m_{\nu_e} g_{\nu_e} + m_{\nu_x} g_{\nu_x})$$

(2.2.6)

with \(g\) the multiplicity factor as masses are measured in eV. Then, considering that the neutrinos that comprise DM are present only in galaxies,

$$m_{\nu_e} g_{\nu_e} + m_{\nu_x} g_{\nu_x} \leq (1.2) \text{ eV}$$

(2.2.7)

obviously a much stringent limit than the previous ones. A lower limit can be obtained by considering that the neutrinos are distributed like isothermal gas spheres; then, their velocity distribution is maxwellian, and the maximum phase-space density is given by

$$\rho_0 m_\nu^{-4}(2\pi \sigma)^{-3/2},$$

with \(\rho_0\) the central density and \(\sigma\) the dispersion velocity (the masses are assumed to be equal). Since the neutrinos are noninteracting, the phase-space density must decrease with time from the maximum at \(2g_\nu h^{-3}\). This gives a lower limit on the mass of

$$m_\nu > (101 \text{ eV}) \left( \frac{100 \text{ km/s}}{\sigma} \right)^{1/4} \left( \frac{1 \text{ kpc}}{r_c} \right)^{1/4} g_\nu^{-1/4}$$

(2.2.8)
CHAPTER 2. DARK MATTER

with \( r_c = (9\sigma^2/4\pi G\rho_0)^{1/2} \) is the core radius of an isothermal sphere. A typical galactic halo has \( r_c \approx 20\text{kpc} \) and \( \sigma \approx 150 \text{ km/s} \). This gives \( m_\nu \gtrsim 20g_\nu^{-1/4} \text{ (eV)} \), inconsistent with Eq. 2.2.7. It is important to consider that, even if the neutrinos are not distributed as an isothermal sphere, there would still be a maximum phase-space density of \( 4g_\nu h^{-3} \) due to the Pauli principle, which gives an upper limit in the mass \( 2^{1/4} \) bigger than the one imposed by the decrease of the maximum phase-space density. This shows that the neutrinos cannot be DM in galaxies, or any other lepton of low mass. Other reasons why the neutrino cannot be the only DM will be shown in the next section.

2.3 Types of dark matter

Usually the types of DM can be classified by their velocities when they decouple from the primordial plasma (and hence their range of masses): low mass/high velocities is dubbed Hot dark matter (HDM), high mass/low velocities is Cold dark matter (CDM) and there is also an intermediate called Warm dark matter (WDM). We will analyse them in more detail.

2.3.1 Hot dark matter

The neutrino, the first DM candidate, is an example of HDM. It decouples from the primordial plasma while still relativistic, so their free streaming length is very high. Any perturbation below this distance are erased. The free streaming length can be estimated by the size of the horizon when the particle becomes non-relativistic. For a particle of a few eV, this happens at \( z \sim 10^5 \). At this point, the horizon is \( \sim 100 \text{ pc} \), which expands to tens of Mpc today. So, the first structures to form in the universe are comparable to superclusters of galaxies. Then, smaller structures would be formed by fragmentation of the larger ones, in a process called “top-down” structure formation. However, this means that galaxy-mass objects wouldn’t form until very late, and since we observe galaxies at high redshift, neutrinos (or any other type of HDM) cannot be the dominant form of DM.
2.3. TYPES OF DARK MATTER

2.3.2 Cold dark matter

After the realization that DM cannot be composed of low-mass particles, the proposition of Lee and Weinberg \[21\] gained more strength. Cold dark matter is composed of particles that are non-relativistic at decoupling, and so have masses ranging from MeV to TeV. Moreover, CDM has become part of the *Concordance* cosmological model, the paradigm today.

CDM is usually composed of WIMPs; since annihilation with its antiparticle is a possibility in this scenario, observing signals from galactic sources could be a way of constraining the cross-section and mass of the WIMPs. Another reason for WIMPs being a good DM candidate is the so called WIMP miracle: the density of thermally produced WIMPs today does not depend directly on the mass of the particle, but only on the cross-section: for typical cross-sections in the weak scale, the density of WIMPs is in agreement with the measurements, making WIMPs a very attractive DM candidate.

Because of the low velocities at decoupling, smaller structures form first then merge to form bigger ones: a bottom-up structure formation. Despite being the most successful model so far, CDM suffers from some problems.

- **The missing satellite problem:** since there is no cutoff scale, it is expected a lot of small-scale structures around galaxies, called satellite galaxies. However, despite some dwarf galaxies found in the milky way halo, the number is still small compared to simulations.

- **The core vs cusps:** CDM simulations, such as the ones done by Navarro, Frenk and White \[9, 10\], show a DM profile with a cusp, i.e., a profile that scales with $r^{-a}$ in the central regions. However, observations from dwarf spheroidal galaxies (dSphs) and low surface brightness galaxies (lsb), systems where DM dominates at all scales \([11, 12, 13]\), seem to favor a core \([14, 15, 16, 17]\).

Despite being the accepted paradigm, CDM apparently suffer from some crucial problems; it is not clear, however, if the problems are on a basic level or just the observations and simulations fail to account for every possible variable.
2.3.3 Warm dark matter

Warm dark matter (WDM) is an intermediate classification, with particle masses usually in the keV scale. At decoupling the velocities are not so large as to erase small-scale structure and not so small as to have no cutoff. WDM behaves just like CDM for large scales, but due to the velocities being larger than zero, can solve the missing satellite problem. So, despite the success of the CDM model, there are still some problems to be explained and WDM appears as a viable alternative. Usually, the mass of the WDM particle is given by the size of galaxies.
Chapter 3

Semidegenerate system of fermions

In this chapter, a unified model for DM halos and central objects is presented as a self-gravitating system of semidegenerate fermions, its characteristics and problems.

3.1 The model

The equilibrium configurations of a self-gravitating semi-degenerate system of fermions were studied by [18] in Newtonian gravity and by [19] in general relativity. It is shown that in any such system the density at large radii scales as $r^{-2}$ quite independently of the values of the central density, providing a flat rotation curve. They are, however, unbound and with infinite mass. In order to account for tidal effects present in galaxies, Ingrosso et.al. [20] imposed a cutoff in the velocity of the particles: at any given point, there is a maximum velocity for the particles in the system which is the velocity occurring for those particles to reach the boundary. This can be formulated in terms of a cutoff in the kinetic energy in the distribution function. Following [19] we are considering spherical symmetry and a static system, so the line element can be written in standard Schwarzschild
coordinates as
\[ ds^2 = -e^\nu(r)c^2dt^2 + e^\lambda(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \] (3.1.1)

Considering a perfect fluid, solving the field equations give us so called the Tolman-Oppenheimer-Volkoff (TOV) equations
\[ \frac{dP(r)}{dr} = -\frac{G}{c^2} \frac{(P(r) + \rho(r)c^2)(M(r) + 4\pi P(r)r^3)}{r(re^2 - 2GM(r))} \] (3.1.2)
\[ \frac{dM(r)}{dr} = 4\pi \rho(r)r^2, \] (3.1.3)
with \( M \) the mass within a radius \( r \), \( \rho \) and \( P \) the energy density and the pressure respectively (all of them depending on \( r \)), given by the Fermi-Dirac integrals
\[ \rho = m \frac{g}{h^3} \int_0^{\epsilon_c} \left( 1 + \frac{\epsilon}{mc^2} \right) \frac{1 - e^{(\epsilon - \epsilon_c(r))/kT}}{e^{(\epsilon - \mu)/kT} + 1} d^3p, \] (3.1.4)
\[ P = \frac{2g}{3h^3} \int_0^{\epsilon_c} \left( 1 + \frac{\epsilon}{2mc^2} \right) \left( 1 + \frac{\epsilon}{mc^2} \right)^{-1} \frac{(1 - e^{(\epsilon - \epsilon_c(r))/kT})\epsilon}{e^{(\epsilon - \mu)/kT} + 1} d^3p, \] (3.1.5)
with \( \epsilon_c(r) \) the cutoff energy, \( g = 2s + 1 \) the multiplicity factor, \( m \) the mass of the particle, \( T \) the temperature and \( \mu \) the chemical potential and it is implied that the last two depend on the radius. These integrals are expressed as a function of the one particle kinetic energy \( \epsilon = \sqrt{m^2c^4 + p^2c^2} - mc^2 \). It can be seen that the usual expression for the Fermi-Dirac integrals is recovered when we let the particles propagate to infinity, i.e, \( \epsilon_c \rightarrow \infty \).

The particle energy is a constant of motion, so
\[ (\epsilon + mc^2)e^{\nu/2} = \text{const}, \] (3.1.6)
while thermodynamical equilibrium (Tolman condition and Klein integral [21]) implies
\[ (\mu + mc^2)e^{\nu/2} = (\mu_R + mc^2)e^{\nu_R/2} \] (3.1.7)
\[ Te^{\nu/2} = T_Re^{\nu_R/2}, \] (3.1.8)
where the quantities with subscript “R” refer to the boundary of the configuration.
For the cutoff energy we have
\[ (\epsilon_c + mc^2)e^{\nu/2} = mc^2e^{\nu_R/2}, \] (3.1.9)
3.1. THE MODEL

since $\epsilon_c(R) = 0$.

Introducing the function $W = \epsilon_c/kT$ and the temperature parameter at the boundary $\beta_R = kT_R/mc^2$, and using eqs. (3.1.8) and (3.1.9) we can find that

$$\frac{mc^2}{kT} = \frac{1 - \beta_RW}{\beta_R}. \tag{3.1.10}$$

Note that the condition $0 \leq \beta_RW < 1$ has to be fulfilled. Using eq. (3.1.8) to substitute the temperature in eq. (3.1.10) we get the relation between the metric function $\nu$ and $W$:

$$e^\nu = e^{\nu_R} (1 - \beta_RW)^2 \tag{3.1.11}$$

so now the spacetime metric is completely determined:

$$e^\nu = e^{\nu_R} [1 - \beta_RW]^2, \quad e^\lambda = \left(1 - \frac{2GM}{rc^2}\right)^{-1} \tag{3.1.12}$$

with $\nu_R + \lambda_R = 0$.

Differentiating eq. (3.1.11) and using the conservation of the energy momentum tensor

$$\frac{dP}{dr} = -\frac{1}{2} (P + \rho c^2) \frac{d\nu}{dr} \tag{3.1.13}$$

gives

$$\frac{dP}{dr} = \frac{\beta_R(P + \rho c^2) dW}{1 - \beta_RW} \frac{dW}{dr} \tag{3.1.14}$$

and we can write eq. (3.1.12) as

$$\frac{dW}{dr} = -\frac{G}{c^2} \frac{1 - \beta_RW}{\beta_R} \frac{M c^2 + 4\pi Pr^3}{r(rc^2 - 2GM)} \tag{3.1.15}$$

In order to numerically integrate the final set of equations (3.1.3) and (3.1.13) with initial conditions $W(0) = W_0$ and $M(0) = 0$, it is useful to transform all of our physical variables into dimensionless ones:

$$\rho = \frac{c^2}{G\chi^2} \hat{\rho} \tag{3.1.16}$$

$$P = \frac{c^4}{G\chi^2} \hat{P} \tag{3.1.17}$$

$$M = \frac{c^2\chi}{G} \hat{M} \tag{3.1.18}$$

$$r = \chi \hat{r}, \tag{3.1.19}$$
CHAPTER 3. SEMIDEGENERATE SYSTEM OF FERMIONS

where

$$\chi = \frac{\hbar}{m_c} \left( \frac{m_p}{m} \right) \left( \frac{8\pi^3}{g} \right)^{1/2}$$

(3.1.20)

has dimension of length and \(m_p = (\hbar c / G)^{1/2}\) is the Planck mass.

It is instructive to write down the characteristic length \(\chi\), that is inversely proportional to the mass of the particle squared, in conventional units

$$\chi = 0.870m^{-2} \text{ pc},$$

(3.1.21)

where \(m\) is measured in keV/c^2; the unit of mass is

$$c^2G = 1.820 \times 10^{13}m^{-2}M_\odot,$$

(3.1.22)

where \(M_\odot = 1.989 \times 10^{33}\) g is the mass of the Sun.

We then obtain the dimensionless equations

$$\frac{dW}{d\tilde{r}} = -\left[ \frac{1 - \beta_R W}{\beta_R} \right] \frac{\dot{M}(r) + 4\pi \rho \tilde{r}^3}{\tilde{r}(\tilde{r} - 2\dot{M}(r))},$$

$$\frac{dM(r)}{dr} = 4\pi \tilde{r}^2,$$

(3.1.23)

where

$$\rho = 4\sqrt{2}\pi \left[ \frac{\beta_R}{1 - \beta_R W} \right]^{3/2} \int_0^W \left[ 1 + \frac{\beta_R x/2}{1 - \beta_R W} \right]^{1/2} \left[ 1 + \frac{\beta_R x}{1 - \beta_R W} \right]^2 \frac{1 - e^{-W}}{e^{-\theta} + 1} x^{1/2} dx,$$

$$\dot{\rho} = \frac{8\sqrt{2}}{3}\pi \left[ \frac{\beta_R}{1 - \beta_R W} \right]^{5/2} \int_0^W \left[ 1 + \frac{\beta_R x/2}{1 - \beta_R W} \right]^{3/2} \frac{1 - e^{-W}}{e^{-\theta} + 1} x^{3/2} dx,$$

(3.1.24)

with \(\theta = \mu / kT\) is the degeneracy parameter and we introduced the variable \(x = \epsilon / kT\). We have for this variable

$$\frac{\epsilon}{mc^2} = \frac{\beta_R x}{1 - \beta_R W}.$$

(3.1.25)

To find a relation between the cutoff parameter and the degeneracy parameter we can use Eq.(3.1.7) and (3.1.8) to get

$$\frac{mc^2}{kT} = \frac{1 - \beta_R(\theta - \theta_R)}{\beta_R}$$

(3.1.26)
where $\theta_R$ is the value of the degeneracy parameter at the boundary. When compared to Eq. (3.1.10) this gives us

$$W(r) = \theta(r) - \theta_R,$$  \hspace{1cm} (3.1.27)

so that $W(R) = 0$. We can relate the parameters in the boundary with those in the center (subscript 0)

$$\begin{align*}
\theta_R &= \theta_0 - W_0 \\
\beta_R &= \frac{\beta_0}{1 + \beta_0 W_0}
\end{align*}$$  \hspace{1cm} (3.1.28)

so that $\beta_R \approx \beta_0$ for $\beta_0 \ll 1$, evidencing the fact that for non-relativistic systems (rest mass larger than the kinetic energy), the temperature of the system is almost constant. Besides that we have

$$\frac{1 - \beta_R W}{\beta_R} = \frac{1 - \beta_0 (W - W_0)}{\beta_0}.$$  \hspace{1cm} (3.1.29)

We can now numerically solve the system (3.1.28) together with

$$\theta = \theta_0 + W - W_0$$  \hspace{1cm} (3.1.30)

with initial conditions $\dot{M}(0) = 0$, $W(0) = W_0$ and using eq. (3.1.24) with three independent parameters: $W_0$, $\theta_0$ and $\beta_0$. The only remaining free parameter is the mass of the particle, which occurs only in the definition of $\beta$ and the characteristic length $\chi$.

### 3.1.1 Properties of the equilibrium configurations

We have solved numerically the system of integral-differential equations given by (3.1.15), the two equations corresponding to $\beta$ and $\theta$ and (3.1.24), with a set of initial conditions $M_0$, $W_0$, $\beta_0$ and $\theta_0$. Galactic dark matter halos have asymptotic rotation velocities of the order of ten to thousands km/s, i.e., they are not relativistic. As that velocities are of the same order as the thermal velocities of the fermionic particles forming the halo, this means that $\beta_R \ll 1$ and consequently $\beta_0 \ll 1$. We plot below the density profile for different values of $\beta_0$ and $\theta_0$ (3.1 and 3.2).
We can see that, despite the wide range of the parameters, the shape of the density profile is universal and composed of a central degenerate core, an inner halo of almost constant density and a tail that scales with $r^{-2}$. Also, from fig. 3.1 it is clear that, for small values of $\theta_0$ we have smaller inner halos, but the drop from the core is also smaller; and vice-versa for large values of $\theta_0$. This means that, for $\theta_0 \leq 0$ (classical systems), we lose the core+inner halo, being left only with the halo; inversely, for very large values of the degeneracy parameter, the drop is so sharp that the halo is practically non-existent. Also, the temperature parameter has very little influence on the density profile. On the velocity curves (3.3 and 3.4) for the same range of parameters as before, we see as expected a universal behaviour:

- Part I: The core with constant density, where $v \propto r$;
- Part II: The first part of the inner halo, where the mass of the core prevails over the mass of the halo and $v \propto r^{-1/2}$;
- Part III: Second part of the inner halo, where now the mass of the halo
3.1. THE MODEL

Figure 3.2: Density profile of the model for different values of $\beta_0$ with $\theta_0 = 40$.

Figure 3.3: Rotation curve of the model for different values of $\theta_0$ with $\beta_0 = 10^{-7}$.
prevails and again $v \propto r$;

- Part IV: The outer halo, where the velocity tends to a constant value $v_0$ after some oscillations of diminishing magnitude.

It’s interesting to notice that the asymptotic value of the velocity does not depend on the degeneracy parameter at the center, only on $\beta_0$, like an isothermal sphere. This fact allow us to determine uniquely $\beta_0$ from observations of flat velocity curves on galaxies. Also we can see the great influence of $\theta_0$ on the inner halo.

In Figs. 3.5, 3.6 we can see the density profile and rotation curve for negative values of the degeneracy parameter. It does not have a core, with just an inner halo and the tail where the density scales with $r^{-2}$, exactly the same behaviour as a pseudo-isothermal sphere. This just shows the fact that for negative degeneracy we are back to a classical solution, so the core (where the quantum effects are noticeable) is gone.
3.2 Comparison with other DM profiles

To compare results obtained with known Dark Matter properties we need to find out the correspondence between fits of circular velocity. There is some controversy in current literature about the undisturbed profile of dark matter in Galaxies and clusters. Cold dark matter simulations suggest the so-called Navarro-Frenk-White profile \[ \rho_{NFW} = \frac{\rho_{0}}{r/r_{NFW}(1 + r/r_{NFW})^2} \] (3.2.31) while the phenomenological pseudoisothermal sphere

\[ \rho = \frac{\rho_{iso}}{1 + (r/r_{iso})^2} \] (3.2.32)

and the Burkert profile \[ \rho = \frac{\rho_{B}}{(1 + r/r_{B})(1 + (r/r_{B})^2)}. \] (3.2.33)
are commonly used for fitting. In order to compare these profiles with the semidegenerate solution, we chose specific values for $\theta_0$ and $\beta_0$ and fit the profiles above to ours. We came to conclusion that NFW and Burkert profiles, while having wrong asymptotics as $r \to \infty$, better reproduce the characteristic "bump" in the circular velocity near the edge of inner halo (fig. 3.7). The best reproduction is obtained for Burkert profile, which is almost coincidental with the semidegenerate one until after the maximum. As most of papers dealing with rotational curve fitting find out that Burkert or other cored profiles are the best fits for dark matter distribution, and that the characteristic scale $r_B$ of the fitted profile is comparable to the full length of fitting range (see, e.g., [23]), that means that the semidegenerate fermion halo can provide the same quality of fits for that galaxies.

For very large radii, the profiles are very different, since both NFW and Burkert don’t have a flat rotation curve, as the density far from the center scales with $r^{-3}$. 

Figure 3.6: Rotation velocity for negative values of $\theta_0$ (same $\beta_0$ as before), resembling a pseudo-isothermal sphere.
Figure 3.7: Dependence of $v_c$ on radius $r$ (black thick line) for $\beta = 10^{-7}$, $\theta_0 = 30$, and a particle mass of 8.5 keV near the edge of the inner halo and its fits by NFW (red dashed line), Burkert (blue thin line) and pseudoisothermal sphere (green dot-dashed line) profiles. The fit range used was from 10 to 85 kpc.

3.3 Scaling Laws

The solutions obtained show remarkable self-similarity properties, so it would be useful to know how the physical parameters of the system scale with the free parameters of the model, $\beta_0$, $\theta_0$, $W_0$ and $m_f$.

For the asymptotic rotation velocity, we have:

$$v_0 = 4.07 \times 10^5 \sqrt{\beta_0} \text{ km/s} \quad (3.3.34)$$

where we can see that it depends on the temperature parameter only, as mentioned before. Also, this is the same law one would find for an isothermal sphere (21), confirming that the tail can be considered to be in the Boltzmann limit.

The core is defined as the region from the center of the system until the first maximum of the rotation velocity curve (region I in fig. 3.4). Near that point the
density of fermions decreases fast. The scaling laws for the mass and radius are

\[ M_c = 4.07 \times 10^{12} (\beta_0 \theta_0)^{0.75} \left( \frac{m_f}{\text{kev}/c^2} \right)^{-2} M_\odot, \]  

(3.3.35)

\[ r_c = 0.1954 (\beta_0 \theta_0)^{-0.25} \left( \frac{m_f}{\text{kev}/c^2} \right)^{-2} \text{pc}, \]  

(3.3.36)

We can see that both the mass and radius of the core are dependent not on \( \beta_0 \) and \( \theta_0 \) separately, but only on their product \( \beta_0 \theta_0 = \mu_0/mc^2 \) at the center of configuration. A scaling law for the density in the core (which is almost constant) can be built from these two:

\[ \rho_c = C m_f^{5/2} \mu_0^{3/2} M_\odot/\text{pc}^3, \]  

(3.3.37)

with \( C \approx (3/4\pi) \times 10^{14.73} \).

For a degenerate Fermi gas with spin 1/2, we can find the particle number density as

\[ n = \int_0^{p_F} \frac{1}{\pi^2 k^3} p^2 dp \]

from which we find that \( p_F = (3\pi^2 n)^{1/3} \). For a non-relativistic gas, \( E_F = \mu = p^2/2m \). Using the fact that the mass density can be written as \( \rho = m(N/V) \), we find that (for a constant density and chemical potential):

\[ \rho \propto \mu_0^{3/2} m^{5/2}, \]  

(3.3.38)

which is (apart from a numerical factor) exactly the same as we found on Eq. 3.3.37. This means that, for low values of \( \beta_0 \) (non-relativistic systems), the core can be treated as a fully degenerate fermion gas.

The halo is defined as the second maximum of the velocity curve (regions II and III in fig. 3.41). The scaling laws for the halo are

\[ M_h = 4.42 \times 10^{13} \beta_0^{0.75} 10^{0.16} \theta_0 (\frac{m_f}{\text{kev}/c^2})^{-2} M_\odot, \]  

(3.3.39)

\[ r_h = 0.83 \beta_0^{-0.25} 10^{0.16} \theta_0 (\frac{m_f}{\text{kev}/c^2})^{-2} \text{pc}. \]  

(3.3.40)

It is interesting to note that the halo properties scale exponentially with \( \theta_0 \) and not linearly like in the core, so that even small changes in this parameter change the halo drastically.
We can again build a scaling law for the density in the halo using Eqs. 3.3.39 and 3.3.40:

$$\rho_h \propto (kT_0)^{3/2} e^{-0.737\mu_0/kT_0} m_f^{5/2}.$$  \hfill (3.3.41)

In the Boltzmann limit, the particle number density is given by

$$n_B = \frac{g(mkT)^{3/2}}{(2\pi)^{3/2} h^3} e^{\mu/kT}.$$  

We can use the fact that $\rho = mn$ to get

$$\rho \propto (kT)^{3/2} m^{5/2} e^{\mu/kT}.$$  \hfill (3.3.42)

Which is very similar to Eq. 3.3.41, except that in the scaling laws the parameters are measured in the center. But for $\beta_0 << 1$ the temperature in the system almost constant, so that $T_h \approx T_0$. The chemical potential, however, is not. But we can find numerically a relation between $\theta_0$ and $\theta_h \equiv \theta(r_h)$ that reads:

$$\theta_h = C(T) - 0.721\theta_0$$  \hfill (3.3.43)

with $C(T)$ a constant that depends on the temperature. We substitute in Eq. 3.3.41 to obtain

$$\rho_h \propto (kT)^{3/2} m^{5/2} e^{1.022\mu/kT},$$  \hfill (3.3.44)

very similar to the Boltzmann relation. Therefore, we can say that the halo is in the Boltzmann regime to a good approximation.

The scaling laws above are exact in $m_f$ and hold in the following physical range of the other parameters: $\log_{10} \beta_0 \in [-11, -5], \theta_0 \in [15, 200], W_0 \in [50, 300]$. Note that $W_0$ does not appear in the scaling laws and so have no influence in the physical properties of the system, serving only to determine the size of the configuration. Another thing to notice is that for $\theta_0 < 15$, the halo mass and radius cannot be described by the simple equations above and we have to solve the system numerically to obtain them. Both $\log M_h$ and $\log r_h$ do not depend linearly on $\theta_0$ anymore; in fact they reach a minimum at $\theta_0 \approx 4.6$, for any $\beta_0$ in the range above. This is exactly the point when the temperature at the core is of the order of the Fermi energy $E_F = \frac{\hbar}{2m} (3\pi^2 n_c)^{2/3}$ where $n_c$ is the number density at the core radius; for $\theta_0 < 5$ we cannot consider the core as degenerate anymore.
3.3.1 Universality Laws

Recently some studies have shown that, despite the wide range of masses and magnitudes, the DM on galaxies have some universal properties. McGaugh et al. [25] plotted all velocity data due to DM of 60 spiral galaxies with different properties and found out that it obeyed a simple law with little spread:

\[ \log(V_{c,DM}/\text{km/s}) = 1.47 + 0.5 \log(r/\text{kpc}), \]  

(3.3.45)

spanning radii from 1 to 74 kpc. Later, [26] extended this law to LSB and Milky Way dSphs. Both data (spirals+dSphs) followed a mass-radius relation of the form \( M_{DM} \propto r^2 \), which implies an acceleration due to DM of \( 3 \times 10^{-9} \text{cm/s}^2 \) for radii from 0.02 to 75 kpc.

By fitting galaxy rotation curves with a Burkert profile, [27, 28] showed that all DM halos have a constant surface density \( \mu = \rho_B r_B \) up to the Burkert radius \( r_B \). This in turn means that the acceleration due to DM at \( r_B \) is also constant, \( a_{DM} \approx 3.2 \times 10^{-9} \text{cm/s}^2 \). Since we already know that our model and the Burkert profile are equivalent when fitting rotation curves, we can use this universality law to try to constrain our parameters. The constancy of the acceleration at \( r_B \) leads to the constancy of the maximal acceleration due to DM at \( r_{max}^B \), which is the same as \( a_{DM}(r_B) \) within 0.1%, with \( r_{max}^B = 0.96r_B \). Since our profile and Burkert are almost coincidental until after the maximum of the rotation curve (which happens for the Burkert profile at \( r \approx 3.3r_B \)), we can apply this to our model by imposing that the maximum acceleration in the halo take the same value as the one given by the universality law. The scaling laws for the maximum acceleration \( a_{max} = a(r_{max}) \) and for \( r_{max} \) are

\[ a_{max} = 1.2 \times 10^3 \beta_0^{1.216} 10^{0.1612\theta_0} m_f^2 \text{cm/s}^2 \]  

(3.3.46)

\[ r_{max} = 0.358 \beta_0^{-0.221} 10^{0.1612\theta_0} m_f^{-2}. \]  

(3.3.47)

Using the values for the Burkert profile that fits best with our model, we find that \( r_{max} = 1.1r_{max}^B \), so we can use the universality law with a good approximation. Plugging in \( a_{max} = 3.2 \times 10^{-9} \) in Eq.3.3.46 and using Eq.3.3.34, we can find a
relation between the asymptotic velocity, \( \theta_0 \) and \( m_f \):

\[
v_0 = 7.1 e^{0.152\theta_0} m^{-0.82}. \tag{3.3.48}
\]

In fig. 3.8, we plot the central degeneracy vs mass, with different curves representing different asymptotic rotation velocities (in km/s), up to a mass of 50 keV (this is the maximum mass for the sterile neutrino, according to cosmological constraints given by [29] and section 3.6). The lower and higher velocities are typical for dSphs and elliptical galaxies, respectively, since this was the range used in [27, 28]. We can see that for every fixed asymptotic velocity, there is a whole family of values for the mass and the degeneracy parameter that makes the system compatible with the universality law presented above; therefore it is not enough measure the

Figure 3.8: \( \theta_0 \) vs mass using the scaling law for the acceleration (eq. 3.3.48), with different curves for different asymptotic velocities. See text for details.
asymptotic rotation velocity and use this law to fix all parameters in the model. We can only set a maximum value for the degeneracy parameter at \( \theta_0 = 50 \), if we interpret the fermion as the sterile neutrino. It is interesting to notice that higher velocity systems will always have larger values of \( \theta_0 \), and that larger masses will imply larger values of the degeneracy parameter. A minimum for \( \theta_0 \) and the particle mass was obtained in [30]. There, they fixed the velocity in the halo \( v_h = 13 \) km/s, the halo radius \( r_h = 584 \) pc and the halo mass \( M_h = 2.2 \times 10^7 M_\odot \), typical values for dSphs (with data from [31]). They found a family of values for \( \theta_0 \) and \( m \) that would give the values for the halo as above. There is a minimum value for \( \theta_0 \) in order for the halo to have the desired dimensions, with a corresponding minimum mass, at \( \theta_0 = 11 \) and \( m = 5.2 \) keV. Fig. 3.9 show some values for \( \theta_0 \) and \( m \) that satisfy the halo dimensions. For any \( \theta_0 < 11 \), there is no possible value for the mass that can give the correct halo mass and radius. For a larger halo, the lower limit for the mass is lower, but \( \theta_0^{\text{min}} = 11 \) always. This gives a more detailed and firmer lower limit to the mass and degeneracy parameter than the universality.

Figure 3.9: Density profiles for different values of \( m \) and \( \theta_0 \) which gives the halo magnitudes as reported in the text. Taken from [30].
3.3. SCALING LAWS

3.3.2 Application to the Milky Way

In this section we apply the model to the Milky Way to see whether the model is in agreement with the DM halo and if it can work as an alternative to the SMBH in the center, analogously to what was done in Munyaneza et.al. [1].

Sofue [32] using data from different sources, fit the rotation curve of the milky way with Burkert, NFW and isothermal profiles, with the baryonic matter composed of a bulge and an exponential disk. It is found that the inner part \( r \lesssim 1 \text{kpc} \) of the rotation curve is dominated by the baryons, so that the DM is only important at large radii where the errors are very large, making it hard to choose a model over the others. For the Burkert profile (the one which is most similar to our model), the best fit is given by (obtained similar values using masers as tracers for the outer rotation curve)

\[
 r_B = 10 \text{kpc}, \quad \rho_B = 2.4 \times 10^{-2} \mathcal{M}_\odot \text{pc}^3
\]  
(3.3.49)
giving a maximum velocity of \( v_{\text{max}} = 167 \text{ km/s} \) at at radius of 32.4 kpc. The mass enclosed in this radius is \( 2.1 \times 10^{11} \mathcal{M}_\odot \). Considering that \( v_{\text{max}} \) is the same for the semidegenerate solution, that the mass above is the halo mass and the mass of the central object is \( 4.4 \times 10^6 \mathcal{M}_\odot \) ([34], [35]), we can use eqs. 3.3.34, 3.3.35 and 3.3.39 to find:

\[
 \beta_0 \approx 1.24 \times 10^{-7}, \\
 \theta_0 \approx 29.7, \\
 m_f \approx 8.8 \text{ keV/c}^2.
\]  
(3.3.50)

Then we take the other two scaling laws (3.3.30) and (3.3.40) to obtain

\[
 r_c \approx 5.7 \times 10^{-2} \text{pc} \\
 r_h \approx 32.1 \text{Kpc}.
\]  
(3.3.51)  
(3.3.52)

Fig. 3.10 shows the excellent agreement between our model and the Burkert profile
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Figure 3.10: Comparison between our profile and Burkert for milky way values.

for the values shown above.

We see that the halo radius is almost the same as in the Burkert profile, while the core radius is two orders of magnitude larger than the periastron of the S02 star ($6 \times 10^{-4}$ pc, [43]). Therefore a central object formed of fermions cannot provide the necessary mass consistent with the observed orbit of this star.

If we try to fix the radii instead of masses as $r_c = 6 \times 10^{-4}$ pc and $r_h = 32.4$ kpc with the same velocity (hence the same value for $\beta_0$), we get

\begin{align*}
\theta_0 & \approx 41.9 \\
m_f & \approx 82.5 \text{keV}/c^2 \quad (3.3.53)
\end{align*}

which gives us

\begin{align*}
M_h & \approx 2.11 \times 10^{11} M_\odot \\
M_c & \approx 6.5 \times 10^4 M_\odot. \quad (3.3.54)
\end{align*}
We see that, while the halo mass is almost the same as the one given by the Burkert profile, the core mass is way too low compared to the inferred mass. Besides that, if one tries to interpret the fermion as a sterile neutrino, the mass is larger than the constraints given by [29].

### 3.4 Critical configurations

We now turn to study different equilibrium configurations up to the last stable one before collapse. In order to do so we fix the mass and the degeneracy parameter in the center, and analyse the different systems in a central density \( \rho_0 \) vs. core mass \( M_c \) diagram.

<table>
<thead>
<tr>
<th>( \theta_0 )</th>
<th>( \beta^{cr}_0 )</th>
<th>( \mu^{cr}_e / (mc^2) )</th>
<th>( r^{cr}_e (pc) )</th>
<th>( M^{cr}<em>e (M</em>\odot) )</th>
<th>( \rho^{cr}<em>0 (M</em>\odot / pc^3) )</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>( 6.45 \times 10^{-2} )</td>
<td>( 6.45 \times 10^{-2} )</td>
<td>( 6.59 \times 10^{-3} )</td>
<td>( 1.59 \times 10^{10} )</td>
<td>( 7.00 \times 10^{16} )</td>
</tr>
<tr>
<td>5</td>
<td>( 2.23 \times 10^{-2} )</td>
<td>0.111</td>
<td>( 4.55 \times 10^{-3} )</td>
<td>( 7.91 \times 10^{9} )</td>
<td>( 7.28 \times 10^{16} )</td>
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<tr>
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<td>0.285</td>
<td>( 2.61 \times 10^{-3} )</td>
<td>( 7.54 \times 10^{9} )</td>
<td>( 3.44 \times 10^{17} )</td>
</tr>
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<td>( 8.33 \times 10^{-3} )</td>
<td>0.333</td>
<td>( 2.38 \times 10^{-3} )</td>
<td>( 7.44 \times 10^{9} )</td>
<td>( 4.57 \times 10^{17} )</td>
</tr>
<tr>
<td>55</td>
<td>( 6.06 \times 10^{-3} )</td>
<td>0.333</td>
<td>( 2.38 \times 10^{-3} )</td>
<td>( 7.44 \times 10^{9} )</td>
<td>( 4.57 \times 10^{17} )</td>
</tr>
<tr>
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<td>0.333</td>
<td>( 2.38 \times 10^{-3} )</td>
<td>( 7.44 \times 10^{9} )</td>
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<tr>
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<tr>
<td>( 3 \times 10^9 )</td>
<td>( 1.11 \times 10^{-10} )</td>
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<td>( 2.38 \times 10^{-3} )</td>
<td>( 7.44 \times 10^{9} )</td>
<td>( 4.57 \times 10^{17} )</td>
</tr>
</tbody>
</table>

Table 3.1: Critical core masses and critical central magnitudes and parameters

Table 3.1 shows the value of the physical parameters of the system (see Sec. 3.3) for different values of the degeneracy and temperature parameter. For each value of \( \theta_0 \) there exists a critical value of \( \beta_0 \) where the mass of the core reaches a maximum, which we call a critical mass \( (M^{cr}_e) \). This is shown in Fig. 3.11 for different fixed values of \( \theta_0 \). Here we are not studying a sequence of equilibrium
configurations for a single system like it was done in [36, 37], but studying different equilibrium configurations up to the last stable one. From the table, we can see two families of configurations:

- For $\theta_C^\tau > 40$, the normalized chemical potential at the center ($\mu_0^\tau / m_\tau c^2$) remains constant at 0.33 with a constant core mass of $M_C^\tau = 7.44 \times 10^9 M_\odot$, with $\beta_0^\tau < 8.3 \times 10^{-3}$ (we will set $m_\tau = 8.5$ keV from now on). This shows that the temperature does not play any role in such systems and we consider them as degenerate. In this case, the core is dominated by the degeneracy pressure (due to high $\theta_0$) and the core mass depends only on the mass of the fermion through $M_C^\tau \propto m_\tau^3 / m^2$. The core radius is this case is about three times the Schwarzschild radius for a black hole of the same mass.

- For lower values of $\theta_0$, the normalized chemical potential at the center changes for different values of $\theta_0$ and $\beta_0$, which increases the critical core mass from the value shown before. Since $\beta_0$ is higher than before and $\theta_0$ is lower, the contribution of the thermal pressure in the core is now appreciable due to the high temperatures (and mildly relativistic velocities).

In Fig.3.11 we show the core mass - central density relation for fixed values of $\theta_0$. Each point in each curve represent different values of $\beta_0$ (and hence temperature) with a corresponding different chemical potential that keeps a constant $\theta_0$. For every value of $\theta_0 > 40$, the curves are superimposed, showing that temperature effects are negligible. In this case, $\mu_0^\tau / m c^2 = 0.33$ and $M_C^\tau \propto m_\tau^3 / m^2$. For lower values of the degeneracy parameter the critical mass changes for different temperatures (as shown by different curves). Now the critical chemical potential changes with the temperature, and the critical masses are higher. This can be explained by looking at Figs.3.13 and 3.12: for higher temperatures, the general relativistic effects on the core are not so relevant, while for lower temperatures (and high degeneracy) it’s the opposite; due to this, the critical mass for hotter systems is larger.
Figure 3.11: Central density-core mass relation for fixed values of $\theta_0$. Each point in the curves represents a different value of the chemical potential and temperature, as shown for $\theta_0 = 1$. 

$$M_c(M_\odot) \quad \rho_0(M_\odot pc^{-3})$$

- $\theta_0 = 1$
- $\theta_0 = 5$
- $\theta_0 > 40$

$T_{0(1)}$, $\mu_{0(1)}$
$T_{0(2)} > T_{0(1)}$, $\mu_{0(2)} < \mu_{0(1)}$
$T_{0(cr)} > T_{0(2)}$, $\mu_{0(cr)} < \mu_{0(2)}$
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Figure 3.12: The time component of the metric $e^{\nu/2}$ and the temperature as a function of the radius for $\theta_0 = 5$ and $\beta_0 = 2.23 \times 10^{-2}$.

3.4.1 Astrophysical application

Since we have $M_c^r \sim 10^9 M_\odot$, (for a particle mass in the keV scale) of the order of SMBHs in AGNs, we will check if the critical configurations could in fact represent AGNs.

Here we plot in Fig. 3.15 the velocity curves for different values of the critical parameters. We see that the velocities for low values of $\theta_0$ are very high in all of the configuration, a few percent of the speed of light, too high compared to the observed values in galaxies. However, the velocity in the flat part gets smaller for higher values of $\theta_0$; extrapolating to $\theta_0 \gtrsim 10^6$, this value would be $200 - 300 \text{ km/s}$, consistent with rotation curves of galaxies. But as can be seen from Fig. 3.10 and 3.14, the mass of the halo for $\theta_0 = 100$ is already $10^{22} M_\odot$ with radius 100 Gpc, larger than any galaxy; it would be even larger and more massive for higher values of $\theta_0$. Even velocities measured by masers in the sup-pc regions in AGNs are at...
most 1000 km/s, still lower than the values we get for any value of $\theta_0$ at that radius.

Therefore, even though the central mass is comparable to those of AGNs, the critical configurations have higher velocities and masses than the observed values and are not a viable alternative.

### 3.5 An analytical formula for the critical mass

It is useful to try to find an analytical formula for the critical mass to try to understand the physics behind it. For this we will use the Newtonian hydrostatic equilibrium equation corresponding to the last stable configuration, where the pressure due to gravity is balanced by a high relativistic semi-degenerate Fermi
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Figure 3.14: Density profile for different values of $\theta_0^\tau$ and $\beta_0^\tau$.

where $P_g^\tau(r)$ is the ultra relativistic approximation of a highly relativistic Fermi gas ($\mu \gg mc^2$) and we expanded the pressure up to second order in temperature around $\mu/kT \gg 1$ (see e.g. [38]). Considering that the density in the core is nearly constant, i.e., $\rho = \rho_0^\tau \approx \text{const.} \forall r \leq r_c^\tau$, we can write the core radius as

$$r_c^\tau = (3M_c^\tau/(4\pi\rho_0^\tau))^{1/3}.$$ With this, we can rewrite (3.5.55) as follows,

$$\frac{P_g(r)}{GM(r)\rho(r)} \approx \frac{\mu^4}{12\pi^2(\hbar c)^3} \left(1 + \frac{2\pi^3/2(kT)^2}{\mu^2}\right) + O(T^4), \quad \text{(3.5.56)}$$

And finally, we write the central mass density as $\rho_0^\tau \approx n^\tau m$, where $n^\tau = \mu^3/(3\pi^2(\hbar c)^3)$. With this expression for $\rho_0^\tau$ in (3.5.56) we can directly give $M_c^\tau$ in

$\text{footnote}$ in (3.5.56) we have used the fermi energy ($\epsilon_f = \mu$) with the rest energy substracted off in consistency with the theoretical formulation of our model
3.5. AN ANALYTICAL FORMULA FOR THE CRITICAL MASS

Figure 3.15: Circular velocity for different values of \( \theta_0 \) with the corresponding \( \beta_0 \) given as before. We can see that the velocities are very high in the flat part but tends to lower values for higher \( \theta_0 \).

terms of \( \theta_0 \) as:

\[
M_{c}^{\text{cr}} \approx \frac{3\sqrt{\pi}}{16} \frac{M_{pl}^3}{m_f^2} \left( 1 + \frac{2\pi^2}{\theta_0^2} \right)^{3/2}.
\] (3.5.57)

It can be seen that the critical mass is basically the mass due to the degeneracy pressure plus a contribution of the temperature that only becomes relevant for \( \theta_0 \lesssim 15 \). For degenerate systems (\( \theta_0 >> 1 \)) we recover the numerical result and the critical mass is proportional to \( m_f^3/m^2 \) albeit with a slight different value: \( M_{c}^{\text{cr}}(\text{analytical}) = 7.503 \times 10^9 M_\odot \). Even for \( \theta_0 = 40 \), \( M_{c}^{\text{cr}}(\text{analytical}) = 7.62 \times 10^9 M_\odot \), only a 2% difference with the numerical value. When we turn to lower values of \( \theta_0 \) our approximation of a semi degenerate ultra relativistic fermi gas breaks down, and the discrepancy between the numerical results and the analytical is large. We have, for example, \( M_{c}^{\text{cr}}(\text{analytical}) = 1.79 \times 10^{10} M_\odot \) for \( \theta_0 = 5 \), more than two times the numerical value.
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3.6 A natural candidate: the sterile neutrino

One of the efforts nowadays is to design experiments that can find out what DM is made of. In this section, we will give a candidate that lies in our range of mass reported in Section 3.3.

Since our particle should have a mass of a few keV in order to reproduce the galactic structures, the sterile neutrino is a natural candidate in this model. The sterile neutrino is a right-handed neutrino, called sterile since it doesn’t feel the weak force and just mixes with the active neutrinos. It was proposed as a DM candidate by Dodelson and Widrow [39]. The number of sterile neutrinos is not very well constrained, but the so called $\nu$MSM, the minimal standard model with neutrino masses, requires three sterile neutrinos; one with a mass in the keV range and the other two with degenerate masses of the order of a few GeV [29]. The $\nu$MSM has a DM candidate in the lightest sterile neutrino and has the...
minimal particle content consistent with baryogenesis without introducing new physics beyond the electroweak scale.

The sterile neutrino can decay via mixing with the active neutrinos into three (anti)neutrinos, producing an x-ray line. This can constrain the mass and mixing angle, by requiring that the lifetime of this neutrino is greater than the age of the universe in order for structure to form. The lack of detection (so far) of this line also helps constraining the parameter space.

![Figure 3.17](image_url)

Figure 3.17: Constraints for the mass and mixing angle of the DM candidate in the \(\nu\)MSM from different sources. Taken from Shaposhnikov et.al. [29].

We can see from Fig. 3.17 that the mass of this DM candidate is constrained to be in the 1 – 50 keV range, which makes it a perfect particle candidate in our model.

There are many models on how the sterile neutrinos are produced; depending on this, they may change the effective number of neutrinos in the early universe, as measured by WMAP and Planck. We can neglect the two massive neutrinos, since by the time of the active neutrino decoupling (around 2 MeV) they will be non-relativistic and therefore will not contribute to the entropy. The effective number
of neutrinos is defined as
\[ N_{\text{eff}} \equiv \frac{\rho_{\text{rel}} - \rho_{\gamma}}{\rho_{\nu}^{\text{th}}}, \]  
(3.6.58)
where \( \rho_{\text{rel}} \) is the energy density of all relativistic particles, \( \rho_{\gamma} \) is the same for photons and \( \rho_{\nu}^{\text{th}} \) is the energy density of neutrinos with a thermal distribution.

The entropy of the universe is conserved, so (considering a flat Friedmann metric) \( sa^3 = \text{const} \), where the entropy is given by
\[ s = \frac{\rho + P - \mu n}{T} \]  
(3.6.59)
with \( \rho \) the energy density, \( P \) the pressure, \( \mu \) the chemical potential and \( n \) the number density. Considering that the sterile neutrino decouples from the plasma before the active ones (\( T \geq 2\text{MeV} \)), we can consider them to be relativistic at the time. For a relativistic species, \( \rho \propto T^4 \) and \( P \approx \rho/3 \). Neglecting the chemical potential, the effective number of neutrinos can be written as\(^2\)
\[ N_{\text{eff}} = 3.046 \left[ 1 + \frac{\Delta N}{3} \left( \frac{10.73}{g_s(T_{D})} \right)^{4/3} \right] \]  
(3.6.60)
with \( \Delta N \) the extra number of neutrinos.

From Fig. 3.18, if the sterile neutrino decouples early enough (\( T_d \gtrsim 30 \text{MeV} \)), \( N_{\text{eff}} \) would be still within the Planck bounds. In fact, if \( T_d > 1 \text{GeV} \), \( \Delta N_{\text{eff}} \approx 0.04 \) is too small to be measured.

It is worth mentioning that these calculations are somewhat dependant on the specific form of \( g_s(T) \). Currently there is a debate over when the quark-hadron phase transition took place (150 or 400 MeV), but the results presented here should not change too much. This shows that, even in the worst case where the chemical potential is negligible, the presence of a sterile neutrino could remain undetected by CMB experiments provided that its decoupling from the primordial plasma happened early enough.

\(^2\)The deviation from the standard value of 3 is due to the reheating of the active neutrinos by the electron-positron annihilation and considerations of non-instantaneous decoupling.
Figure 3.18: The effective number of neutrinos is plotted in blue, for one sterile neutrino. The red dashed line is the contribution of the sterile neutrino alone, and the black lines are the Planck bounds.
Chapter 4

Active galactic nuclei and blazars

Active galactic nuclei is a name given to very energetic phenomena occurring in the center of some galaxies that cannot be attributed to stars. The emission of an AGN is usually thermal due to an accretion disk, while a small fraction of them (around 10%) show a non-thermal component in the form of a jet. In the early years of AGN research, several models were proposed to explain the powerful thermal central emission, but today it is widely accepted that an accreting black hole is the responsible. When the AGN present a jet, it can swamp the accretion disk emission and the resulting observed energy spectrum is mainly non-thermal. Due to the presence of an obscuring torus around the central engine, properties of AGNs can change drastically with the viewing angle.

Figure 4.1 show a representation of an AGN. In the center there is the SMBH, with the accretion disk and the obscuring torus around it. The broad line region (BLR) is close to the black hole, with matter rotating at very high speeds. At large distances from the center the material is colder, generating narrow lines. Depending on the radio emission, AGNs can be classified as radio loud or quiet (this does not mean no radio emission, just weak radio emission).

Other than the presence or not of a radio jet, AGNs were classified based on their angle with our line of sight and it was thought that the physical process behind the emission for different classes was different. However, this takes into account only the pointing direction and not the physical properties of the sources.
Figure 4.1: Representation of an AGN, showing that the classification of AGNs depends on the angle to the line of sight.

Understanding the physics behind the emission processes can lead to unification schemes (e.g. [40, 41]); the basic premise of such schemes is that, essentially, all AGNs are the same and the differences we observe can be ascribed to different angles to our line of sight, so that a simple model can be used to explain the variety of AGNs. However, due to the difficulty of observing the inner regions of AGNs and our lack of understanding of the emission process, there is no single unification scheme today.

Blazars are a class of AGN characterized by rapid and large amplitude spectral variability, assumed to be due to the presence of a jet with the material moving at relativistic speeds pointing very close to the line of sight. Usually the observed radiation shows extreme properties, mostly coming from relativistic amplification effects. The observed Spectral Energy Distribution (SED) presents a general shape composed of two bumps in $\nu - \nu F_\nu$ space (see Fig. 4.2). All SEDs were built on the
ASDC online SED builder, one in the infrared (IR) to soft X-ray band and the other one in the hard X-ray to $\gamma$-rays. According to the standard picture [e.g. 42], the first peak is associated with the emission of synchrotron radiation due to relativistic electrons moving in a magnetic field, and the second peak is mainly associated with synchrotron photons that are Inverse-Compton (IC) scattered to higher energies by the same relativistic electron population that generates them (Synchrotron Self Compton model, SSC). The seed photons undergoing IC scattering can also come from outside regions, like the accretion disk and the broad line region, and can add an extra ingredient (External Compton models, EC) for modelling the observed SED.

Blazars can be divided in two classes: BL Lacs and flat-spectrum radio quasars (FSRQ). BL Lacs are characterized by an almost featureless optical spectrum, occasionally with some narrow emission lines (with equivalent width in the rest-frame no larger than 5 Å) while FSRQs have strong emission lines. However, this classification is not very robust: on some occasions, BL Lacs can exhibit emission lines broader than the usual threshold, and FSRQs can also appear nearly

\[^{1}\text{http://tools.asdc.asi.it/SED.}\]
featureless in flare states [13].

Figure 4.3: Optical spectrum of PKS 1222+216, an FSRQ. We can see clearly strong emission lines.

Figs. 4.3 and 4.4 show the optical spectrum of two blazars, one FSRQ and one BL Lac respectively. It can be seen that the thermal emission of the galaxy in the BL Lac case is completely swamped by the non-thermal emission of the jet, and thus the spectrum is featureless, whereas in the FSRQ case strong emission lines can be seen.

If the peak frequency of the synchrotron bump ($\nu_{\text{peak}}$) is larger than $10^{15}$ Hz, a blazar is usually called High Synchrotron Peaked (HSP) BL Lac, or HBL in the original BL Lac classification of [14], which was later extended to all blazars by [15]. If this peak is below $10^{14}$ Hz the blazar is called a Low Synchrotron Peaked (LSP) blazar; for values in the middle the blazar is called an Intermediate Synchrotron Peaked (ISP).
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Figure 4.4: Optical spectrum of BZBJ0809, a BL Lac. No lines can be seen as the spectrum is completely dominated by the non-thermal emission of the AGN.

Figure 4.5: SED of the same BL Lac shown before. The first peak is at $10^{15.5}$ Hz.

Figure 4.6: SED of the same FSRQ shown before. The synchrotron peak is at $10^{13.8}$ Hz.

Usually FSRQs are LSPs, while BL Lacs span a wider range of frequencies. Observations have shown that HSPs are also bright sources of high energy TeV
photons and that they may be the dominant component of a putative extragalactic TeV background (TB) \[46\].

The very high energy (VHE) $\gamma$-rays from blazars may be absorbed due to interaction with extragalactic background light (EBL) photons ($\gamma_{VHE} + \gamma_{EBL} \rightarrow e^+ + e^-$). The resulting electron positron pairs cool by scattering cosmic microwave background (CMB) photons to $\gamma$-ray energies, which are offset by a small angle w.r.t. the line of sight when the pairs are deflected in the possible presence of intergalactic magnetic fields (IMFs) [e.g., \[47\]]. Studying the development of the cascade through intergalactic distances may provide a tool to constrain the EBL fluxes at the IR range \[48\] and also impose lower limits to the IMF. The attenuation due to the EBL may leave a characteristic imprint which is dependent on the redshift of the source and the observed energy band, but a true understanding of such a process demands a clear description of the intrinsic SED generated by the AGN’s central engine.

There is a clear need for a large number of TeV targets in order to gain insight into the underlying physics. It is therefore important to build a large sample of HSP objects to provide bright targets for $\gamma$-ray and TeV detections. This will also permit the study of variability in different energy bands to search for fundamental correlations. Within the motivations for identifying extreme AGNs there is also the possibility of studying jet properties in extreme conditions and determining the population distribution of HSPs. Since AGNs can be detected in a broad range of redshifts, extreme bright blazars may also be an efficient tool for studying cosmological structures formation and evolution \[49\].

Another interesting aspect from blazars is the possibility of contamination of the CMB, as proposed by \[16\]. This is mainly due to LSPs, since their radio and microwave emission are more intense. Blazars that are not detected by CMB experiments (and therefore not removed from the map) may increase the errors or even change the power spectrum. Again, in order to estimate the blazar contribution to the CMB, a large sample is necessary. In the next chapter I will present the largest sample of HSP blazars up to date.
Chapter 5

A Multiwavelength catalog of TeV candidate blazars

In this chapter, I will show the 1WHSP, a catalog of HSP blazars with more than 700 sources based on the Wide-field Infrared Survey Explorer (WISE), describe its properties and the physics we can do with such a large catalog.

5.1 The WISE mission

The Wide-field Infrared Survey Explorer (WISE) \cite{50} is an infrared astronomical satellite launched by NASA in December 2009, with the first all-sky data release in March 2012. It detected over 563 million sources\(^1\) (the WISE All-sky data release) in its four bands, W1 (3.4 \(\mu\)m), W2 (4.6 \(\mu\)m), W3 (12 \(\mu\)m) and W4 (22 \(\mu\)m), with a sensitivity a hundred times better than its predecessor, the Infrared Astronomical Satellite (IRAS). In order to select HSP blazars from all the sources detected by WISE, we devised a method that combines data from WISE with multi-frequency selection criteria.

Throughout this chapter we adopt a Flat-LCDM cosmology with the following parameters: \(\Omega_M=0.308\), \(\Omega_L=0.692\), \(H_0 = 67 \text{ km s}^{-1} \text{ Mpc}^{-1}\) \cite{8}.

\(^1\)Another data release in November 2013, the All WISE, added another \(\sim200\) million sources. We do not use those sources in this work.
5.2 Building a large sample of HSP blazars

An effective way of building blazar samples is to work with data from multi-frequency (possibly all-sky) surveys and apply selection criteria based on spectral features that are known to be specific to blazar SEDs (e.g.\cite{51,52}). The recent work by\cite{53} and\cite{54}, based on data from WISE, has shown that blazars concentrate in a distinct region of the IR colour-colour diagram, which has been named the \textit{WISE blazar strip} (WBS). This region can be used to identify $\gamma$-ray sources\cite{55,56} and to select new large IR flux limited samples of blazars.

The peculiar colours of the WBS reflect the IR slope of the non-thermal radiation from the jet, which in the case of HSP objects is a power law with energy index approximately in the range $0.4 - 0.8$. However, in a significant number of blazars, like e.g. MKN 421 and MKN 501, the non-thermal IR continuum is contaminated by the presence of the host galaxy, and the source moves away from the blazar strip. This was already noted in the original paper by\cite{53} (see their Fig. 2), although no quantitative estimation of the fraction of HSP sources that lied outside the non-thermal blazar strip was attempted.

To assess the efficiency of this procedure we applied it to the largest homogeneous sample of certified HSP blazars that has been assembled previous to this work, that is the Sedentary survey catalogue of extreme HBLs (for details see\cite{51} and\cite{57}). Figure\cite{5.1} shows the WBS delimited by two dashed black lines, with the Sedentary sources as blue filled circles. Surprisingly, more than half of those objects were not found inside the WBS. This is partly due to the fact that 33 sources in the Sedentary survey do not meet the requirement of being detected with signal-to-noise ratio (snr) > 2 in the WISE channels considered, and partly because 42 other objects have infrared colours that are outside the WBS as defined in\cite{53}) (probably because in many of these sources the host galaxy light contaminates the IR colours). We conclude that selecting blazar candidates only from sources that are within the original WBS leads to significant incompleteness in general, and particularly at faint IR fluxes where the strong requirement of a detection in three bands is frequently not met.

To partly remedy this we extended the search for candidates to a wider portion
of the colour-colour diagram defining an *enlarged WBS* as a polygon (red dashed lines in Fig. 5.1) limited by the following corners:

Corner 1 \( (c_{4.6-12} = 1.0, \ c_{3.4-4.6} = 0.2) \),
Corner 2 \( (c_{4.6-12} = 1.9, \ c_{3.4-4.6} = 0.14) \),
Corner 3 \( (c_{4.6-12} = 3.81, \ c_{3.4-4.6} = 1.17) \),
Corner 4 \( (c_{4.6-12} = 3.29, \ c_{3.4-4.6} = 1.67) \),

where \( c_{3.4-4.6} = m_{[3.4 \mu m]} - m_{[4.6 \mu m]} \)
and \( c_{4.6-12} = m_{[4.6 \mu m]} - m_{[12.0 \mu m]} \), see Fig. 5.1.

This enlarged WBS area includes over 3.7 million WISE objects that are above the Galactic plane \( (|b| > 20^\circ) \) and are detected with snr > 2 in all the 3.4 \( \mu m \), 4.6 \( \mu m \), and the 12 \( \mu m \) WISE channels. Even though we have increased the IR color-color selection area, there are still 23 (15\%) HSPs/HBLs in the Sedentary survey that are clearly outside the *enlarged WBS*. This is because in these blazars the optical/IR flux is dominated by the host galaxy and not by the non-thermal emission.

Although the size of this sample is only about 5\% of that of all the WISE sources located at \( |b| > 20^\circ \), it still includes a very large fraction of non-blazar sources, and is clearly far too large to be considered for optical spectroscopy follow up. To remove as many as possible non-blazars objects from this initial set of WISE candidates we imposed a number of additional restrictions based on well-known broad-band spectral peculiarities of blazars. This was done by performing a cross-match between the position of the WISE colour selected objects with a number of radio [NVSS, FIRST and SUMSS: 58, 59, 60] and X-ray [IPC, ROSAT BSC and FSC, XMM, SWIFT: 61, 62, 63, 64, 65, 66] catalogues and then applying the following constraints:

\[
0.05 < \alpha_{1.4 \text{GHz}-3.4 \mu m} < 0.45 \quad (5.2.1)
\]
\[
0.4 < \alpha_{4.6 \mu m-1 \text{keV}} < 1.1 \quad (5.2.2)
\]
\[
-1.0 < \alpha_{3.6 \mu m-12.0 \mu m} < 0.7 \quad (5.2.3)
\]

\(^2\)WISE magnitudes are in the Vega system.
5.2. BUILDING A LARGE SAMPLE OF HSP BLAZARS

Figure 5.1: The WISE colour-colour diagram similar to that from Massaro et.al. (53). The red dots are the Sedentary sources, while the blue dots are the WISE ones.

where

$$\alpha_{\nu_1-\nu_2} = -\frac{\log(f_{\nu_1}/f_{\nu_2})}{\log(\nu_1/\nu_2)}.$$  \hspace{1cm} (5.2.4)

A radius of 0.1 arcmin was adopted for the cross-correlations unless the positional uncertainty of a catalogued source (as e.g. in the case of many X-ray detections in the RASS survey) was reported to be larger than 0.1 arcmin. In such cases we used the 95% uncertainty radius (or ellipse major axis) of each source as maximum distance for the cross-match. In addition, to avoid selecting objects with misaligned jets (which are expected to be radio-extended), the spatial extension of radio counterparts (as reported in the original catalogues) was limited to 1 arcmin. This procedure was carried out whenever possible, based on the 1.4 GHz radio image from NVSS, which includes the entire sky north of $-40^\circ$ declination.

The parameter ranges considered above are derived from the shape of the SED of HSP blazars, which is assumed to be close to those of the three well known HSPs MKN 421, MKN 501 and PKS 2155−304 shown in Fig. 5.2, which also displays the limiting slopes ($\alpha_{1.4GHz-3.4\mu m}$ and $\alpha_{4.6\mu m-1keV}$) used for the selection. The condition on $\alpha_{3.4\mu m-12.0\mu m}$ is used to exclude low energy peaked (LSP) blazars.
from the blazar strip [see 53, for details].

![Averaged and rescaled SED of MKN421, MKN501, and PKS2155–304.](image)

Figure 5.2: Average and rescaled to $10^{10}$ Hz SEDs of three well known and representative HSP blazars: MKN 421, MKN 501, and PKS 2155–3304. The solid lines represent the radio to infrared and infrared to X-ray slope limits used in our selection criteria.

The above multi-frequency conditions reduce the size of the sample to 936 objects. These were then studied individually to clean the sample, leaving HSP sources only.

We realize that we are far from being able to select all HSP objects present in the WISE catalogue, mainly due to two reasons: One is that in order to be complete, we would need deep X-ray coverage for the whole sky. Unfortunately, deep X-ray catalogues current available (SWIFT, XMM, Chandra) have very limited sky coverage (10-15%). So, we lose many faint objects with X-ray fluxes below the shallow sensitivity limits of the ROSAT all sky survey (out of the regions with deep coverage). The second main reason is that the IR color-color selection demands the sources to be detected in all 3.4$\mu$m, 4.6$\mu$m, and 12.0$\mu$m WISE channels,
a requirement that is less and less fulfilled by faint sources, specially in the low energy channels from WISE.

In fact, even with the procedure based on the enlarged WBS, 65 HSPs in the Sedentary Survey (about 43% of the sample) still remain unselected. Of these, 33 were not detected with good snr in at least one of the three WISE channels that we consider, 23 (or \( \sim 15\% \)) were instead left out because their IR colours fall outside the enlarged WBS, and 4 were not selected because X-ray variability caused them to fail the requirement \( \alpha_{w2-x} < 1.1 \). Finally, five Sedentary sources were not detected in any of the WISE channels.

### 5.2.1 Deriving the synchrotron peak frequency and classifying the sources

To make sure that all sources in our sample are HSPs we built the radio to \( \gamma \)-ray SED of each object using the ASDC on-line SED builder tool, which gives access to multi-frequency flux measurements from a large number of catalogues and databases. We determined \( \nu_{\text{peak}} \) by fitting a third degree polynomial function to the data that can be associated to synchrotron emission. Whenever the IR and optical data were due to the host galaxy, these were excluded from the fit. When available, XRT and UVOT data (obtained from the SWIFT public archive) were added to the SEDs, to better characterise the \( \nu_{\text{peak}} \) and \( f_{\nu} \) values. In almost all cases the available data were sufficient for a good determination of \( \nu_{\text{peak}} \). As an example, Fig. 5.3 shows the spectral energy distribution of 1WHSP J022716.5+020200, illustrating how \( \nu_{\text{peak}} \) and \( \nu_{\text{peak}} f_{\nu_{\text{peak}}} \) are determined through a third degree polynomial fit to the data associated to the synchrotron emission. The large variability that is a defining feature of blazars, and is observed in many objects, clearly plays an important role. In the specific case of 1WHSP J022716.5+020200, both \( \nu_{\text{peak}} \) and \( \nu_{\text{peak}} f_{\nu_{\text{peak}}} \) change by large factors, illustrating the uncertainties that are intrinsic to these measurements. Our polynomial fits are applied to all available data and therefore the parameter estimations reflect the average value of all the flux measurements in the database, smoothing out the effect of variability.

Sources can be classified as HSPs only if they have \( \nu_{\text{peak}} > 10^{15} \) Hz. In addi-
Figure 5.3: Spectral energy distribution of 1WHSP J022716.5+020200 illustrating how $\nu_{\text{peak}}$ and $\nu_{\text{peak}} \nu_{\text{peak}}$ are determined through a third degree polynomial fit. The plot also shows how the large variability observed in many objects influences the determination of $\nu_{\text{peak}}$.

...tion, in a number of cases objects were removed from the final sample for a variety of reasons. Nearly 70 sources had not enough data to allow us to estimate $\nu_{\text{peak}}$, about 100 other objects were removed because they were either identified with known FSRQs (and therefore very likely LSPs) or the fit gave $\nu_{\text{peak-obs}} < 10^{15}$ Hz, and a few cases (about 7) were radio extended and therefore likely misaligned jets. Another reason for removing sources was source confusion and the corresponding mismatch of the radio/optical/X-ray positions, especially for faint objects having larger error boxes associated with the radio (SUMSS) and X-ray coordinates. Finally, after analysing the 936 SEDs case by case, the selection criteria listed above give a clean sub-sample of 651 confirmed HSP blazars or blazar candidates. To build the final sample we have added 62 IR detected HSPs that were not selected...
by our scheme, giving a total of 711 objects. Therefore, considering that many of
the uncertain cases could well turn out to be HSPs, our automatic search method
(based on WISE colors and SED slopes) is at least \( \approx 70\% \) efficient.

The above results give us confidence that the majority of our candidates are in-
deed genuine HSPs but also tell us that we should expect a small contamination by
spurious sources or LSP FSRQs that appear as HSPs, since the emission from the
blue bump sometimes mimics synchrotron emission and affects the determination
of \( \nu_{\text{peak}} \). Finally, we should find more examples of the so far elusive intermediate
HSP blazars where the underlying broad lines are not completely swamped by
non-thermal emission from the jet, as predicted by [12] and [17].

Most of the previously identified HSPs showed BL Lac-like spectra. Reliable
redshift measurements were available when absorption features and/or the Ca
H&K break were visible. In some cases, dedicated observing campaigns had been
carried out trying to detect some galaxy absorption/emission features [e.g., 68].

5.3 The catalogue

5.3.1 List of HSP blazars and candidates

Based on the blazar candidates from the enlarged WBS, a total of 649 sources were
selected, from which 301 were already known blazars (in the BZCAT catalogue [69,
70], Milliquas\(^3\) or listed as confirmed BL Lac in NED), 70 are new spectroscopically
confirmed HSP blazars, and 278 objects showed a blazar-like SED but have no
optical spectrum available. The optical identification of these HSP candidates
could result in a considerable number of sources to be added to blazar catalogues.
This sample was enhanced with 62 known HSPs which our selection had missed
(60 from the Sedentary survey and 2 HSP-TeV sources from the TeVCat). The
full catalogue contains 711 HSP blazars and candidates.

\(^3\)quasars.org/milliquas.htm
5.3.2 Fermi-LAT and TeV sources

About 25% (163 objects) of the sources in our catalog have a counterpart in one of the Fermi-LAT AGN catalogues that have been published so far \[71, 72\], which were based on one or two years of integration time. Since a new catalogue based on four years of Fermi-LAT data is in preparation we had the opportunity to cross-match the positions of the remaining 1WHSP objects with those of a consolidated (although still unpublished) version of the upcoming 3LAC γ-ray catalogue. As all sources in our sample are HSPs their γ-ray spectral indices are expected to be flat and their error regions should be small.

We found that 86 sources, in this list of previously undetected γ-ray sources, matched one of the entries of the new Fermi-LAT catalogue within a radius of 10 arc-minutes. The average value of the 95% error ellipse major axis was only 4.9 arc-minutes. To assess the statistical reliability of this result we shifted all the positions of one of the two catalogues by a few degrees in different directions and re-run the cross-matching several times. An average value of 1.5 sources were still within the 10 arc-minutes radius after the coordinates shifts, thus implying that we should not expect more than about 2 spurious associations (or \(\approx 2.3\%\)).

In addition, all matching sources were individually inspected using the "Error Circle Explorer" tool available at the ASDC web site \(^4\) to visually verify that all of them were within the γ-ray error ellipse and that no other bright confusing source was present. We stress that these new associations should be considered only preliminary. Final associations of all four-year Fermi-LAT catalogue (3FGL) entries will be published by the Fermi team in a dedicated paper.

An independent confirmation of the goodness of the association process comes from the distribution of the γ-ray photon spectral indices shown in Fig. 5.4 for all γ-ray associations (solid histogram) and for the subsample of the 86 additional sources discussed above (dashed histogram). In agreement with previous results the distributions are centred on values slightly less than 2, typical of HSP blazars [e.g., 45]. The two distributions are clearly very similar and fully consistent with the hypothesis that they come from the same parent distribution, as confirmed by a

\(^4\)http://www.asdc.asi.it
5.3. THE CATALOGUE

Figure 5.4: The distribution of γ-ray photon spectral index for the subsample of 1WHSP sources included in one of the Fermi-LAT published catalogues (solid histogram) and for the new preliminary associations with the γ-ray sources of the upcoming four year Fermi-LAT catalogue (dashed histogram). See text for details.

KS test giving a p-value of 0.62. Given the fact that blazars are the class of sources with the largest number of associations between TeV and 2FGL counterparts \(^{13}\) and that HSPs are strong emitters in the VHE band, it is likely that many of the sources currently not included in TeV catalogues should be detectable by the present and especially by future generations of Cherenkov telescope arrays (see Sect. 5.4.5).
5.4 Properties

5.4.1 The synchrotron $\nu_{\text{peak}}$ distribution

Fig. 5.5 displays the distribution of the observed values of $\nu_{\text{peak}}$ for our WHSP sample. This peaks just below $10^{16}$ Hz, reflecting our selection method. For comparison, the dashed histogram shows the $\nu_{\text{peak}}$ distribution for the Sedentary BL Lacs, which peaks at a frequency approximately ten times larger, consistently with the fact that the Sedentary survey focused on the most extreme HSPs. In both surveys the maximum observed values of $\nu_{\text{peak}}$ are at $\approx 2 - 3 \times 10^{18}$ Hz. In the SED of a very small percentage of objects, the X-ray measurements were not sufficient to determine $\nu_{\text{peak}}$ because the X-ray flux was still increasing at $\nu = 2 \times 10^{18}$ Hz. For these rare sources we plot lower limits on $\nu_{\text{peak}}$.

Figure 5.5: The distribution of the observed synchrotron peak energy in the full sample of WHSP blazars (solid and dotted black histograms) and in the Sedentary survey (dashed green histogram).
There are a few extreme cases where \( \nu_{\text{peak\,-\,rest}} \) is larger than \( 10^{18} \) Hz. One example of a well defined SED (where the X-ray peak is covered by SWIFT’s XRT and BAT instruments) is that of WISEJ102212.62+512400.5 at \( z = 0.14 \) with \( \nu_{\text{peak\,-\,rest}} \approx 2 \times 10^{18} \) Hz. Given the many sources with unknown redshift, \( \nu_{\text{peak\,-\,rest}} \) might reach even larger values. The largest observed values of \( \nu_{\text{peak\,-\,rest}} \) set strong constraints on the maximum energy at which electrons can be accelerated in blazar jets. Assuming a standard SSC model, a magnetic field of \( B = 0.1 \) Gauss, and a Doppler factor = 10, a maximum value of \( \nu_{\text{peak\,-\,rest}} \) of \( 5 \times 10^{18} \) Hz translates into a Lorentz factor \( \sim 10^6 \).

### 5.4.2 The redshift distribution and the blazar sequence

Our sample of confirmed HSPs includes 435 sources, 231 of which have reliable redshift measurements, covering the 0.03 – 0.81 range, and 204 (47\%) have instead featureless optical spectra and therefore no redshift determination. For 114 sources with no redshift it was possible to assign a lower limit taken from [73] or calculated by us ([see 73, for details]) and for another 11 we could only assign upper limits based on [74]. There still remains 51 objects (11\% of the confirmed HSPs) for which no \( z \) nor any limit could be estimated and another 28 with uncertain values from literature.

Most of the redshift values come from BZCAT [69], from the optical spectroscopy work of [73] and [76], from the Sloan Digital Sky Survey\(^5\) (SDSS) Data Release 10, and from NED. Figure 5.6 shows the redshift distribution for our HSP sample, which has \( \langle z \rangle = 0.28 \) and \( \sigma = 0.13 \), similar to that reported for BZCAT HBLs (\( \langle z \rangle \approx 0.30 \)). Figure 5.7 includes also the distribution of lower limits, which extends well beyond the range of observed values, in agreement with the predictions of the simplified view of blazars put forward by [12, 67].

It is worth recalling that the large \( \nu_{\text{peak}} \) values of HSPs translate into relatively high non-thermal jet components, which might swamp any accretion disk or host galaxy emission, making the redshift determination very uncertain or even impossible. Therefore, the redshift distribution from such a population is often

\(^5\)http://www.sdss.org/
incomplete. Said otherwise, high power – high $\nu_{\text{peak}}$ blazars are going to be very hard to identify because of their featureless optical spectra and, therefore, lack of redshift. This is because when both radio power and $\nu_{\text{peak}}$ are large, the dilution by the non-thermal continuum becomes extreme and all optical features are washed away [e.g., 42, 77].

[78] have shown that the claim of the existence of a blazar sequence, that is of a negative correlation between bolometric luminosity and $\nu_{\text{peak}}$, might have also been based on this effect, which could have led to an artificial lack of sources in the high power – high $\nu_{\text{peak}}$ region (see their Fig. 6). Our new sample of HSP blazars provides further strong support for this idea.

We calculate the peak luminosity due to synchrotron emission by using

$$L_{\nu,s} = 4\pi f_{\nu,s} d_L^2$$

(5.4.5)
with \( d_L \) the luminosity distance given by

\[
d_L = \frac{c(1 + z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1 + z')^3 + \Omega_\Lambda}} \tag{5.4.6}
\]

In order to compare the bolometric luminosity, \( L_{\text{bol}} = \nu L_{\nu S} + \nu L_{\nu IC} \), of our sources with that of other blazars, we have taken \( L_{\text{bol}} \) to be \( 1.5\nu L_{\nu S} \) [13]. This correction had to be applied since we lack high energy data and cannot determine properly the IC peak for our SEDs.

As shown in Fig. 5.7, the new HSP sample adds many sources populating the region with high \( L_{\text{bol}} \) and high \( \nu_{\text{peak}} \). Sources with lower limits on their redshift [calculated or taken from [28]] are shown with upwards arrows. Those objects could very likely populate the upper right corner of the plot with high luminosity HSPs, reaching the area occupied by the four sources discovered by [28].
In addition, figure 5.8 show a strong candidate of high power HSP, BZB1503-1541. It is a confirmed GeV source (therefore an extreme object) but has no known redshift since the spectra is completely featureless (BL Lac type). This kind of source could turn out to be a bright and extreme HSP; It is important to keep in mind their existence and the role they can play in drawing conclusions about the blazar sequence.

Figure 5.8: Spectral energy distribution of WISEJ150340.67-154114.0 = BZBJ1503-1541, an HSP blazar with no known redshift, that is detected by Fermi-LAT in the highest energy channels.

5.4.3 The infrared LogN-LogS of HSP blazars

The 1WHSP blazar sample has been assembled by means of a colour selection of the sources listed in the WISE catalogue, with the application of additional multi-frequency restrictions to ensure that the SED of all the selected objects is typical of HSP blazars. The sample is therefore flux limited in the IR band, although
its level of completeness varies and can be poor at low infra-red fluxes where the WISE colours are often not available due to the different sensitivity limits in the four WISE channels.

The number of sources per square degree (N) as a function of the flux density (S) is an important tool to measure the cosmic evolution and also to estimate the completeness of the sample. We can also use the LogN-LogS to estimate the total number of sources of a certain type expected in the sky.

To estimate the infrared LogN-LogS of HSP blazars it is necessary to determine the area of sky where the sources could be found at any given flux density. Since WISE covers the entire sky and we are requiring an X-ray detection for each candidate, the area useful for the LogN-LogS determination at a given flux density ($f_{\text{IR}}$) is defined by the part of the sky covered by X-ray observations deeper than the X-ray flux corresponding to a source with flux density equal to $f_{\text{IR}}$ and having the steepest $\alpha_{4.6\mu m-1\text{keV}}$ allowed by the selection criteria. For the selection of our sample we used all available X-ray catalogues; however, to avoid complications, for the estimation of the LogN-LogS we only use the subsample of sources included in the RASS X-ray survey, which covers the whole sky.

In practice, we use six flux density values at $4.6\mu m$ (W2) band as reference. For each of the calculated X-ray fluxes, we determined the corresponding area covered by the RASS and we used it to estimate the surface density of HSP sources with IR flux above the chosen value. The resulting Log N - Log S is shown in Fig. 5.9 below.

The LogN-LogS plot shows a break at a flux density $\approx 5 \times 10^{-3} \text{ Jy}$, reflecting the onset of severe incompleteness at faint fluxes. If the Euclidean line is extrapolated down to $S_{4.6\mu m} \approx 10^{-4} \text{ Jy}$, the expected density of HSP blazars is of $\approx 0.2 \text{ deg}^{-2}$, corresponding to a total of $\approx 8,000$ objects in the sky that may be within the detection capabilities of next generation of detectors. The IR LogN-LogS for HSP blazars, combined with the distribution of $\nu_{\text{peak}}$ shown in Fig. 5.12, may be used to estimate the contribution of such sources to the cosmic X-ray Background (CXB) light, especially because the integrated flux of blazars may produce a considerable fraction of it, as shown by [46]. Moreover, HSPs are the dominant extragalactic
sources of TeV photons, and a LogN-LogS may provide a way to estimate the intensity of the TeV background.

5.4.4 The $\gamma$-ray spectral index vs $\nu_{\text{peak}}$

[15] showed that the $\gamma$-ray spectral index of blazars is correlated with $\nu_{\text{peak}}$, which they estimated using an approximate method based on radio, optical and X-ray flux density ratios. To test this dependence using our sample of HSP blazars we plot in Fig. 5.10 the average values of the $\gamma$-ray spectral index in various bins of $\nu_{\text{peak}}$ as a function of $\nu_{\text{peak}}$. To avoid the cases with bad photometry (upper limits in the Fermi channels) we have considered only the sources with spectral index
5.4. PROPERTIES

error lower than 0.24. A clear correlation is present confirming the finding of [15].

Figure 5.10: The average Fermi-LAT γ-ray spectral index of all HSPs in our sample detected by Fermi-LAT, binned in six intervals of log(ν_{peak}), is plotted as a function of log(ν_{peak}). The best fit line corresponds to \( \Gamma_{Fermi-LAT} = -0.073 \log(\nu_{peak}) + 3.03 \)

5.4.5 The TeV band

At the time of writing, the list of sources detected in the TeV band includes 145 objects\(^6\) and is rapidly growing thanks to the increasing sensitivity provided by the implementation of stereoscopic arrays of Imaging Atmospheric Cherenkov Telescopes (IACTs) like HESS, VERITAS and MAGIC-II, which are flux limited to \( \approx 1 - 3 \times 10^{-13} \text{ erg/cm}^2/\text{s} \) at 1 TeV. About one third (44) of the known TeV sources are located outside the Galactic plane (\(|b| > 20^\circ\)), like our sample of HSP blazars. These include 4 low/intermediate synchrotron peaked (LSP/ISP) blazars, 1 nebula, 2 starburst galaxies, 2 radio galaxies, and 3 FSRQs. Of the remaining 32 TeV sources, all classified as HSPs, 30 were included in our selection scheme, which translates into an \( \approx 94\% \) selection efficiency. This is a considerable improvement

\(^6\)http://tevcat.uchicago.edu
compared to the Sedentary survey, which includes only 13 TeV detected sources. The 2 TeV HSP blazars that our selection criteria missed (1217 52.10 +30 07 00.4 and 03 03 26.40 -20 07 11.2) show an SED with $\nu_{\text{peak}} \lesssim 10^{15}$ Hz, that is borderline between an ISP and HSP classification.

Figure 5.11: The distribution of synchrotron peak frequencies for the 1WHSP sources that are still undetected in the TeV band (top panel) and of those that have already been detected (bottom panel) by Cherenkov telescopes.

Figure 5.12 shows that the $\nu_{\text{peak}}$ distribution of the subsample of 1WHSP sources detected at TeV energies (bottom panel) spans the entire range ($10^{15} - 10^{18}$ Hz) covered by still undetected HSPs (top panel) thus implying that all objects in our list could be detected at TeV energies, given that enough sensitivity is reached. In addition we note that a few blazars of the ISP type ($10^{14}$ Hz $< \nu_{\text{peak}} < 10^{15}$Hz) have been detected by Cherenkov telescopes, extending the above conclusion to even lower synchrotron peak energies.

Figure 5.12 gives the distribution of the synchrotron peak fluxes, $\nu_{\text{peak}}\nu_{\text{peak}}^{-1}$. 
5.4. PROPERTIES

Figure 5.12: The distribution of synchrotron peak fluxes for the 1WHSP sources that have been detected so far (bottom panel) and that are still undetected in the TeV band (top panel). For each bin in $nF_{\nu \text{peak}}$ we report the percentage of 1WHSP sources that are already TeV detected.

of the 1WHSP blazars that have been detected in the TeV band (bottom panel) and of those that are still undetected (top panel). For each bin in $\nu_{\text{peak}} F_{\nu \text{peak}}$, we have calculated the percentage of 1WHSP sources that are already TeV detected. From this, one clearly concludes that the already TeV detected sources are the brighter objects. Note that the peak flux of the undetected blazars in many cases is just below that of the detected ones, and is never fainter than about a factor ten than the faintest detected object. Given that variability of one order of magnitude or even larger is often observed in the X-ray and TeV bands, most of the HSP blazars in our sample (with the exception of those -if any- at very high redshift) may be detectable during flaring episodes by the present generation of Cherenkov telescopes, and certainly by the future CTA. Since the lower energy threshold
of Cherenkov telescopes is decreasing significantly, enlarging their reach to larger redshifts, we conclude that probably all our HSPs are good targets for present and future VHE observations.

To provide a quantitative measure of potential detectability by TeV instruments we define a Figure Of Merit (FOM) as the ratio between the synchrotron peak flux $\nu_{\text{peak}} f_{\nu_{\text{peak}}}$ of a source and that of the faintest blazar in our sample that has already been detected in the TeV band. This FOM gives an objective way of assessing the likelihood that a given HSP is also a TeV source. A total of 127 sources have $\text{FOM} \geq 1.0$, meaning that their synchrotron peak flux are as bright as the faintest HSPs already detected as TeV sources. Note that there have been 32 TeV detected 1WHSPs up to now, and here we highlight 95 potential TeV sources that may be within reach of present generation of detectors.
### 5.4. PROPERTIES

Table 5.1: **1WHSP Sample.** The column *Type* indicates if the source is a spectroscopically confirmed HSP (0 for previously known blazar, 0\(^+\) for newly identified blazar), a candidate HSP (1), a complementary source from the Sedentary survey (2), if it is classified as extended by WISE (3), and complementary source from TeVCat (4). The columns *BzCat* and *TeVCat* show if the source is part of those catalogs. The column *FOM* represents the likelihood of detection in the TeV band (see section 4.4). Entries marked with an asterisk are uncertain.

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<th>Log((\nu f(\nu_p)))</th>
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Conclusions

In chapter 2 it was presented a model of self-gravitating semidegenerate fermions as DM on galactic halos. Since the system also has a core, it could be an alternative to the supermassive black hole in the center of galaxies. It provides naturally a flat rotation curve at large distances from the center as it is currently observed. We showed that our density profile is similar to other profiles used in the literature, such as Burkert and NFW and that any fit done with this profiles could be done with ours too.

Since our velocity curves are self-similar, we were able to built scaling laws for the observables in galaxies, i.e., mass and radii, to see how they change with our free parameters. We find out that the cutoff parameter does not influence these properties, so our parameter space is essentially decreased by one. However, the scaling laws are valid only for a limited range on the degeneracy parameter, since for $\theta_0 < 15$ the behaviour of the mass and halo radius is more complicated. We found out that the scaling laws for the core can be understood by treating it as a non-relativistic degenerate fermi gas. Instead, the halo can be considered to be in the Boltzmann regime to a good approximation. Finally, we can determine the asymptotic velocity just from $\beta_0 = kT/me^2$, just like an isothermal sphere.

To try to constrain our parameter space, we make use of the universality law presented by Gentile et.al. and Donato et.al., where they found that the surface density of galaxies spanning several orders of magnitude in luminosity is constant at the Burkert radius. Since we know that our velocity curve is almost equal to a Burkert one, we can apply this result to our model. We find that, for any given velocity, there is a family of values of the particle mass and $\theta_0$ that satisfy the
universality law. A lower limit for the particle mass and degeneracy parameter was found by Ruffini et al. using the model with fixed halo mass, radius and velocity typical for a small dSph. For any $\theta_0 < 11$, they could not obtain the desired values for the halo, imposing a lower limit on the mass and degeneracy parameter.

We showed that, for every value of $\theta_0$, there is a value of $\beta_0$ where the core mass reaches a maximum. For high values of the degeneracy parameter, the critical core mass depends only on the mass of the particle, $M_{c}^{cr} \propto \frac{n_{0}^{3}}{m_{p}}$. This shows that this is due to the high degeneracy in the center since the temperature has no effect. For lower values of $\theta_0$, the corresponding $\beta_0^{cr}$ increases and the core mass changes for different values of the parameters, but it is always higher than before. This can be understood since for hotter systems, general relativistic effects that would lower the mass are less relevant in the core than before. We show that, at least for the degenerate case, an analytical formula for the critical mass can be obtained by considering a relativistic fermi gas in newtonian hydrostatic equilibrium.

Since we find that the mass of the particle should be in the keV scale, the sterile neutrino is a viable candidate in our model. This neutrino could, in principle, change the effective number of neutrinos measured by CMB experiments. However, if this neutrino decoupled from the primordial plasma early enough, the effect would be very small and not measurable.

An interaction between these fermions could compress the core and bring new physics into play and large scale structure could help constrain the strength of the interaction. Fitting this model to individual rotation curves is also important, and some work is being done in this direction. Since dSphs are the most DM dominated objects we see, data coming from them could help us understand better the properties of DM halos.

In chapter 4 I presented the 1WHSP, a catalog of HSP blazars based on WISE data. It has 713 sources between known and newly spectroscopically confirmed blazars and blazar candidates, making it the largest and most homogeneous one. This was done mainly for three reasons:

- estimate the surface density of HSP blazars down to relatively faint infrared
We show that using the so called WISE blazar strip to select blazars from the WISE sources leads to severe incompleteness of the sample, since blazars whose emission is contaminated by the host galaxy are left out. We used an enlarged version of the WBS and applied multifrequency selection criteria to arrive at the final sample.

Even though this catalog has 711 sources, our Log N-Log S results imply that there are in fact thousands of HSPs above the WISE flux limit. We also provide some evidence against the blazar sequence by finding blazars with high luminosity and high synchrotron peak frequency.

The catalog was called 1WHSP where the W stands for WISE and HSP for High Synchrotron Peaked blazars. We intend to release more lists as more x-ray data become available, and the fact that the requirement that one source be detected in three of the WISE channels can be neglected. At the time of the writing, a new WISE catalog became available so we can enhance the sample we already have.

Another catalog of LSPs is also in the plans, so we can try to estimate their contamination to the CMB. Since there are a lot more of those than of HSPs, the sample left after the multifrequency selection is still very large for individual checking, so a statistical analysis is required.

The catalog was sent to the Fermi team (since we used some unpublished data) and it is currently being refereed by them. It will be available soon.
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